1.Define the following terms and theorems

a.Extreme value theorem

b.Tangent line

c.Limit

d.Indeterminate form

e.Squeeze theorem

f.Intermediate value theorem

g.Constant multiple rule

h.Product rule

i.Quotient rule

j.Chain rule

k.Critical number

l.Rolle’s theorem

m.Mean value theorem

n.Point of inflection

o.Point-slope form

p.Antiderivative

q.Sigma notation

r.Riemann sum

s.Mean value theorem for integrals

t.Fundamental theorem of calculus

u.Average value of F(x) on the closed interval [a,b]

v.Second fundamental theorem of calculus

w.Trapezoidal rule

x.Midpoint formula

y.General power rule for integration

z.Change of variables for definite integrals

aa.Natural logarithmic function

bb.Natural exponential function

cc.The first derivative test

dd.The second derivative test

2.Explain the difference between the following

a.Secant line vs. tangent line

b.Removable vs. non removable discontinuity

c.Explicit vs implicit differentiation

d.Relative extrema vs. absolute extrema

e.General solution vs. particular solution

f.Indefinite integral vs. definite integral

g.Inscribed rectangle vs. circumscribed rectangle

h.Purpose of the first derivative test vs. purpose of the second derivative test

i.U-substitution vs. change of variables

3.What are 3 common types of behaviors associated with the nonexistence of a limit?

4.Complete the following limits

a. $\lim_{x \to c} f(g(x))$

b. $\lim_{x \to 0} \frac{sinx}{x}$

c. $\lim_{x \to 0} \frac{1-cosx}{x}$

5.What conditions need to be met for a function to be considered continuous?

6.What is the limit definition of a tangent?

7.Where is a function not differentiable?

8.What is the relationship between the ability to differentiate a function and the continuity of the function?

9.List the derivatives of the 6 trig functions and their inverses

10.Complete the table with the appropriate symbols/notations

First derivative	y’	f’(x)	$\frac{dy}{dx}$	$\frac{d}{dx}$ [f(x)]	D_x[y]
Second derivative					
Third derivative					
Fourth derivative					
nth derivative					

11.Describes the steps taken to solve an implicit differentiation problem

12. Describe the steps taken to solve a related rates problem

13. Describe how to find extrema and points of inflection

14. What is the test for increasing/decreasing functions?

15. What is the test for concavity?

16. Describe the process used to determine limits at infinity

17. Describe the process used to solve applied minimum and maximum problems. What are these types of problems known as?

18. How do you find the inverse of a function?

19. How do you find the inverse derivative of a function?

20. Complete the following basic integration formulas

a. $\int 0 dx =$

b. $\int K dx =$

c. $\int K f(x) dx =$

d. $\int [f(x) \pm g(x)] dx =$

e. $\int x^n dx =$

f. $\int cos(x) dx =$

g. $\int sin(x) dx =$

h. $\int sec^2(x) dx =$

i. $\int sec(x) tan(x) dx =$

j. $\int csc^2(x) dx =$

k. $\int csc(x) cot(x) dx =$

21. Complete the following equations

a. $\sum_{i=1}^{n} k a_i =$

b. $\sum_{i=1}^{n} (a_i \pm b_i) =$

c. $\sum_{i=1}^{n} c =$

d. $\sum_{i=1}^{n} i =$

e. $\sum_{i=1}^{n} i^2 =$

f. $\sum_{i=1}^{n} i^3 =$

22. What is the equation used to approximate the area under a curve using the left endpoints? using the right endpoints?

23.How do you get the area under the curve using the limit-sum formula?

24.What is the relationship between the ability to integrate a function and the continuity of the the function?

25.How do you find the average value of a function F on the closed interval [a,b]

26.What are the steps taken to make a change of variables?

27.Complete the following. Assume f is a function integrable on the closed interval [-a,a]

a.If f is an even function, $\int_{-a}^{a} f(x)dx =$

b.If f is an odd function, $\int_{-a}^{a} f(x)dx =$

28.What are the 3 properties of a natural logarithmic function?

29.Complete the following

a.ln(1)=

b.ln(ab)=

c.ln(a^n)=

d.ln($\frac{a}{b}$)=

e.$e^a e^b$=

f. $\frac{e^a}{e^b}$ =

30.What is the derivative of the natural log?

31.Complete the following statements

a.If f is continuous on its domain, then f□-^{1} is...

b.If f is increasing on its domain, then f□-1^1 is...

c.If f is decreasing on its domain, then f□-^{1} is...

d.If f is differentiable at c and f(c)≠0, then f^{-1} is...

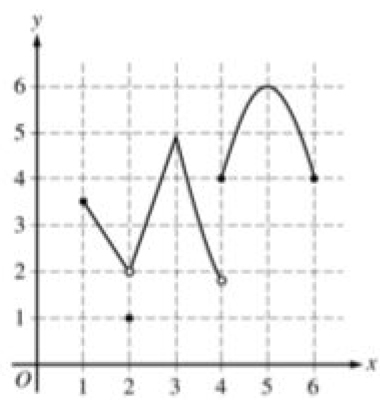
32.What are the 4 properties of the natural exponential function?

