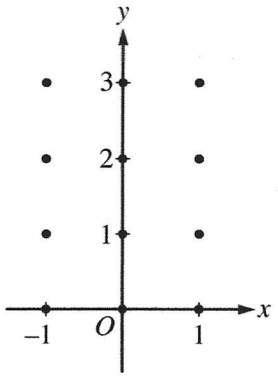
NO CALCULATOR ALLOWED

4. Consider the differential equation $\frac{dy}{dx} = \frac{x(y-1)}{4}$.

(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.



(b) Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(1) = 3$. Write an equation for the line tangent to the graph of f at the point $(1, 3)$ and use it to approximate $f(1.4)$.

Do not write beyond this border.

Continue problem 4 on page 17.

AP® CALCULUS AB
2015 SCORING GUIDELINES

Question 4

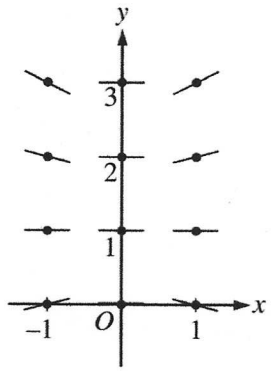
Consider the differential equation $\frac{dy}{dx} = \frac{x(y-1)}{4}$.

(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.

(b) Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(1) = 3$. Write an equation for the line tangent to the graph of f at the point $(1, 3)$ and use it to approximate $f(1.4)$.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(1) = 3$.

(a)



$2 : \begin{cases} 1 : \text{zero slopes} \\ 1 : \text{other slopes} \end{cases}$

(b) $\left.\frac{dy}{dx}\right|_{(x,\,y)=(1,\,3)} = \frac{(1)(2)}{4} = \frac{1}{2}$

An equation for the line tangent to the graph of f at the point $(1, 3)$ is $y = \frac{1}{2}(x-1) + 3$.

$f(1.4) \approx \frac{1}{2}(0.4) + 3 = 3.2$

$2 : \begin{cases} 1 : \text{tangent line} \\ 1 : \text{approximation} \end{cases}$

(c) $\frac{dy}{dx} = \frac{x(y-1)}{4}$

$\int \frac{dy}{y-1} = \int \frac{x}{4}\,dx$

$\ln|y-1| = \frac{x^2}{8} + C$

$\ln 2 = \frac{1}{8} + C \Rightarrow C = \ln 2 - \frac{1}{8}$

Because $f(1) = 3$, $y > 1$, so $|y-1| = y-1$.

$\ln(y-1) = \frac{x^2}{8} + \ln 2 - \frac{1}{8}$

$y = 2e^{(x^2-1)/8} + 1$ for all x

$5 : \begin{cases} 1 : \text{separation of variables} \\ 2 : \text{antiderivatives} \\ 1 : \text{constant of integration and uses initial condition} \\ 1 : \text{solves for } y \end{cases}$

Note: max 3/5 [1-2-0-0] if no constant of integration

Note: 0/5 if no separation of variables