

Looking Ahead to Chapter 5

Focus

In Chapter 5, you will learn how to write linear equations in a variety of forms. You will learn what the different pieces of a linear equation represent, and how they apply to real-life situations.

Chapter Warm-up

Answer these questions to help you review skills that you will need in Chapter 5.

Evaluate the equation $y = 3x - 17$ for each value of x .

1. $x = 3$

2. $x = 0$

3. $x = -8$

Evaluate the equation $y = -7x + 24$ for each value of x .

4. $x = -4$

5. $x = 14$

6. $x = -9$

Solve each inequality.

7. $3x - 5 < 19$

8. $8x + 20 \geq 36$

9. $15 < 4 - 7x$

Read the problem scenario below.

In order to entice people to work for them, a local used car lot advertises that it will pay a commission to its employees in the amount of 12% of the employee's total sales for each month.

10. How much in total sales would an employee have to sell in one month to earn \$2700 in commission?
11. How much would an employee earn in commission if they sold \$76,000 worth of used cars in one month?

Key Terms

linear equation ■ p. 204

linear function ■ p. 204

function notation ■ p. 204

x-intercept ■ p. 208

y-intercept ■ p. 208

unit rate ■ p. 212

slope ■ p. 215

vertical change ■ p. 215

horizontal change ■ p. 215

rate of change ■ p. 215

slope-intercept form ■ p. 222

point-slope form ■ p. 229

domain ■ p. 235

range ■ p. 235

piecewise function ■ p. 237

standard form ■ p. 241

interest ■ p. 245

principal ■ p. 245

simple interest ■ p. 245

interest rate ■ p. 245

literal equation ■ p. 246

Writing and Graphing Linear Equations



One of the most frequently performed full-length plays by high schools in the United States is William Shakespeare's *A Midsummer Night's Dream*. In Lesson 5.7, you will determine the costs to produce flyers to advertise a school play.

5.1 Widgets, Dumbbells, and Dumpsters

Multiple Representations of Linear Functions ■ p. 197

5.2 Selling Balloons

Finding Intercepts of a Graph ■ p. 205

5.3 Recycling and Saving

Finding the Slope of a Line ■ p. 211

5.4 Running in a Marathon

Slope-Intercept Form ■ p. 219

5.5 Saving Money

Writing Equations of Lines ■ p. 227

5.6 Spending Money

Linear and Piecewise Functions ■ p. 233

5.7 The School Play

Standard Form of a Linear Equation ■ p. 239

5.8 Earning Interest

Solving Literal Equations ■ p. 245

5.1

Widgets, Dumbbells, and Dumpsters

Multiple Representations of Linear Functions

Objective

In this lesson, you will:

- Represent linear functions using equations, tables, and graphs.



SCENARIO

In today's business world, it is not uncommon for one company to own several companies. These other companies are called *subsidiaries* and often, you will find that the subsidiaries of a company do not have to produce similar products or provide similar services.

Key Terms

- linear equation
- linear function
- function notation



Problem 1

Making and Selling Widgets

One subsidiary of a company makes and sells widgets. The subsidiary sells the widgets for \$4 each and adds a \$9 shipping charge to the cost of an order.

- Use complete sentences to describe how you would find the total cost of an order of 12 widgets.
- Use complete sentences to describe how you would find the total cost of an order of 257 widgets.
- Use complete sentences to describe how you would find the total cost of an order of any number of widgets.

Investigate Problem 1

- Write an equation that gives the total cost of an order in terms of the number of widgets. Use x to represent the number of ordered widgets and use y to represent the total cost in dollars.
- Use your equation to find the total cost of an order of 12 widgets. Show all your work and use a complete sentence in your answer.

Use your equation to find the total cost of an order of 257 widgets. Show all your work and use a complete sentence in your answer.

Investigate Problem 1

3. Use your equation to find the number of widgets that can be ordered for \$89. Show all your work and use a complete sentence in your answer.

Use your equation to find the number of widgets that can be ordered for \$2069. Show all your work and use a complete sentence in your answer.

Use your equation to find the number of widgets that can be ordered for \$7789. Show all your work and use a complete sentence in your answer.

Use complete sentences to describe how you found your answers in Question 3.