33.What is the derivative of e^x?

34.Let f be the function given by $f(x)=300x-x^3$. On which of the intervals is f increasing?

a.(-∞, 10] and [10, ∞)
b.[-10, 10]
c.[0,10] only
d.[0, $10\sqrt{3}$] only
e.[0, ∞)

35.The graph of the function f is shown above. Which of the following statements is false?



Graph of f

a. $\lim_{x\to 2} f(x)$ exists
b. $\lim_{x\to 3} f(x)$ exists
c. $\lim_{x\to 4} f(x)$ exists
d. $\lim_{x\to 5} f(x)$ exists
e.The function f is continuous at x=3

36.

t (hours)	4	7	12	15
$R(t)$ (liters/hour)	6.5	6.2	5.9	5.6

A tank contains 50 liters of oil at time $t = 4$ hours. Oil is being pumped into the tank at a rate $R(t)$, where $R(t)$ is measured in liters per hour, and t is measured in hours. Selected values of $R(t)$ are given in the table above. Using a right Riemann sum with three subintervals and data from the table, what is the approximation of the number of liters of oil that are in the tank at time $t = 15$ hours?

a.64.9
b.68.2
c.114.9
d.116.6
e.118.2

37. Using the substitution $u = \sqrt{x}$, $\int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$ is equal to which of the following?

a.2 $\int_1^{16} e^u du$

b.2 $\int_1^{4} e^u du$

c.2 $\int_1^{2} e^u du$

d. $\frac{1}{2}\int_1^{2} e^u du$

e. $\int_1^{4} e^u du$

38.Let f(x)=$(2x+1)^3$ and let g be the inverse function of f. Given that f(0)=1, what is the value of g'(1)?
a.- $\frac{2}{27}$
b. $\frac{1}{54}$
c. $\frac{1}{27}$
d. $\frac{1}{6}$
E.6

39.Let f be a function that is continuous on the closed interval [2, 4] with f(2)=10 and f(4)=20. Which of the following is guaranteed by the Intermediate Value Theorem?

a.f(x)=13 has at least one solution in the open interval (2, 4)
b.f(3)=15
c.f attains a maximum on the open interval (2, 4)
d.f'(x)=5 has at least one solution in the open interval (2, 4)
e.f'(x)>0 for all x in the open interval (2, 4)

40.

People enter a line for an escalator at a rate modeled by the function r given by

$$r(t) = \begin{cases} 44\left(\frac{t}{100}\right)^3\left(1 - \frac{t}{300}\right)^7 & \text{for } 0 \le t \le 300 \\ 0 & \text{for } t > 300, \end{cases}$$

where $r(t)$ is measured in people per second and t is measured in seconds. As people get on the escalator, they exit the line at a constant rate of 0.7 person per second. There are 20 people in line at time $t = 0$.

a.How many people enter the escalator during the time interval $0 \le t \le 300$?

b.During the time interval $0 \le t \le 300$, there are always people in line for the escalator. How many people are in line at t=300?

c.For t>300, what is the first time t that there are no people in line for the escalator

d.For $0 \le t \le 300$, at what time t is the number of people in line a minimum? To the nearest whole number, find the number of people in line at this time

41. Two particles move along the x-axis. For $0 \leq t \leq 8$, the position of particle P at time t is given by $x_P(t)=\ln(r^2-2r+10)$, while the velocity of particle Q at time t is given by $v_P(t)=t^2-8t+15$. Particle Q is at position x=5 at time t=0

a. For $0 \leq t \leq 8$, when is particle P moving to the left?

b. For $0 \leq t \leq 8$, find all times t during which the two particles travel in the same direction

c. Find the acceleration of particle Q at time t=2. Is the speed of particle Q increasing, decreasing, or neither at time t=2? Explain your reasoning

d. Find the position of particle Q the first time it changes direction

42. Find the values of c that satisfy Rolle's Theorem for each problem

a. $y=-x^3+2x^2+x-6$; [-1, 2]

b. $y=\frac{-x^2+2x+15}{x+4}$; [-3, 5]

c. $y=-2\sin(2x)$; $[-\pi, \pi]$

43. For each problem, determine if Rolle's Theorem can be applied. If it can, find all values of c that satisfy the theorem. If it cannot, explain why not?

a. $y=\frac{-x^2-2x+8}{-x+3}$; [-4, 2]

b. $y=2\tan(x)$; $[-\pi, \pi]$

44. A boat is being pulled into a dock by a rope attached to it and passing through a pulley on the dock positioned 5 meters higher than the boat. If the rope is being pulled at a rate of 2 m/s, how fast is the boat approaching the dock when it is 12 meters away from the dock?

45. A right cylinder of a constant volume is being flattened. At the moment when its radius is 3 cm, the height is 4 cm, and the height is decreased at a rate of 0.2 cm/sec. At that moment, what is the rate of change of the radius?

