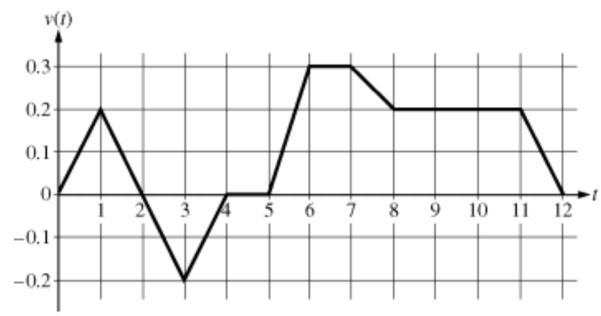
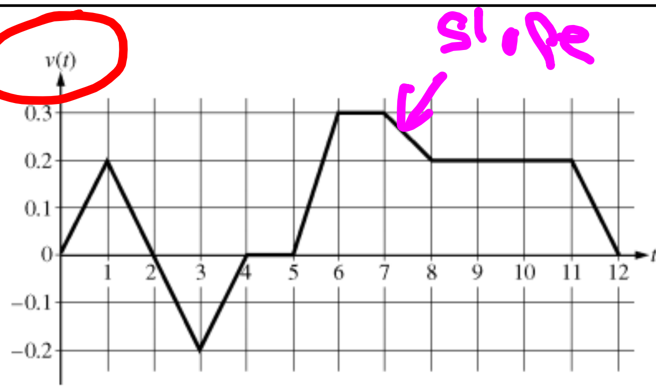
April 21, 2020

AP® CALCULUS AB
2009 SCORING GUIDELINES

Question 1



Caren rides her bicycle along a straight road from home to school, starting at home at time $t = 0$ minutes and arriving at school at time $t = 12$ minutes. During the time interval $0 \le t \le 12$ minutes, her velocity $v(t)$, in miles per minute, is modeled by the piecewise-linear function whose graph is shown above.



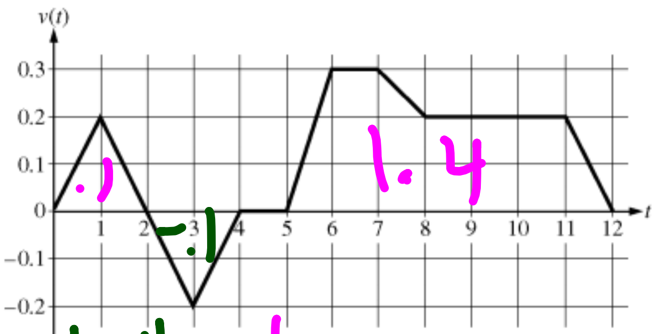
v'(t) = accel
slope of v(t)
at 7.5

(a) Find the acceleration of Caren's bicycle at time $t = 7.5$ minutes. Indicate units of measure.

(a) $a(7.5) = v'(7.5) = \dfrac{v(8) - v(7)}{8 - 7} = -0.1$ miles/minute^2

Pay attention to
units $\Delta y = .1$

$\int |v(t)|$: total distance any direction



$v > 0$ right

$v < 0$ left

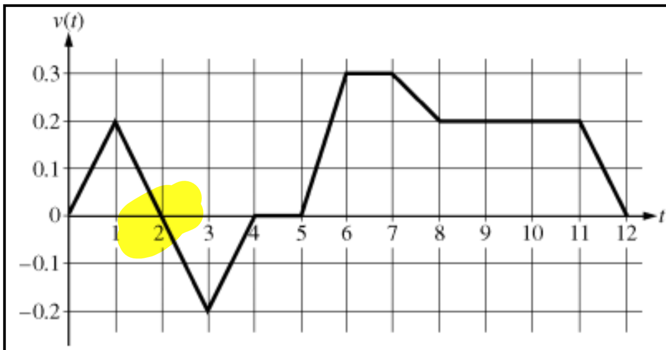
$|-.1| = .1$

(b) Using correct units, explain the meaning of $\int_0^{12} |v(t)|\, dt$ in terms of Caren's trip. Find the value of $\int_0^{12} |v(t)|\, dt$.

(b) $\int_0^{12} |v(t)|\, dt$ is the total distance, in miles, that Caren rode during the 12 minutes from $t = 0$ to $t = 12$.

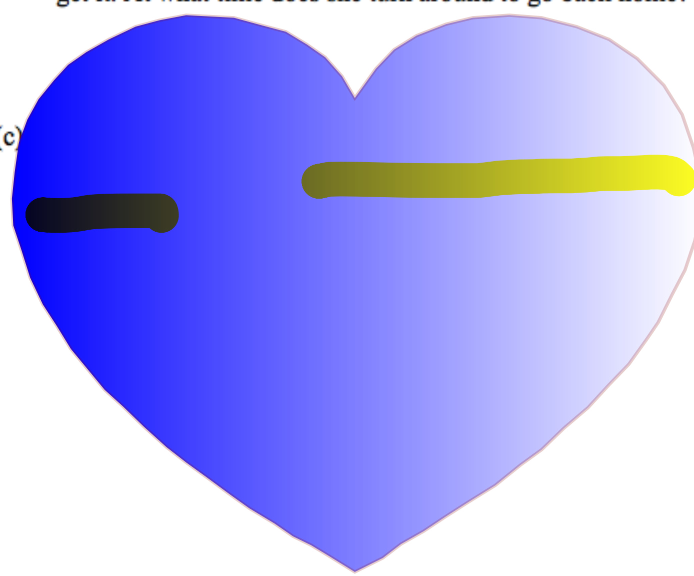
$$\int_0^{12} |v(t)|\, dt = \int_0^{2} v(t)\, dt - \int_2^{4} v(t)\, dt + \int_4^{12} v(t)\, dt$$

