Key 2021

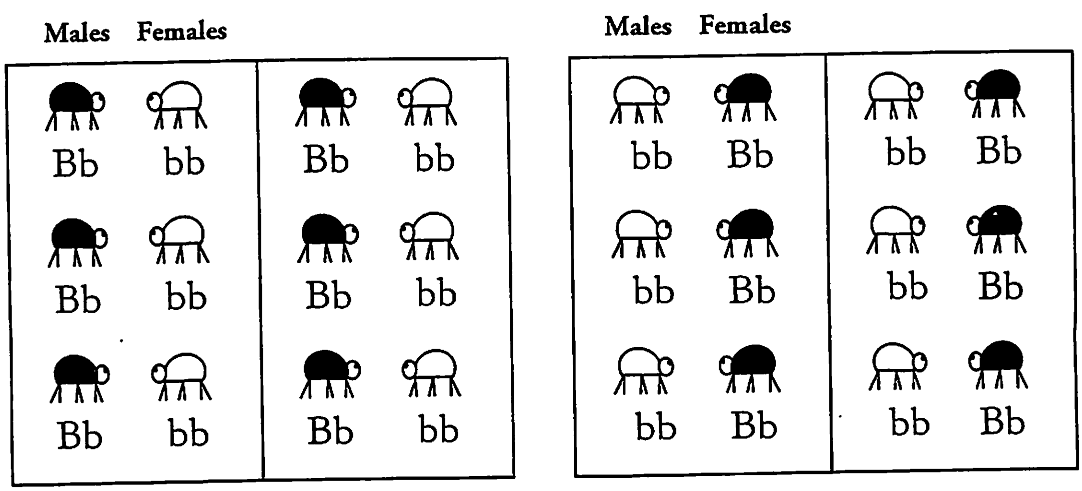
The Hardy-Weinberg Equation

How can we make predictions about the characteristics of a population?

Why?

Punnett squares provide an easy way to predict the possible genotypes for an offspring, but it is not practical to perform a Punnett square analysis on all possible combinations of all members of a population to predict what the population might look like in the future. For that we must turn to statistics. The Hardy-Weinberg equation is a tool biologists use to make predictions about a population and to show whether or not evolution is occurring in that population.

Model 1 – Controlled (Selective) Mating



1. How many mating pairs are illustrated in Model 1?

   There are twelve mating pairs.

2. Describe the parents in each mating pair in Model 1. Use terms such as homozygous, heterozygous, dominant, and recessive.

   One parent is heterozygous and the other is homozygous recessive.

3. Use two Punnett squares to determine the possible genotypes for offspring from the pairs.

	B	*b*
b	*Bb*	*bb*
b	*Bb*	*bb*

	b	*b*
B	*Bb*	*Bb*
b	*bb*	*bb*

4. If each mating pair has one offspring, predict how many of the first generation offspring will have the following genotypes.

BB	Bb	bb
0	*6*	*6*

5. Imagine the 24 beetles in Model 1 as a population in an aquarium tank.

   a. How likely is the pairing scenario in Model 1 to take place during the natural course of things within that tank?

   This pairing scenario is too organized. It is not very likely.

   b. Why is Model 1 labeled "Selective Mating"?

   Each heterozygous beetle was mated with a homozygous beetle purposefully.

6. List two other pairings that might occur in the population in Model 1 if the beetles were allowed to mate naturally.

   Bb with Bb *bb with bb*

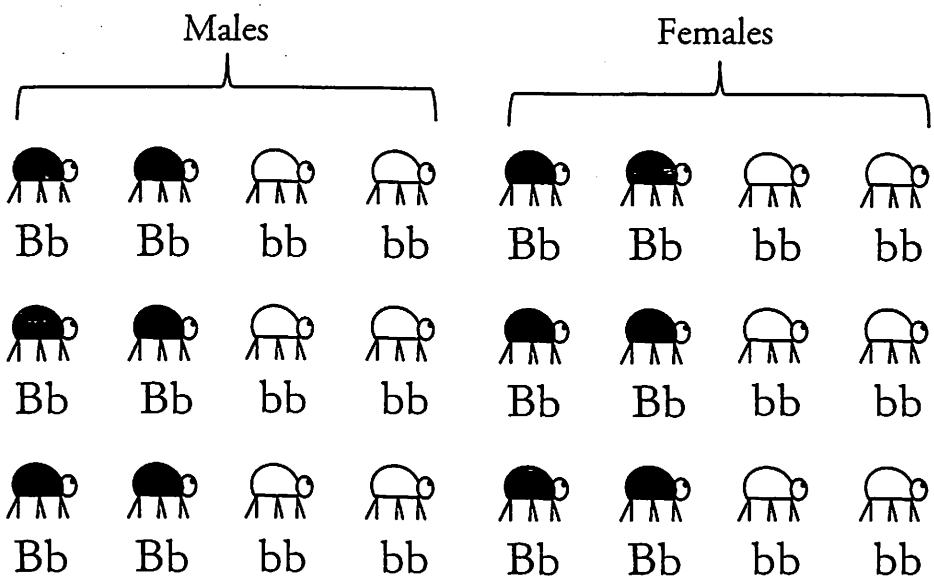
7. If the population of beetles in Model 1 mated naturally would your prediction for the offspring in Question 4 still be valid? Explain.