46. Evaluate the following indefinite integrals

a. $\int(\sqrt{u^3} - \frac{1}{2}u^{-2} + 5)du$

b. $\int \frac{x^3+3x^2-9x-2}{x-2} dx$

c. $\int \frac{8x-5}{\sqrt[3]{x}} dx$

d. $\int \frac{(1+cot^2 z)cotz}{cscz} dz$

e. $\int e^x\sqrt{1-e^x} dx$

47.Evaluate the definite integrals

a. $\int_{1}^{8} \sqrt{\frac{2}{x}}dx$

b. $\int_{-8}^{-1} \frac{x-x^2}{2\sqrt[3]{x}}dx$

c. $\int_{0}^{3} |x^2 - 4x + 3|dx$

d. $\int_{0}^{\pi/4} \frac{1-sin^2\theta}{con^2\theta}d\theta$

e. $\int_{0}^{\pi/2} e^{sin\pi x}cos\pi x\, dx$

48.Integrate the following using u-substitution

a. $\int \frac{x^2+2x}{x^2+2x+1}dx$

b. $\int \frac{sec^2x}{(1+tanx)^3}dx$

c. $\int (tan2x + cot2x)^2 dx$

d. $\int \frac{csc^2x}{cot^3x}dx$

ANSWER KEY

1.Define the following terms and theorems

a.Extreme value theorem

If f is continuous on the closed interval [a, b], then f has both a minimum and a maximum on the interval

b.Tangent line

Touches the curve at one point and is the instantaneous rate of change at that point

c.Limit

The limit of f(x) as x approaches c is the value that f(x) approaches as x approaches c

d.Indeterminate form

0/0 expression produced by direct substitution. The limit must be rewritten to produce an actual value when you get an indeterminate form

e.Squeeze theorem

If h(x)≤f(x)≤g(x), for all x in an open interval containing c, except possibly at c itself, and if $\lim_{x \to c} h(x) = L = \lim_{x \to c} g(x)$, then $\lim_{x \to c} f(x)$ exists and is equal to L

f.Intermediate value theorem

If f is continuous on the closed interval [a, b] and k is any number between f(a) and f(b), then there is at least one number c in [a, b] such that f(c)=k

g.Constant multiple rule

If f is a function that is differentiable and c is a real number, then cf is also differentiable and $\frac{d}{dx}$ [cf(x)]=cf’(x)

h.Product rule

$\frac{d}{dx}$ [f(x)g(x)]=f’(x)+g’(x)

i.Quotient rule

$\frac{d}{dx}[\frac{f(x)}{g(x)} = \frac{g(x)f'(x)-f(x)g'(x)}{[g(x)]2}$

j.Chain rule

$\frac{d}{dx}$ [f(g(x))]=f’(g(x))g’(x)

k.Critical number

A number when an extrema of a function could exist. It is found at a point where the derivative of the function is equal to zero or non differentiable

l.Rolle’s theorem

Let f be continuous on the closed interval [a, b] and differentiable on the open interval (a, b). If f(a)=f(b), then there is at least one number c in (a, b) where f'(c)=0

m.Mean value theorem

If f is continuous on the closed interval [a, b] and differentiable on the open interval (a, b), then there exists a number c in (a, b) such that f'(c)= $\frac{f(b)-f(a)}{b-a}$

n.Point of inflection

A point where a change of concavity occurs

o.Point-slope form

$y-y_1=m(x-x_1)$

p.Antiderivative

A function F is an antiderivative of f on an interval *I* if F '(x)=f(x) for all x in *I*

q.Sigma notation

The sum of n terms $a_1, a_2, a_3, \ldots, a_n$ is written as

$$\sum_{i=1}^{n} a_i = a_1 + a_2 + a_3 + \cdots + a_n$$

where i is the **index of summation,** a_i is the ***i*th term** of the sum, and the **upper and lower bounds of summation** are n and 1.

r.Riemann sum

Let f be defined on the closed interval $[a, b]$, and let Δ be a partition of $[a, b]$ given by

$$a = x_0 < x_1 < x_2 < \cdots < x_{n-1} < x_n = b$$

where Δx_i is the width of the ith subinterval. If c_i is *any* point in the ith subinterval, then the sum

$$\sum_{i=1}^{n} f(c_i)\,\Delta x_i, \qquad x_{i-1} \le c_i \le x_i$$

is called a **Riemann sum** of f for the partition Δ.

s.Mean value theorem for integrals

If f is continuous on the closed interval [a, b], then there exists a number c in the closed interval [a, b] such that $\int_a^b f(x)dx$ =f(c)(a-b) t.Fundamental theorem of calculus

If a function f is continuous on the closed interval $[a, b]$ and F is an antiderivative of f on the interval $[a, b]$, then

$$\int_a^b f(x)\,dx = F(b) - F(a).$$

u.Average value of F(x) on the closed interval [a,b]

If f is integrable on the closed interval $[a, b]$, then the **average value** of f on the interval is

$$\frac{1}{b-a}\int_a^b f(x)\,dx.$$

v.Second fundamental theorem of calculus

If f is continuous on an open interval I containing a, then, for every x in the interval,

$$\frac{d}{dx}\left[\int_a^x f(t)\,dt\right] = f(x).$$

w.Trapezoidal rule

Let f be continuous on $[a, b]$. The Trapezoidal Rule for approximating $\int_a^b f(x)\,dx$ is given by

$$\int_a^b f(x)\,dx \approx \frac{b-a}{2n}[f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-1}) + f(x_n)].$$