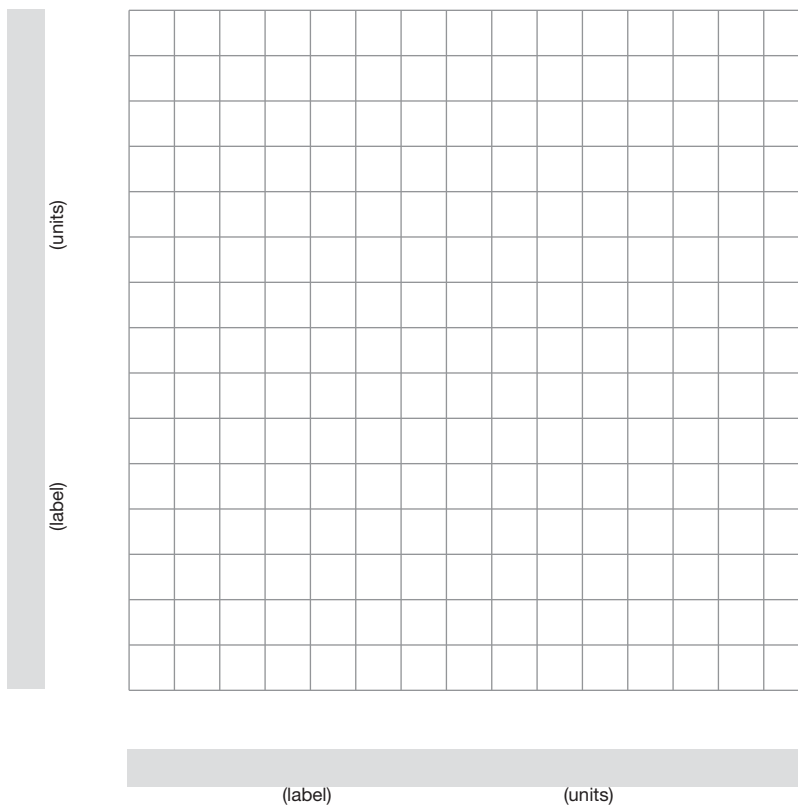
4. Complete the table of values that shows the relationship between the total cost and the number of widgets ordered.

Quantity Name	Widgets ordered	Total cost
Unit		
Expression		
	12	
		89
		2069

Investigate Problem 1

5. Use the grid below to create a graph of the data from the table on the previous page. First, choose your bounds and intervals. Be sure to label your graph clearly.

Variable quantity	Lower bound	Upper bound	Interval



6. Use your graph to approximate the total cost of ordering 900 widgets. Then explain how you found your answer, using complete sentences.

Use your graph to approximate the number of widgets that can be ordered for \$6000. Then explain how you found your answer, using complete sentences.



Problem 2

Making and Selling Dumbbells



Another subsidiary of the company makes and sells dumbbells. The subsidiary sells the dumbbells by the pound for \$.50 per pound. Because dumbbells are heavy, the subsidiary adds an \$18 shipping-and-handling charge to the cost of an order.

- A. Complete the table of values that shows the relationship between the total cost and the number of pounds of dumbbells ordered.

Quantity Name	Dumbbells ordered	Total cost
Unit		
Expression		
	120	
		89
	300	

- B. Use complete sentences to describe how to find the total cost when you are given the number of pounds of dumbbells ordered.
- C. Use complete sentences to describe how to find the number of pounds of dumbbells ordered when you are given the total cost of the order.