   No, the prediction from the Punnett squares would not include the offspring from the other two types of pairings.

8. Discuss in your group the limitations of Punnett square predictions when it comes to large populations. Summarize the key points of your discussion here.

   Answers will vary. Punnett squares only give the predicted offspring ratios for one type of pairing. In large populations it would be cumbersome to do a Punnett square for every possible pairing scenario. There is no way to know how many of each type of pairing will occur naturally in a large population.

Model 2 – Population Genetics



9. Compare the organisms in the population in Model 1 with the organisms in the population in Model 2.

   The populations are exactly the same in number of males and females, heterozygous, and homozygous recessive.

10. Individually match up twelve mating pairs from the population in Model 2 that might occur in a natural, random mating situation.

    Answers will vary.

4 pairs of Bb × Bb	*2 pairs of Bb × Bb*	*5 pairs of Bb × Bb*
4 pairs of Bb × bb	*8 pairs of Bb × bb*	*2 pairs of Bb × bb*
4 pairs of bb × bb	*2 pairs of bb × bb*	*5 pairs of bb × bb*

11. Compare your set of mating pairs with other members of your group. Did your mating scheme match anyone else's in the group?

    Students should see some variation in the crosses listed by students in their group.

Read This!

When it comes to mating in natural populations with hundreds or even millions of individuals, it is difficult, maybe even impossible, to think of all the mating scenarios. After several generations of leaving things up to nature, the alleles that are present in the population will become more and more randomized. Statistics can help biologists predict the outcome of the population when this randomization has occurred. If the population is particularly nonrandom to start, this randomization may take several generations.

12. How many total alleles are in the population in Model 2?

    Each organism has 2 alleles. $2 \times 24 = 48$ *alleles.*

13. What is the probability of an offspring from the Model 2 population getting a dominant allele?

    $12/48 = 0.25$

14. What is the probability of an offspring from the Model 2 population getting a recessive allele?

    $36/48 = 0.75$

15. If p is used to represent the frequency of the dominant allele and q is used to represent the frequency of the recessive allele, then what will p + q equal?

    $0.25 + 0.75 = 1$

16. Use your knowledge of statistics to calculate the probability of an offspring from the Model 2 population having each of these genotypes. Support your answers with mathematical equations.

    (Don't forget there are two ways to get a heterozygous offspring—Bb or bB.)

BB	Bb	bb
$0.25 \times 0.25 = 0.0625$	$0.25 \times 0.75 = 0.188$ $0.75 \times 0.25 = 0.188$ 0.376	$0.75 \times 0.75 = 0.563$

17. Check your answers in Question 16 by adding the three values together. Your sum should be equal to one. Explain why the sum of the three answers in Question 16 should be equal to one.

    Since there are only three possible outcomes for the genotype of an offspring, the sum of those possible outcomes would represent 100% or 1.0.

18. Using p and q as variables, write formulas for calculating the probability of an offspring from a population having each of the following genotypes.

BB	Bb	bb
$p \times p = p^2$	$pq + qp = 2pq$	$q \times q = q^2$

19. Complete the equation:

    $p^2 + 2pq + q^2 = 1$

STOP

Read This!

The equations you have just developed, $p + q = 1$ and $p^2 + 2pq + q^2 = 1$, were first developed by G. H. Hardy and Wilhelm Weinberg. They represent the distribution of alleles in a population when

- The population is large.
- Mating is random.
- All genotypes are equally likely to reproduce (there is no natural selection).
- No organisms enter or leave the population (there is no immigration or emigration).
- No mutations occur.