Moreover, as $n \to \infty$, the right-hand side approaches $\int_a^b f(x)\,dx$.

x.Midpoint formula

Area=(distance between two points)x(middle y value of the two points)

y.General power rule for integration

If g is a differentiable function of x, then

$$\int [g(x)]^n g'(x)\,dx = \frac{[g(x)]^{n+1}}{n+1} + C, \qquad n \neq -1.$$

Equivalently, if $u = g(x)$, then

$$\int u^n\,du = \frac{u^{n+1}}{n+1} + C, \qquad n \neq -1.$$

z.Change of variables for definite integrals

If the function $u = g(x)$ has a continuous derivative on the closed interval $[a, b]$ and f is continuous on the range of g, then

$$\int_a^b f(g(x))g'(x)\,dx = \int_{g(a)}^{g(b)} f(u)\,du.$$

aa.Natural logarithmic function

ln(x)

bb.Natural exponential function

e^x

cc.The first derivative test

Let c be a critical number of a function f that is continuous on an open interval I containing c. If f is differentiable on the interval, except possibly at c, then $f(c)$ can be classified as follows.

1. If $f'(x)$ changes from negative to positive at c, then f has a *relative minimum* at $(c, f(c))$.
2. If $f'(x)$ changes from positive to negative at c, then f has a *relative maximum* at $(c, f(c))$.
3. If $f'(x)$ is positive on both sides of c or negative on both sides of c, then $f(c)$ is neither a relative minimum nor a relative maximum.

dd.The second derivative test

Let f be a function such that $f'(c) = 0$ and the second derivative of f exists on an open interval containing c.

1. If $f''(c) > 0$, then f has a relative minimum at $(c, f(c))$.
2. If $f''(c) < 0$, then f has a relative maximum at $(c, f(c))$.

If $f''(c) = 0$, the test fails. That is, f may have a relative maximum at c, a relative minimum at $(c, f(c))$, or neither. In such cases, you can use the First Derivative Test.

2.Explain the difference between the following

a.Secant line vs. tangent line

The secant line is the average rate of change between two points while the tangent line the instantaneous rate of change at a point

b.Removable vs. non removable discontinuity

A removable discontinuity is a hole and a non removable discontinuity is a jump or a vertical asymptote

c.Explicit vs implicit differentiation

Explicit differentiation deals with equations in standard form(y=mx+b), while implicit differentiation deals with equations in implicit form(xy=1)

d.Relative extrema vs. absolute extrema

Relative extrema are the most extreme points relative to their neighbors while absolute extrema are the most extreme points in the interval

e.General solution vs. particular solution

The general solution is a equation + C, while a particular solution knows what C is. Finding the particular solution requires knowing the general solution and the initial condition

f.Indefinite integral vs. definite integral

Definite is between two points, while indefinite is more of a general solution. A definite integral is a number, but an indefinite integral is a family of functions

g.Inscribed rectangle vs. circumscribed rectangle

Inscribed is inside the curve and circumscribed is outside the curve. Inscribed gives you the lower sum and circumscribed is the upper sum

h.Purpose of the first derivative test vs. purpose of the second derivative test

The first derivative test tells you about the extrema and whether the function is increasing or decreasing. The second derivative test tells you about the turning points of the function and its concavity

i.U-substitution vs. change of variables

There is no difference

3.What are 3 common types of behaviors associated with the nonexistence of a limit?

1. f (x□) approaches a different number from the right side of c than it approaches from the left side
2. f(x□) increases or decreases without bound as x approaches c
3. f(x□) oscillates between two fixed values as x approaches c

4.Complete the following limits

a. $\lim_{x \to c} f(g(x))$ = f($\lim_{x \to c}$ g(x))

b. $\lim_{x \to 0} \frac{sinx}{x}$ = 1

c. $\lim_{x \to 0} \frac{1-cosx}{x}$ = 0

5.What conditions need to be met for a function to be considered continuous?

1. f(c) is defined
2. $\lim_{x \to c}$ f(x) exists
3. $\lim_{x \to c}$ f(x) = f(c)

6.What is the limit definition of a tangent?

f'(x)= $\lim_{\Delta x \to 0} \frac{f(c+\Delta x)-f(c)}{\Delta x}$

7.Where is a function not differentiable?

At cusps and corners

8.What is the relationship between the ability to differentiate a function and the continuity of the function?

