1.4 Perimeter and Area in the Coordinate Plane

Essential Question How can you find the perimeter and area of a polygon in a coordinate plane?

EXPLORATION 1 Finding the Perimeter and Area of a Quadrilateral

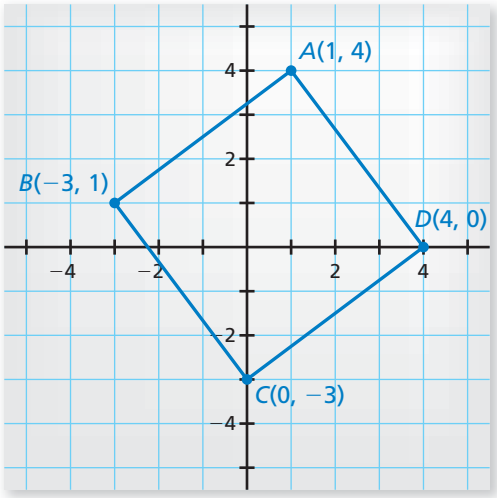
Work with a partner.

a. On a piece of centimeter graph paper, draw quadrilateral $ABCD$ in a coordinate plane. Label the points $A(1, 4)$, $B(-3, 1)$, $C(0, -3)$, and $D(4, 0)$.

b. Find the perimeter of quadrilateral $ABCD$.

c. Are adjacent sides of quadrilateral $ABCD$ perpendicular to each other? How can you tell?

d. What is the definition of a square? Is quadrilateral $ABCD$ a square? Justify your answer. Find the area of quadrilateral $ABCD$.



LOOKING FOR STRUCTURE

To be proficient in math, you need to visualize single objects as being composed of more than one object.

EXPLORATION 2 Finding the Area of a Polygon

Work with a partner.

a. Partition quadrilateral $ABCD$ into four right triangles and one square, as shown. Find the coordinates of the vertices for the five smaller polygons.

b. Find the areas of the five smaller polygons.

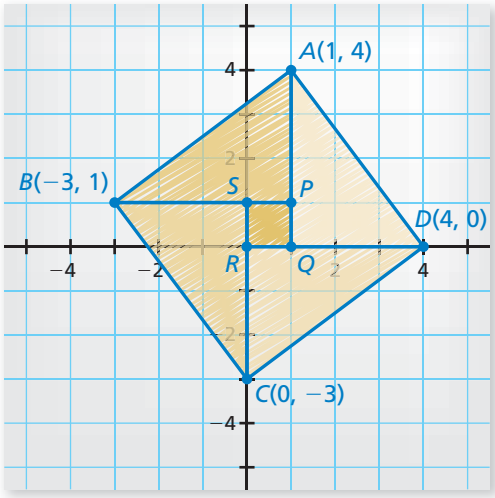
Area of Triangle BPA:

Area of Triangle AQD:

Area of Triangle DRC:

Area of Triangle CSB:

Area of Square $PQRS$:



c. Is the sum of the areas of the five smaller polygons equal to the area of quadrilateral $ABCD$? Justify your answer.

Communicate Your Answer

3. How can you find the perimeter and area of a polygon in a coordinate plane?

4. Repeat Exploration 1 for quadrilateral $EFGH$, where the coordinates of the vertices are $E(-3, 6)$, $F(-7, 3)$, $G(-1, -5)$, and $H(3, -2)$.

1.4 Lesson

Core Vocabulary

Previous
polygon
side
vertex
n-gon
convex
concave

What You Will Learn

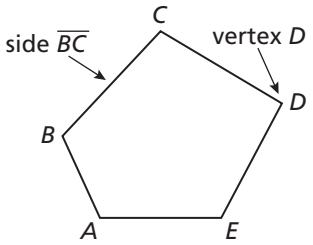
- Classify polygons.
- Find perimeters and areas of polygons in the coordinate plane.

Classifying Polygons

Core Concept

Polygons

In geometry, a figure that lies in a plane is called a plane figure. Recall that a *polygon* is a closed plane figure formed by three or more line segments called *sides*. Each side intersects exactly two sides, one at each *vertex*, so that no two sides with a common vertex are collinear. You can name a polygon by listing the vertices in consecutive order.

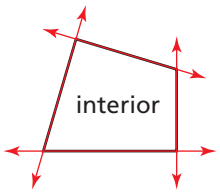


polygon *ABCDE*

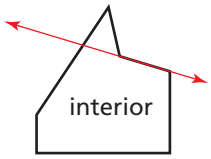
The number of sides determines the name of a polygon, as shown in the table.