In other words, the group of alleles available in the population must be very stable from generation to generation. If the distribution of genotypes in a population matches that predicted by the Hardy-Weinberg equation, then the population is said to be in **Hardy-Weinberg equilibrium**. If the distribution of genotypes in a population does not match that predicted by the Hardy-Weinberg equation, then the population is said to be evolving.

20. Consider the requirements for a population to be in Hardy-Weinberg equilibrium. In the natural world, are populations likely to be in Hardy-Weinberg equilibrium? Justify your reasoning.

    It is very unlikely that all of the criteria in the Read This! box would be met in a population, so finding a population in Hardy-Weinberg equilibrium would be nearly impossible.

21. Sickle-cell anemia is a genetic disease. The Sickle-cell allele is recessive, but individuals with the homozygous recessive genotype (ss) often die prematurely due to the disease. This affects approximately 9% of the population in Africa. Use the Hardy-Weinberg equations to calculate the following:

    a. The frequency of the recessive allele in the population (q).

    $q = \sqrt{0.09} = 0.30$

    b. The frequency of the dominant allele in the population (p).

    $1 = 0.30 + p \qquad p = 0.70$

    c. The frequency of homozygous dominant individuals in the African population.

    $0.70 \times 0.70 = 0.49$

    d. The frequency of heterozygous individuals in the African population.

    $2 \times 0.70 \times 0.30 = 0.42$

    e. Based on this analysis, is the African population in Hardy-Weinberg equilibrium? Justify your answer.

    No. Because the members of the population that contract sickle cell because they are homozygous recessive will likely die before reproducing, the frequency of alleles in the population is not stable. There is natural selection taking place.

22. Individuals with the heterozygous genotype (Ss) for Sickle-cell exhibit resistance to Malaria, a serious disease spread by mosquitoes in Africa and other tropical regions.

    a. Discuss with your group how this might affect the frequency of the recessive allele in the African population. Summarize your group's conclusions here.

    This may increase the frequency of the recessive allele because heterozygous individuals have better fitness in the African environment while homozygous dominant individuals would be more susceptible to Malaria and may die before reproducing.

    b. How might this trait affect the values calculated in Question 21 and the population's tendency toward Hardy-Weinberg equilibrium?

    The African population may be closer to Hardy-Weinberg equilibrium than calculated in Question 21 because the resistance to Malaria will increase the q value and decrease the p value.

STOP

23. Consider the beetle population in Model 2. Imagine a change occurred in the beetle's ecosystem that made it easier for predators to spot the white beetles and six of the white beetles were lost. Predict the genotype frequency in the population after this event.

    $p = 0.33$ $q = 0.66$

    BB (p^2) = 0.109 Bb ($2pq$) = 0.4356 bb (q^2) = 0.4356

24. Compare your answers to Question 22 with those of Question 16. How do your answers support the conclusion that the population is not in Hardy-Weinberg equilibrium?

	BB	*Bb*	*bb*
Before natural selection	*0.0625*	*0.376*	*0.563*
After natural selection	*0.109*	*0.4356*	*0.4356*

Because the genotype frequencies in the population are not constant, the population is evolving.

Extension Questions

25. The ability to taste PTC is due to a single dominant allele "T." You sampled 215 individuals and determined that 150 could detect the bitter taste of PTC and 65 could not. Calculate the following frequencies.

    a. The frequency of the recessive allele.

    $q^2 = 65 \div 215$ $\quad q^2 = \sqrt{0.30}$

    $q^2 = 0.30$ $\quad q = 0.55$

    b. The frequency of the dominant allele.

    $p = 1 - q$ $\quad p = 1 - 0.55$ $\quad p = 0.45$

    c. The frequency of the heterozygous individuals.

    $\chi = 2pq$ $\quad \chi = 2(0.45 \times 0.55)$ $\quad \chi = 0.495$

26. Sixty flowering plants are planted in a flowerbed. Forty of the plants are red-flowering homozygous dominant. Twenty of the plants are white-flowering homozygous recessive. The plants naturally pollinate and reseed themselves for several years. In a subsequent year, 178 red-flowered plants, 190 pink-flowered plants, and 52 white-flowered plants are found in the flowerbed. Use a chi-square analysis to determine if the population is in Hardy-Weinberg equilibrium.