If f is differentiable at x=c, f is continuous at x=c, but f isn't necessarily differentiable just because f is continuous

9.List the derivatives of the 6 trig functions and their inverses

$$\frac{d}{dx}\left(\sin^{-1}x\right)=\frac{1}{\sqrt{1-x^2}}, x\neq\pm1$$

$$\frac{d}{dx}\left(\cos^{-1}x\right)=\frac{-1}{\sqrt{1-x^2}}, x\neq\pm1$$

$$\frac{d}{dx}\left(\tan^{-1}x\right)=\frac{1}{1+x^2}$$

$$\frac{d}{dx}\left(\cot^{-1}x\right)=\frac{-1}{1+x^2}$$

$$\frac{d}{dx}\left(\sec^{-1}x\right)=\frac{1}{|x|\sqrt{x^2-1}}, x\neq\pm1,0$$

$$\frac{d}{dx}\left(\csc^{-1}x\right)=\frac{-1}{|x|\sqrt{x^2-1}}, x\neq\pm1,0$$

$$\frac{d}{dx}[\sin x]=\cos x \qquad \frac{d}{dx}[\tan x]=\sec^2 x \qquad \frac{d}{dx}[\sec x]=\sec x\tan x$$

$$\frac{d}{dx}[\cos x]=-\sin x \qquad \frac{d}{dx}[\cot x]=-\csc^2 x \qquad \frac{d}{dx}[\csc x]=-\csc x\cot x$$

10.Complete the table with the appropriate symbols/notations

First derivative	y’	f’(x)	$\frac{dy}{dx}$	$\frac{d}{dx}$ [f(x)]	D_x[y]
Second derivative	y’’	f’’(x)	$\frac{d^2y}{dx^2}$	$\frac{d^2}{dx^2}$ [f(x)]	D_x^2[y]
Third derivative	y’’’	f’’’(x)	$\frac{d^3y}{dx^3}$	$\frac{d^3}{dx^3}$ [f(x)]	D_x^3[y]
Fourth derivative	$y^{(4)}$	$f^{(4)}$(x)	$\frac{d^4y}{dx^4}$	$\frac{d^4}{dx^4}$ [f(x)]	D_x^4[y]
nth derivative	$y^{(n)}$	$f^{(n)}$(x)	$\frac{d^ny}{dx^n}$	$\frac{d^n}{dx^n}$ [f(x)]	D_x^n[y]

11.Describes the steps taken to solve an implicit differentiation problem

1. Differentiate both sides of the equation *with respect to x*.
2. Collect all terms involving dy/dx on the left side of the equation and move all other terms to the right side of the equation.
3. Factor dy/dx out of the left side of the equation.
4. Solve for dy/dx.

12.Describe the steps taken to solve a related rates problem

1. Identify all *given* quantities and quantities *to be determined*. Make a sketch and label the quantities.
2. Write an equation involving the variables whose rates of change either are given or are to be determined.
3. Using the Chain Rule, implicitly differentiate both sides of the equation *with respect to time t*.
4. *After* completing Step 3, substitute into the resulting equation all known values for the variables and their rates of change. Then solve for the required rate of change.

13.Describe how to find extrema and points of inflection

Extrema:

Identify all critical numbers(points where f'(x)=0 or is undefined. Then, plug x-values on each side of the critical number into the derivative equation to make sure that there is a change of sign at the critical number, since extrema only exist at critical numbers where a change of the sign of the derivative equation of the function occurs at the critical number. If the sign changes from positive to negative, the point is a maximum. If the sign changes from negative to positive, the point is a minimum

Points of inflection:

Identify all points where the second derivative of the function is equal to zero or is undefined. Then, confirm that a change of sign of the second derivative occurs at the point.

14.What is the test for increasing/decreasing functions?

Let f be a function that is continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b).

1. If $f'(x) > 0$ for all x in (a, b), then f is increasing on $[a, b]$.
2. If $f'(x) < 0$ for all x in (a, b), then f is decreasing on $[a, b]$.
3. If $f'(x) = 0$ for all x in (a, b), then f is constant on $[a, b]$.

15.What is the test for concavity?

Let f be a function whose second derivative exists on an open interval I.

1. If $f''(x) > 0$ for all x in I, then the graph of f is concave upward in I.
2. If $f''(x) < 0$ for all x in I, then the graph of f is concave downward in I.

16.Describe the process used to determine limits at infinity

Guidelines for Finding Limits at $\pm\infty$ of Rational Functions

1. If the degree of the numerator is *less than* the degree of the denominator, then the limit of the rational function is 0.
2. If the degree of the numerator is *equal to* the degree of the denominator, then the limit of the rational function is the ratio of the leading coefficients.
3. If the degree of the numerator is *greater than* the degree of the denominator, then the limit of the rational function does not exist.

17.Describe the process used to solve applied minimum and maximum problems. What are these types of problems known as?

These are optimization problems. The process used to solve them is:

Guidelines for Solving Applied Minimum and Maximum Problems

1. Identify all *given* quantities and quantities *to be determined.* If possible, make a sketch.
2. Write a **primary equation** for the quantity that is to be maximized or minimized. (A review of several useful formulas from geometry is presented inside the front cover.)
3. Reduce the primary equation to one having a *single independent variable.* This may involve the use of **secondary equations** relating the independent variables of the primary equation.
4. Determine the feasible domain of the primary equation. That is, determine the values for which the stated problem makes sense.
5. Determine the desired maximum or minimum value by the calculus techniques discussed in Sections 3.1 through 3.4.

18.How do you find the inverse of a function?