Number of sides	Type of polygon
3	Triangle
4	Quadrilateral
5	Pentagon
6	Hexagon
7	Heptagon
8	Octagon
9	Nonagon
10	Decagon
12	Dodecagon
n	*n*-gon

You can also name a polygon using the term *n*-gon, where *n* is the number of sides. For instance, a 14-gon is a polygon with 14 sides.



convex polygon



concave polygon

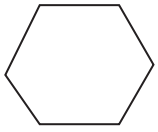
A polygon is *convex* when no line that contains a side of the polygon contains a point in the interior of the polygon. A polygon that is not convex is *concave*.

EXAMPLE 1 Classifying Polygons

Classify each polygon by the number of sides. Tell whether it is *convex* or *concave*.

a.

b.



SOLUTION

a. The polygon has four sides. So, it is a quadrilateral. The polygon is concave.

b. The polygon has six sides. So, it is a hexagon. The polygon is convex.

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Classify the polygon by the number of sides. Tell whether it is *convex* or *concave*.

1.

2.

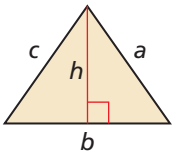
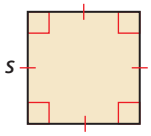
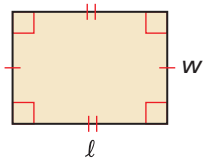
Finding Perimeter and Area in the Coordinate Plane

You can use the formulas given below and the Distance Formula to find the perimeters and areas of polygons in the coordinate plane.

REMEMBER

Perimeter has linear units, such as feet or meters. Area has square units, such as square feet or square meters.

Perimeter and Area

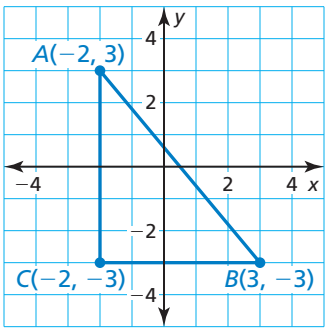
Triangle	Square	Rectangle
		
$P = a + b + c$ $A = \frac{1}{2}bh$	$P = 4s$ $A = s^2$	$P = 2\ell + 2w$ $A = \ell w$

EXAMPLE 2 **Finding Perimeter in the Coordinate Plane**

READING

You can read the notation $\triangle ABC$ as "triangle A B C."

Find the perimeter of $\triangle ABC$ with vertices $A(-2, 3)$, $B(3, -3)$, and $C(-2, -3)$.



SOLUTION

Step 1 Draw the triangle in a coordinate plane. Then find the length of each side.

Side $\overline{AB}$

$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ Distance Formula

$= \sqrt{[3 - (-2)]^2 + (-3 - 3)^2}$ Substitute.

$= \sqrt{5^2 + (-6)^2}$ Subtract.

$= \sqrt{61}$ Simplify.

≈ 7.81 Use a calculator.

Side $\overline{BC}$

$BC = |-2 - 3| = 5$ Ruler Postulate (Postulate 1.1)

Side $\overline{CA}$

$CA = |3 - (-3)| = 6$ Ruler Postulate (Postulate 1.1)

Step 2 Find the sum of the side lengths.

$AB + BC + CA \approx 7.81 + 5 + 6 = 18.81$

So, the perimeter of $\triangle ABC$ is about 18.81 units.

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Find the perimeter of the polygon with the given vertices.

3. $D(-3, 2)$, $E(4, 2)$, $F(4, -3)$

4. $G(-3, 2)$, $H(2, 2)$, $J(-1, -3)$

5. $K(-1, 1)$, $L(4, 1)$, $M(2, -2)$, $N(-3, -2)$

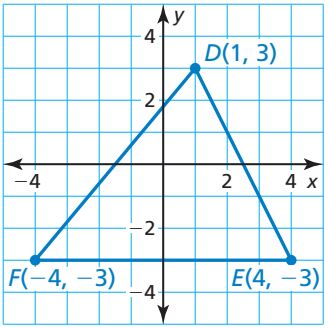
6. $Q(-4, -1)$, $R(1, 4)$, $S(4, 1)$, $T(-1, -4)$

EXAMPLE 3 Finding Area in the Coordinate Plane

Find the area of $\triangle DEF$ with vertices $D(1, 3)$, $E(4, -3)$, and $F(-4, -3)$.

SOLUTION

Step 1 Draw the triangle in a coordinate plane by plotting the vertices and connecting them.



Step 2 Find the lengths of the base and height.