    In the original population:

    $p = 80/120 = 0.66$ $\quad q = 40/120 = 0.33$

    The predicted genotype frequencies for the population once it has reached Hardy-Weinberg equilibrium are:

    $p^2 = 0.4356$ $\quad 2pq = 0.4356$ $\quad q^2 = 0.1089$

    The number of plants with each type of flower in a population of 420 is:

    homozygous dominant = *185* *heterozygous* = *185* *homozygous recessive* = *50*

	Observed data (o)	Expected (e)	(o – e)	(o – e)^2	$\frac{(o-e)^2}{e}$
p^2	178	185	–7	49	0.26
2pq	190	185	5	25	0.135
q^2	52	50	2	4	0.08

Chi-square = 0.474

With 2 degrees of freedom, this chi-square value gives a P value of 0.7–0.8. It is not significant. The flowering population appears to be in Hardy-Weinberg equilibrium.

Teacher Resources – Hardy-Weinberg Equilibrium

Learning Objectives

1. State the formulas that describe the genotype distributions of a population in Hardy-Weinberg equilibrium and describe the meaning of each variable.
2. List the five conditions under which a population will remain in Hardy-Weinberg equilibrium.
3. Calculate the change in population frequencies due to genetic drift or selection using the Hardy-Weinberg equations.

Prerequisites

1. Students should know how to use a Punnett square to predict the genotype frequencies of the next generation from a mating pair. See the *Statistics of Inheritance* activity in POGIL Activities for AP* Biology.
2. Students should have a basic understanding of probability and be able to calculate the probability of an event.
3. Students should know that evolution is a process that affects entire populations, not individuals.
4. Students should have a basic knowledge and understanding of genetic terminology, such as genotype, phenotype, heterozygous, and homozygous.
5. For the *Extension Questions,* students will need to perform a cross for two genes and perform a chi-square analysis. See the *Chi-Square Analysis* activity in POGIL Activities for AP* Biology.

Assessment Questions

1. In the Hardy-Weinberg equations p and q refer to the
   a. Frequencies of phenotypes in a population.
   b. Frequencies of genotypes in a population.
   c. Frequencies of alleles in a population.
   d. None of the above.
2. If a population is in Hardy-Weinberg equilibrium, which of the following is NOT true?
   a. The population is not evolving.
   b. The allele frequencies remain constant from one generation to the next.
   c. The number of individuals each generation is the same.
   d. The genotype frequencies remain constant from one generation to the next.
3. What are the five conditions that must be met in order for a population to be considered in Hardy-Weinberg equilibrium?

Assessment Target Responses

1. *c.*
2. *c.*
3. *Large population, random mating, no natural selection, no migration, no mutations.*

Teacher Tips

- Students' understanding of how a population might reach Hardy-Weinberg equilibrium would be enhanced with a classroom simulation of several generations. One possible activity is to use a set of cards with either a dominant or recessive allele written on one side. Give each student in the class 10 allele cards to represent 5 heterozygous organisms in the population. Calculate the initial genotype frequencies in the population. Next, ask students to walk around the class and "mate" with 5 organisms by blindly swapping allele cards. Recalculate the genotype frequencies in the population. Students should see the frequencies change. Continue the mating and calculating frequencies cycles until there is no more change in the genotype frequencies.
- Students will need direction to understand the distinction between allele frequencies and genotype frequency. The change in frequency and the resulting evolution occurs within populations, not individuals.
- The PTC gene was located in 2003 on the q arm of Chromosome 7. Specifically, Chr7 141.67 Mb. The ability to taste PTC varies among individuals due to other influencing factors such as dry mouth. The Flinn Scientific laboratory kit, *Genetics of Taste,* Catalog No. FB1431, uses PTC test paper, as well as thiourea and sodium benzoate test papers to discuss the genetics of taste.

Alignment to AP Biology Framework

- **Essential knowledge 1.A.1:** Natural selection is a major mechanism of evolution.

  Learning Objective 1.1 The student is able to convert a data set from a table of numbers that reflect a change in the genetic makeup of a population over time and to apply mathematical methods and conceptual understandings to investigate the cause(s) and effect(s) of this change.

  Learning Objective 1.3 The student is able to apply mathematical methods to data from a real or simulated population to predict what will happen to the population in the future.

- **Essential knowledge 1.A.3:** Evolutionary change is also driven by random processes.

  Learning Objective 1.6 The student is able to use data from mathematical models based on the Hardy-Weinberg equilibrium to analyze genetic drift and effects of selection in the evolution of specific populations.

  Learning Objective 1.7 The student is able to justify data from mathematical models based on the Hardy-Weinberg equilibrium to analyze genetic drift and the effects of selection in the evolution of specific populations.