Swap x and y in the equation of the function and solve for y

19.How do you find the inverse derivative of a function?

$$(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}$$

20.Complete the following basic integration formulas

a. $\int 0dx$ = C

b. $\int Kdx$ = Kx+C

c. $\int Kf(x)dx = K\int f(x)dx$

d. $\int [f(x) \pm g(x)]dx = \int f(x)dx \pm \int g(x)dx$

e. $\int x^n dx = \frac{x^{n+1}}{n+1} + C,\ n \neq -1$

f. $\int cos(x)dx$ = sinx+C

g. $\int sin(x)dx$ = -cosx+C

h. $\int sec^2(x)dx$ = tanx+C

i. $\int sec(x)tan(x)dx$ = secx+C

j. $\int csc^2(x)dx$ = -cotx+C

k. $\int csc(x)cot(x)dx$ = -cscx+C

21.Complete the following equations

a. $\sum_{i=1}^{n} ka_i = k\sum_{i=1}^{n} a_i$

b. $\sum_{i=1}^{n} (a_i \pm b_i) = \sum_{i=1}^{n} a_i \pm \sum_{i=1}^{n} b_i$

c. $\sum_{i=1}^{n} c$ = cn

d. $\sum_{i=1}^{n} i = \frac{n(n+2)}{2}$

e. $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

f. $\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$

22.What is the equation used to approximate the area under a curve using the left endpoints? using the right endpoints?

Left endpoints: $\sum_{i=1}^{n} f(\Delta x(i-1)+x_0)\Delta x$

Right endpoints: $\sum_{i=1}^{n} f(\Delta x+x_0)\Delta x$

23.How do you get the area under the curve using the limit-sum formula?

Let f be continuous and nonnegative on the interval $[a, b]$. The area of the region bounded by the graph of f, the x-axis, and the vertical lines $x = a$ and $x = b$ is

$$\text{Area} = \lim_{n\to\infty} \sum_{i=1}^{n} f(c_i)\,\Delta x, \qquad x_{i-1} \le c_i \le x_i$$

where $\Delta x = (b - a)/n$ (see Figure 4.14).

24.What is the relationship between the ability to integrate a function and the continuity of the the function?

If a function f is continuous on the closed interval [a, b], then f is integrable on [a, b]

25.How do you find the average value of a function F on the closed interval [a,b]

$\frac{1}{b-a}\int_a^b f(x)dx$ or $\frac{\int_a^b F'(x)dx}{b-a}$ or $\frac{F(b)-F(a)}{b-a}$

26.What are the steps taken to make a change of variables?

Guidelines for Making a Change of Variables

1. Choose a substitution $u = g(x)$. Usually, it is best to choose the *inner* part of a composite function, such as a quantity raised to a power.
2. Compute $du = g'(x)\,dx$.
3. Rewrite the integral in terms of the variable u.
4. Find the resulting integral in terms of u.
5. Replace u by $g(x)$ to obtain an antiderivative in terms of x.
6. Check your answer by differentiating.

27.Complete the following. Assume f is a function integrable on the closed interval [-a,a]

a.If f is an even function, $\int_{-a}^{a} f(x)dx = 2\int_0^a f(x)dx$

b.If f is an odd function, $\int_{-a}^{a} f(x)dx = 0$

28.What are the 3 properties of a natural logarithmic function?

1. The domain is $(0, \infty)$ and the range is $(-\infty, \infty)$

2. The function is increasing, continuous, one-to-one
3. The graph is concave down

29.Complete the following

a.ln(1)=0

b.ln(ab)=ln(a)+ln(b)

c.ln(a^n)=n(ln(a))

d.ln($\frac{a}{b}$)=ln(a)-ln(b)

e.$e^a e^b$=e^{a+b}

f. $\frac{e^a}{e^b}$ =e^{a-b}

30.What is the derivative of the natural log?

$\frac{1}{x}$, x>0

31.Complete the following statements

a.If f is continuous on its domain, then f□-^{1} is...continuous on its domain

b.If f is increasing on its domain, then f□-1^1 is...increasing on its domain

c.If f is decreasing on its domain, then f□-^{1} is...decreasing on its domain

d.If f is differentiable at c and f(c)≠0, then f^{-1} is...differentiable at f(c)

32.What are the 4 properties of the natural exponential function?

1. The domain of f(x)=e^x is (-∞, ∞) and the range is (0, ∞)
2. The function f(x)=e^x is continuous, increasing, and one-to-one on its entire domain
3. The graph of f(x)=e^x is concave up on its entire domain
4. $\lim_{x \to -\infty}$ e^x=0 and $\lim_{x \to \infty}$ e^x=∞

33.What is the derivative of e^x?

e^x

34.Let f be the function given by f(x)=300x-x^3. On which of the intervals is f increasing?

a.(-∞, 10] and [10, ∞)

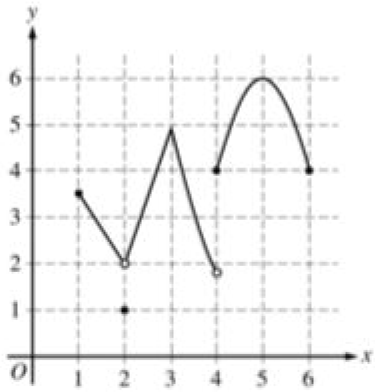
b.[-10, 10]

c.[0,10] only

d.[0, 10$\sqrt{3}$] only

e.[0, ∞)

35.The graph of the function f is shown above. Which of the following statements is false?