Base

The base is $\overline{FE}$. Use the Ruler Postulate (Postulate 1.1) to find the length of $\overline{FE}$.

$$FE = |4 - (-4)| \qquad \text{Ruler Postulate (Postulate 1.1)}$$
$$= |8| \qquad \text{Subtract.}$$
$$= 8 \qquad \text{Simplify.}$$

So, the length of the base is 8 units.

Height

The height is the distance from point D to line segment $\overline{FE}$. By counting grid lines, you can determine that the height is 6 units.

Step 3 Substitute the values for the base and height into the formula for the area of a triangle.

$$A = \tfrac{1}{2}bh \qquad \text{Write the formula for area of a triangle.}$$
$$= \tfrac{1}{2}(8)(6) \qquad \text{Substitute.}$$
$$= 24 \qquad \text{Multiply.}$$

▶ So, the area of $\triangle DEF$ is 24 square units.

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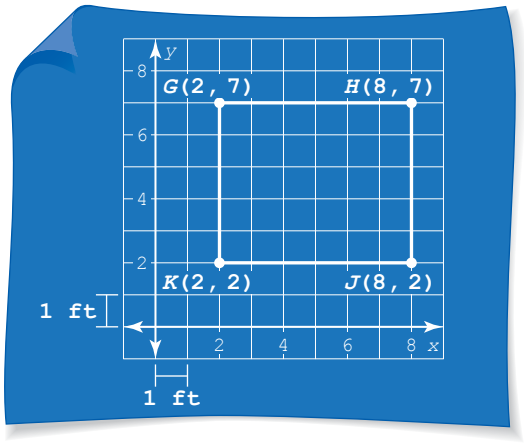
Find the area of the polygon with the given vertices.

7. $G(2, 2)$, $H(3, -1)$, $J(-2, -1)$
8. $N(-1, 1)$, $P(2, 1)$, $Q(2, -2)$, $R(-1, -2)$
9. $F(-2, 3)$, $G(1, 3)$, $H(1, -1)$, $J(-2, -1)$
10. $K(-3, 3)$, $L(3, 3)$, $M(3, -1)$, $N(-3, -1)$

EXAMPLE 4 Modeling with Mathematics

You are building a shed in your backyard. The diagram shows the four vertices of the shed. Each unit in the coordinate plane represents 1 foot. Find the area of the floor of the shed.



SOLUTION

1. **Understand the Problem** You are given the coordinates of a shed. You need to find the area of the floor of the shed.
2. **Make a Plan** The shed is rectangular, so use the coordinates to find the length and width of the shed. Then use a formula to find the area.
3. **Solve the Problem**

Step 1 Find the length and width.

Length $GH = |8 - 2| = 6$ Ruler Postulate (Postulate 1.1)

Width $GK = |7 - 2| = 5$ Ruler Postulate (Postulate 1.1)

The shed has a length of 6 feet and a width of 5 feet.

Step 2 Substitute the values for the length and width into the formula for the area of a rectangle.

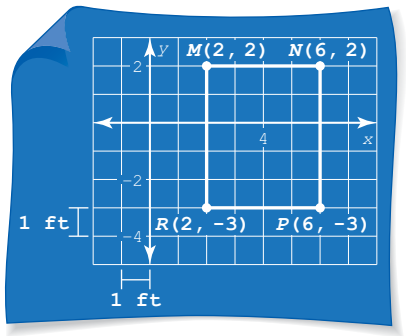
$A = \ell w$ Write the formula for area of a rectangle.

$= (6)(5)$ Substitute.

$= 30$ Multiply.

▶ So, the area of the floor of the shed is 30 square feet.

4. **Look Back** Make sure your answer makes sense in the context of the problem. Because you are finding an area, your answer should be in square units. An answer of 30 square feet makes sense in the context of the problem. ✓



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11. You are building a patio in your school's courtyard. In the diagram at the left, the coordinates represent the four vertices of the patio. Each unit in the coordinate plane represents 1 foot. Find the area of the patio.

1.4 Exercises

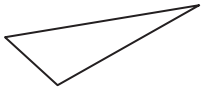
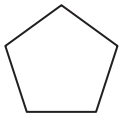
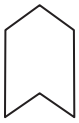
Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

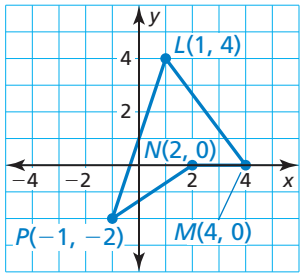
1. **COMPLETE THE SENTENCE** The perimeter of a square with side length s is $P =$ ____.
2. **WRITING** What formulas can you use to find the area of a triangle in a coordinate plane?