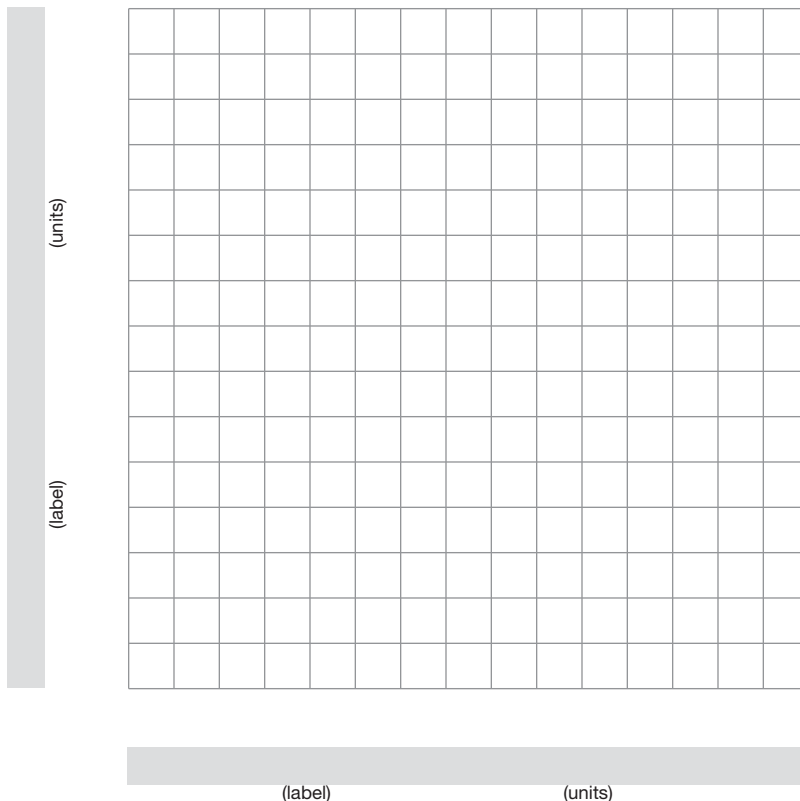
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Investigate Problem 2

1. Use the grid on the next page to create a graph of the data from the table above. First, choose your bounds and intervals. Be sure to label your graph clearly.

Variable quantity	Lower bound	Upper bound	Interval

Investigate Problem 2



2. Use your graph to determine whether the total cost of an order of 220 pounds of dumbbells should be \$115. Use complete sentences to explain.

Use your graph to determine whether the total cost of an order of 260 pounds of dumbbells should be \$155. Use complete sentences to explain.

Use your graph to determine whether the total cost of an order of 200 pounds of dumbbells should be \$118. Use complete sentences to explain.



3. Describe a different way in which you could determine whether the ordered weights and corresponding total costs are correct. Use complete sentences in your answer.

Problem 3

Dumpster Rentals



Yet another subsidiary of the company is a company that provides a dumpster rental service. The company will deliver a dumpster to a customer's site for a one-time fee of \$150. The company also charges a fee each day that the dumpster is rented. The table below shows the cost per week (7 days) without the delivery charge. You will complete the last column of the table in part (B).

Length of rental	Cost	Cost per day
weeks	dollars	dollars per day
1	192.50	
2	385.00	
3	577.50	
4	770.00	
5	962.50	

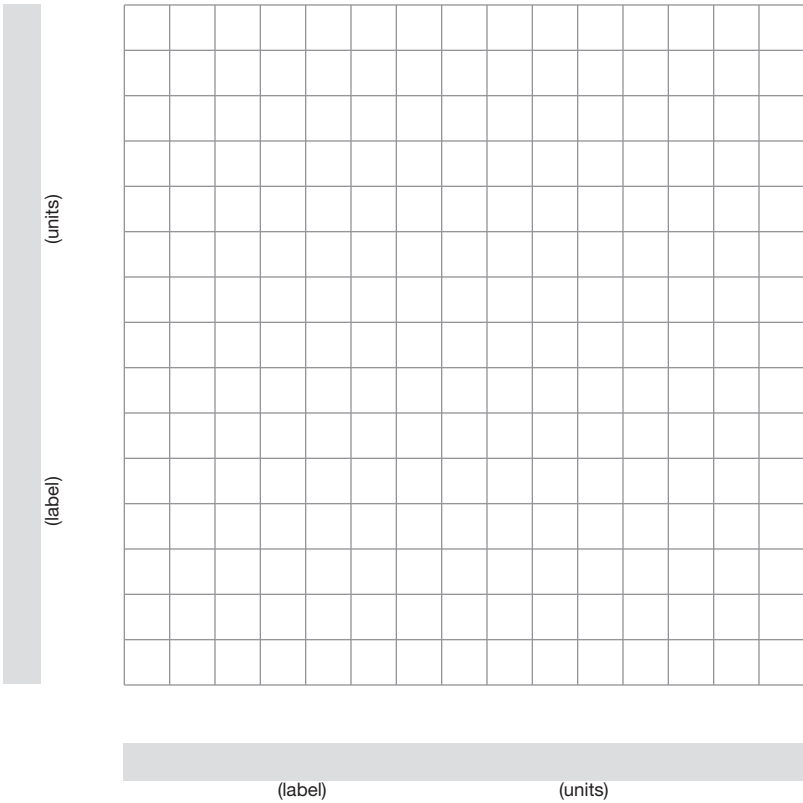
- A.** You want to create a new table that shows the rental cost for different numbers of days. What information do you need to create this table? Use a complete sentence in your answer.
- B.** How can you determine the cost of a rental for one day by using the rows in the table above? Use a complete sentence in your answer. Then complete the last column of the table.
- C.** Complete the table of values below that shows the cost of a rental for 1 to 7 days. Include the one-time cost of dumpster delivery.