Graph of f

a. $\lim_{x \to 2} f(x)$ exists

b. $\lim_{x \to 3} f(x)$ exists

c. $\lim_{x \to 4} f(x)$ exists

d. $\lim_{x \to 5} f(x)$ exists

e.The function f is continuous at x=3

36.

t (hours)	4	7	12	15
$R(t)$ (liters/hour)	6.5	6.2	5.9	5.6

A tank contains 50 liters of oil at time $t = 4$ hours. Oil is being pumped into the tank at a rate $R(t)$, where $R(t)$ is measured in liters per hour, and t is measured in hours. Selected values of $R(t)$ are given in the table above. Using a right Riemann sum with three subintervals and data from the table, what is the approximation of the number of liters of oil that are in the tank at time $t = 15$ hours?

a.64.9

b.68.2

c.114.9

d.116.6

e.118.2

37. Using the substitution $u = \sqrt{x}$, $\int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$ is equal to which of the following?

a. $2\int_1^{16} e^u du$

b. $2\int_1^{4} e^u du$

c. $2\int_1^{2} e^u du$

d. $\frac{1}{2}\int_1^{2} e^u du$

e. $\int_1^{4} e^u du$

38. Let $f(x)=(2x+1)^3$ and let g be the inverse function of f. Given that f(0)=1, what is the value of g'(1)?

a. $-\frac{2}{27}$
b. $\frac{1}{54}$
c. $\frac{1}{27}$
d. $\frac{1}{6}$
E. 6

39. Let f be a function that is continuous on the closed interval [2, 4] with f(2)=10 and f(4)=20. Which of the following is guaranteed by the Intermediate Value Theorem?

a. f(x)=13 has at least one solution in the open interval (2, 4)
b. f(3)=15
c. f attains a maximum on the open interval (2, 4)
d. f'(x)=5 has at least one solution in the open interval (2, 4)
e. f'(x)>0 for all x in the open interval (2, 4)

40. People enter a line for an escalator at a rate modeled by the function r given by

$$r(t) = \begin{cases} 44\left(\frac{t}{100}\right)^3\left(1 - \frac{t}{300}\right)^7 & \text{for } 0 \le t \le 300 \\ 0 & \text{for } t > 300, \end{cases}$$

where $r(t)$ is measured in people per second and t is measured in seconds. As people get on the escalator, they exit the line at a constant rate of 0.7 person per second. There are 20 people in line at time $t = 0$.

a. How many people enter the escalator during the time interval 0≤t≤300?

$\int_0^{300} r(t)\,dt = 270$

According to the model, 270 people enter the line for the escalator during the time interval $0 \le t \le 300$.

b. During the time interval $0 \le t \le 300$, there are always people in line for the escalator. How many people are in line at $t=300$?

$$20+\int_0^{300}(r(t)-0.7)\,dt = 20+\int_0^{300} r(t)\,dt - 0.7\cdot 300 = 80$$

According to the model, 80 people are in line at time $t = 300$.

c. For $t>300$, what is the first time t that there are no people in line for the escalator

Based on part (b), the number of people in line at time $t = 300$ is 80.

The first time t that there are no people in line is

$300+\frac{80}{0.7} = 414.286$ (or 414.285) seconds.

d. For $0 \le t \le 300$, at what time t is the number of people in line a minimum? To the nearest whole number, find the number of people in line at this time

The total number of people in line at time t, $0 \le t \le 300$, is modeled by $20+\int_0^t r(x)\,dx - 0.7t$.

$r(t)-0.7 = 0 \Rightarrow t_1 = 33.013298,\ t_2 = 166.574719$

t	People in line for escalator
0	20
t_1	3.803
t_2	158.070
300	80

The number of people in line is a minimum at time $t = 33.013$ seconds, when there are 4 people in line.

41. Two particles move along the x-axis. For $0 \le t \le 8$, the position of particle P at time t is given by $x_P(t)=\ln(r^2-2r+10)$, while the velocity of particle Q at time t is given by $v_P(t)=t^2-8t+15$. Particle Q is at position x=5 at time t=0

a. For $0 \le t \le 8$, when is particle P moving to the left?

(a) $x_P'(t) = \frac{2t-2}{t^2-2t+10} = \frac{2(t-1)}{t^2-2t+10}$

$t^2-2t+10 > 0$ for all t.

$x_P'(t) = 0 \Rightarrow t = 1$

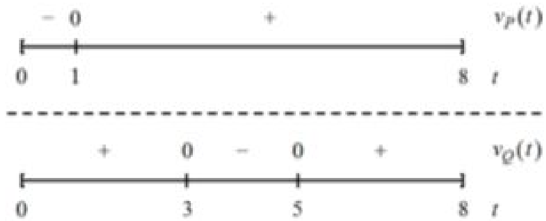
$x_P'(t) < 0$ for $0 \le t < 1$.