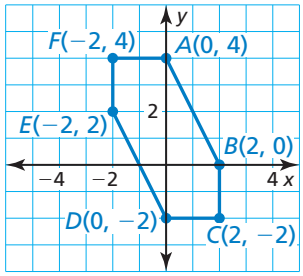
Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, classify the polygon by the number of sides. Tell whether it is *convex* or *concave*. *(See Example 1.)*

3. 
4. 
5. 
6. 

In Exercises 7–12, find the perimeter of the polygon with the given vertices. *(See Example 2.)*

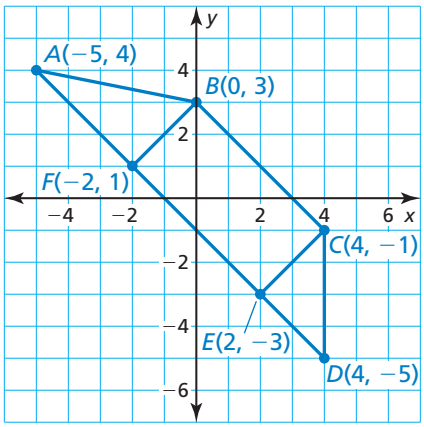
7. $G(2, 4)$, $H(2, -3)$, $J(-2, -3)$, $K(-2, 4)$
8. $Q(-3, 2)$, $R(1, 2)$, $S(1, -2)$, $T(-3, -2)$
9. $U(-2, 4)$, $V(3, 4)$, $W(3, -4)$
10. $X(-1, 3)$, $Y(3, 0)$, $Z(-1, -2)$
11. 

12. 

In Exercises 13–16, find the area of the polygon with the given vertices. *(See Example 3.)*

13. $E(3, 1)$, $F(3, -2)$, $G(-2, -2)$
14. $J(-3, 4)$, $K(4, 4)$, $L(3, -3)$
15. $W(0, 0)$, $X(0, 3)$, $Y(-3, 3)$, $Z(-3, 0)$
16. $N(-2, 1)$, $P(3, 1)$, $Q(3, -1)$, $R(-2, -1)$

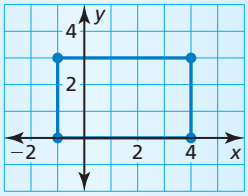
In Exercises 17–24, use the diagram.



17. Find the perimeter of $\triangle CDE$.
18. Find the perimeter of rectangle $BCEF$.
19. Find the perimeter of $\triangle ABF$.
20. Find the perimeter of quadrilateral $ABCD$.
21. Find the area of $\triangle CDE$.
22. Find the area of rectangle $BCEF$.
23. Find the area of $\triangle ABF$.
24. Find the area of quadrilateral $ABCD$.

ERROR ANALYSIS **In Exercises 25 and 26, describe and correct the error in finding the perimeter or area of the polygon.**

25.



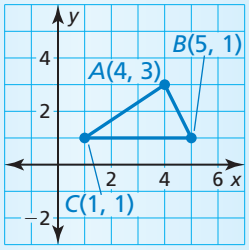
$P = 2\ell + 2w$

$= 2(4) + 2(3)$

$= 14$

The perimeter is 14 units.

26.



$b = |5 - 1| = 4$

$h = \sqrt{(5 - 4)^2 + (1 - 3)^2}$

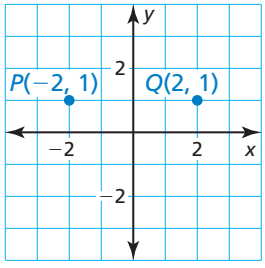
$= \sqrt{5}$

≈ 2.2

$A = \frac{1}{2}bh \approx \frac{1}{2}(4)(2.2) = 4.4$

The area is about 4.4 square units.

In Exercises 27 and 28, use the diagram.