$$= 0.2 + 0.2 + 1.4 = 1.8 \text{ miles}$$



(c) Shortly after leaving home, Caren realizes she left her calculus homework at home, and she returns to get it. At what time does she turn around to go back home? Give a reason for your answer.

(c)



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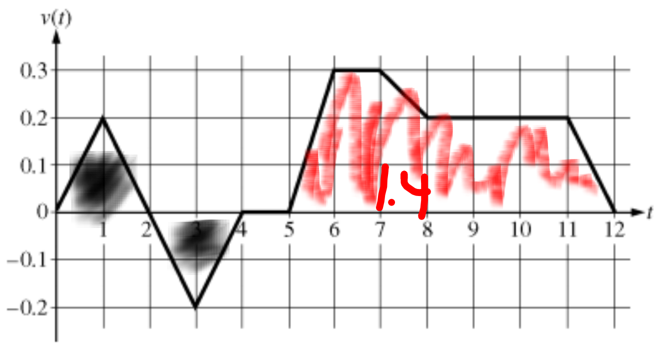

(d) Larry also rides his bicycle along a straight road from home to school in 12 minutes. His velocity is modeled by the function w given by $w(t) = \frac{\pi}{15}\sin\left(\frac{\pi}{12}t\right)$, where $w(t)$ is in miles per minute for $0 \le t \le 12$ minutes. Who lives closer to school: Caren or Larry? Show the work that leads to your answer.

(b) $\int_0^{12} |v(t)|\, dt$ is the total distance, in miles, that Caren rode during the 12 minutes from $t = 0$ to $t = 12$.

$\int_0^{12} |v(t)|\, dt = \int_0^{2} v(t)\, dt - \int_2^{4} v(t)\, dt + \int_4^{12} v(t)\, dt$

$= 0.2 + 0.2 + 1.4 = 1.8$ miles

1.4

(d) $\int_0^{12} w(t)\, dt = 1.6$; Larry lives 1.6 miles from school.

$\int_0^{12} v(t)\, dt = 1.4$; Caren lives 1.4 miles from school.

Therefore, Caren lives closer to school.

Question 3

Mighty Cable Company manufactures cable that sells for \$120 per meter. For a cable of fixed length, the cost of producing a portion of the cable varies with its distance from the beginning of the cable. Mighty reports that the cost to produce a portion of a cable that is x meters from the beginning of the cable is $6\sqrt{x}$ dollars per meter. (Note: Profit is defined to be the difference between the amount of money received by the company for selling the cable and the company's cost of producing the cable.)

(a) Find Mighty's profit on the sale of a 25-meter cable.

(b) Using correct units, explain the meaning of $\int_{25}^{30} 6\sqrt{x}\, dx$ in the context of this problem.

(c) Write an expression, involving an integral, that represents Mighty's profit on the sale of a cable that is k meters long.

(d) Find the maximum profit that Mighty could earn on the sale of one cable. Justify your answer.

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(a) Find Mighty's profit on the sale of a 25-meter cable.

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RATE

$$\int \frac{\text{cost}}{\text{meter}} \, d\,\text{meter}$$

Received − cost = Change in Cost

(a) $\text{Profit} = 120 \cdot 25 - \int_0^{25} 6\sqrt{x}\,dx = 2500$ dollars

(b) Using correct units, explain the meaning of $\int_{25}^{30} 6\sqrt{x}\,dx$ in the context of this problem.

(b) $\int_{25}^{30} 6\sqrt{x}\,dx$ is the difference in cost, in dollars, of producing a cable of length 30 meters and a cable of length 25 meters.

$$\int \frac{\text{cost}}{\text{meter}}\, d\,\text{meter} =$$ Change in Cost

$=$

(c) Write an expression, involving an integral, that represents Mighty's profit on the sale of a cable that is k meters long.

Generalizing

(c) Profit $= 120k - \int_0^k 6\sqrt{x}\,dx$ dollars

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(d) Find the maximum profit that Mighty could earn on the sale of one cable. Justify your answer.

maximize anything = derivative

(d) Let $P(k)$ be the profit for a cable of length k.
$P'(k) = 120 - 6\sqrt{k} = 0$ when $k = 400$.
This is the only critical point for P, and P' changes from positive to negative at $k = 400$.
Therefore, the maximum profit is $P(400) = 16{,}000$ dollars.

changes from + to -

k	144	400	900
$P'(k)$	+	0	−

Find critical points

derivative = 0 or undefined

$\sqrt{k} = 20$ $k = 400$ Now check signs

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$$p^{-}(k) = 120 - 6\sqrt{k}$$