Therefore, the particle is moving to the left for $0 \le t < 1$.

b.For 0≤t≤8, find all times t during which the two particles travel in the same direction

(b) $v_Q(t) = (t-5)(t-3)$

$v_Q(t) = 0 \Rightarrow t = 3, t = 5$



Both particles move in the same direction for $1 < t < 3$ and $5 < t \le 8$ since $v_P(t) = x_P'(t)$ and $v_Q(t)$ have the same sign on these intervals.

c.Find the acceleration of particle Q at time t=2. Is the speed of particle Q increasing, decreasing, or neither at time t=2? Explain your reasoning

(c) $a_Q(t) = v_Q'(t) = 2t - 8$

$a_Q(2) = 2 \cdot 2 - 8 = -4$

$a_Q(2) < 0$ and $v_Q(2) = 3 > 0$

At time $t = 2$, the speed of the particle is decreasing because velocity and acceleration have opposite signs.

d.Find the position of particle Q the first time it changes direction

(d) Particle Q first changes direction at time $t = 3$.

$$x_Q(3) = x_Q(0) + \int_0^3 v_Q(t)\,dt = 5 + \int_0^3 \left(t^2 - 8t + 15\right) dt$$

$$= 5 + \left[\frac{1}{3}t^3 - 4t^2 + 15t\right]_{t=0}^{t=3} = 5 + (9 - 36 + 45) = 23$$

42.Find the values of c that satisfy Rolle's Theorem for each problem

a.y=$-x^3+2x^2+x-6$; [-1, 2]

$\left\{ \frac{2-\sqrt{7}}{3}, \frac{2+\sqrt{7}}{3} \right\}$

b.y= $\frac{-x^2+2x+15}{x+4}$; [-3, 5]

{1}

c.y=-2sin(2x); [-π, π]

$\{-\frac{3\pi}{4}, -\frac{\pi}{4}, \frac{\pi}{4}, \frac{3\pi}{4}\}$

43. For each problem, determine if Rolle's Theorem can be applied. If it can, find all values of c that satisfy the theorem. If it cannot, explain why not?

a. $y=\frac{-x^2-2x+8}{-x+3}$; [-4, 2]

$\{3-\sqrt{7}\}$

b. y=2tan(x); [-π, π]

The function is not continuous on [-π, π]

44. A boat is being pulled into a dock by a rope attached to it and passing through a pulley on the dock positioned 5 meters higher than the boat. If the rope is being pulled at a rate of 2 m/s, how fast is the boat approaching the dock when it is 12 meters away from the dock?

$\frac{13}{6}$ m/sec

45. A right cylinder of a constant volume is being flattened. At the moment when its radius is 3 cm, the height is 4 cm, and the height is decreased at a rate of 0.2 cm/sec. At that moment, what is the rate of change of the radius?

$\frac{3}{40}$ cm/sec

46. Evaluate the following indefinite integrals

a. $\int(\sqrt{u^3} - \frac{1}{2}u^{-2} + 5)du$

$\frac{2u^{5/2}}{5} + \frac{u^{-1}}{2} + 5u + C$

b. $\int \frac{x^3+3x^2-9x-2}{x-2}dx$

$\frac{x^3}{3} + \frac{5x^2}{2} + x + C$

c. $\int \frac{8x-5}{\sqrt[3]{x}}dx$

$\frac{24x^{5/3}}{5} - \frac{15x^{2/3}}{2} + C$

d. $\int \frac{(1+cot^2 z)cot(z)}{csc z}dz$

-csc(z)+C

e. $\int e^x\sqrt{1-e^x}dx$

$-\frac{2}{3}(1-e^x)^{3/2}+C$

47.Evaluate the definite integrals

a. $\int_{1}^{8} \sqrt{\frac{2}{x}} dx$

$8-2\sqrt{2}$

b. $\int_{-8}^{-1} \frac{x-x^2}{2\sqrt[3]{x}} dx$

$\frac{1523}{20}$

c. $\int_{0}^{3} |x^2 - 4x + 3| dx$

4

d. $\int_{0}^{\pi/4} \frac{1-sin^2\theta}{con^2\theta} d\theta$

$\frac{\pi}{4}$

e. $\int_{0}^{\pi/2} e^{sin\pi x} cos\pi x \, dx$

$\frac{1}{\pi}[e^{\sin(\frac{\pi^2}{2})}-1]$

48.Integrate the following using u-substitution

a. $\int \frac{x^2+2x}{x^2+2x+1} dx$

$x+(x+1)^{-1}+C$

b. $\int \frac{sec^2x}{(1+tanx)^3} dx$

$-\frac{1}{2}(1+\tan(x))^{-2}+C$

c. $\int (tan2x + cot2x)^2 dx$

$\frac{1}{2}tan2x - \frac{1}{2}cot2x + C$

d. $\int \frac{csc^2x}{cot^3x} dx$

$\frac{1}{2}(\sec^2x-1)+C$