27. Determine which point is the remaining vertex of a triangle with an area of 4 square units.

Ⓐ $R(2, 0)$

Ⓑ $S(-2, -1)$

Ⓒ $T(-1, 0)$

Ⓓ $U(2, -2)$

28. Determine which points are the remaining vertices of a rectangle with a perimeter of 14 units.

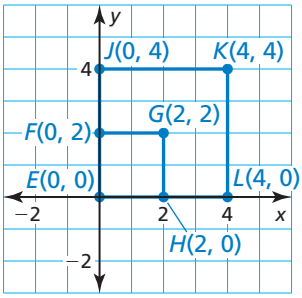
Ⓐ $A(2, -2)$ and $B(2, -1)$

Ⓑ $C(-2, -2)$ and $D(-2, 2)$

Ⓒ $E(-2, -2)$ and $F(2, -2)$

Ⓓ $G(2, 0)$ and $H(-2, 0)$

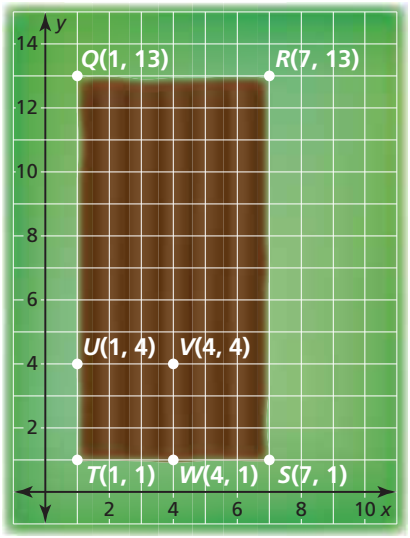
29. **USING STRUCTURE** Use the diagram.



a. Find the areas of square $EFGH$ and square $EJKL$. What happens to the area when the perimeter of square $EFGH$ is doubled?

b. Is this true for every square? Explain.

30. **MODELING WITH MATHEMATICS** You are growing zucchini plants in your garden. In the figure, the entire garden is rectangle $QRST$. Each unit in the coordinate plane represents 1 foot. *(See Example 4.)*



a. Find the area of the garden.

b. Zucchini plants require 9 square feet around each plant. How many zucchini plants can you plant?

c. You decide to use square $TUVW$ to grow lettuce. You can plant four heads of lettuce per square foot. How many of each vegetable can you plant? Explain.

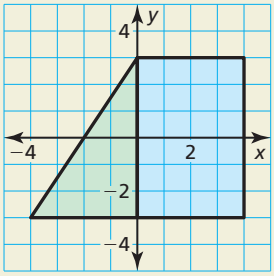
31. MODELING WITH MATHEMATICS You are going for a hike in the woods. You hike to a waterfall that is 4 miles east of where you left your car. You then hike to a lookout point that is 2 miles north of your car. From the lookout point, you return to your car.

a. Map out your route in a coordinate plane with your car at the origin. Let each unit in the coordinate plane represent 1 mile. Assume you travel along straight paths.

b. How far do you travel during the entire hike?

c. When you leave the waterfall, you decide to hike to an old wishing well before going to the lookout point. The wishing well is 3 miles north and 2 miles west of the lookout point. How far do you travel during the entire hike?

32. HOW DO YOU SEE IT? Without performing any calculations, determine whether the triangle or the rectangle has a greater area. Which one has a greater perimeter? Explain your reasoning.



33. MATHEMATICAL CONNECTIONS The lines $y_1 = 2x - 6$, $y_2 = -3x + 4$, and $y_3 = -\frac{1}{2}x + 4$ are the sides of a right triangle.

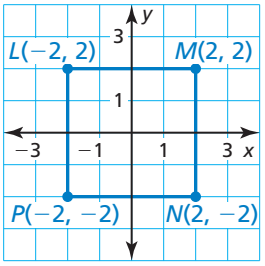
a. Use slopes to determine which sides are perpendicular.

b. Find the vertices of the triangle.

c. Find the perimeter and area of the triangle.

34. THOUGHT PROVOKING Your bedroom has an area of 350 square feet. You are remodeling to include an attached bathroom that has an area of 150 square feet. Draw a diagram of the remodeled bedroom and bathroom in a coordinate plane.

35. PROBLEM SOLVING Use the diagram.

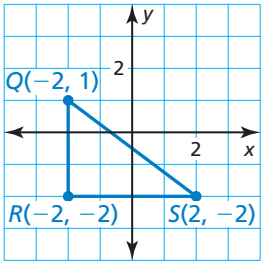


a. Find the perimeter and area of the square.

b. Connect the midpoints of the sides of the given square to make a quadrilateral. Is this quadrilateral a square? Explain your reasoning.

c. Find the perimeter and area of the quadrilateral you made in part (b). Compare this area to the area you found in part (a).

36. MAKING AN ARGUMENT Your friend claims that a rectangle with the same perimeter as $\triangle QRS$ will have the same area as the triangle. Is your friend correct? Explain your reasoning.



37. REASONING Triangle ABC has a perimeter of 12 units. The vertices of the triangle are $A(x, 2)$, $B(2, -2)$, and $C(-1, 2)$. Find the value of x.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Solve the equation. *(Skills Review Handbook)*

38. $3x - 7 = 2$

39. $5x + 9 = 4$

40. $x + 4 = x - 12$

41. $4x - 9 = 3x + 5$

42. $11 - 2x = 5x - 3$

43. $\frac{x + 1}{2} = 4x - 3$

44. Use a compass and straightedge to construct a copy of the line segment. *(Section 1.2)*