Quantity Name	Duration of rental	
Unit	days	
	1	
	2	
	3	
	4	
	5	
	6	
	7	

Investigate Problem 3

- 1. Write an equation that you can use to find the total cost in terms of the number of days of the rental. Identify your variables and use a complete sentence in your answer.
- 2. Use the grid below to create a graph of the data from the table on the previous page. First, choose your bounds and intervals. Be sure to label your graph clearly.

Variable quantity	Lower bound	Upper bound	Interval



- 3. Use your graph to approximate the number of days that a dumpster can be rented for \$680. Use complete sentences to explain how you got your answer.

Investigate Problem 3

Use your graph to approximate the total cost of renting a dumpster for 20 days. Use complete sentences to explain how you found your answer.



4. Is your graph in Question 2 the graph of a function? Use complete sentences to explain your reasoning.



5. If a graph is a straight non-vertical line, is it the graph of a function? Use complete sentences to explain your reasoning.

6. **Just the Math: Linear Equations and Linear Functions** When the graph of an equation is a straight line, then the equation is a *linear equation*. A **linear equation** in two variables is an equation in which each of the variables is raised to the first power (such as x , rather than x^2) and each variable appears at most once. All linear equations, except those in the form $x = a$ (where a is any number), are also **linear functions**. However, not every equation is a function. You will discover this later in this text.

Is the equation that you wrote for Investigate Problem 1, Question 1 a linear function? Use complete sentences to explain your reasoning.

Is the graph for Investigate Problem 2, Question 1 the graph of a linear function? Use complete sentences to explain your reasoning.

If the graph in Investigate Problem 2, Question 1 is that of a linear function, write the equation using function notation.

7. Look at the three equations that you have written so far in this lesson. What do you think is a general form of a linear equation? Use a complete sentence in your answer.

Objectives

In this lesson,
you will:

- Interpret the meaning of intercepts in a problem situation.
- Find intercepts graphically.
- Find intercepts algebraically.



SCENARIO Your local community group wants to raise money to fix one of the playgrounds in your area. Your group decides to sell balloons because they are popular with young children.

Problem 1**Making a Difference**

Your group buys a box of Mylar balloons with zoo animals printed on them for \$10 and decides to sell the balloons for \$2 each.

- A. What is your profit if your group sells 30 balloons? Show all your work and use a complete sentence in your answer.
- B. What is your profit if your group sells 5 balloons? Show all your work and use a complete sentence in your answer.
- C. What is your profit if your group sells 3 balloons? Show all your work and use a complete sentence in your answer.
- D. What does your answer to part (C) mean in terms of the problem situation? Use complete sentences in your answer.
- E. Write an equation for the problem situation. Use x to represent the number of balloons sold. Use y to represent the profit in dollars.

Key Terms

- x -intercept
- y -intercept

Take Note

Recall from Lesson 1.2 that the *profit* is the amount of money that you have left after you subtract the costs from the amount of money that you make.

Investigate Problem 1

1. Use your equation to find the value of y when x is 7. Show all your work and use a complete sentence in your answer.

Use your equation to find the value of y when x is -9 . Show all your work and use a complete sentence in your answer.

Investigate Problem 1

Does an x -value of -9 make sense in the problem situation?
Use complete sentences to explain.

Use your equation to find the value of y when x is 25 . Show all your work and use a complete sentence in your answer.

2. Use your equation to find the value of x when y is 10 . Show all your work and use a complete sentence in your answer.

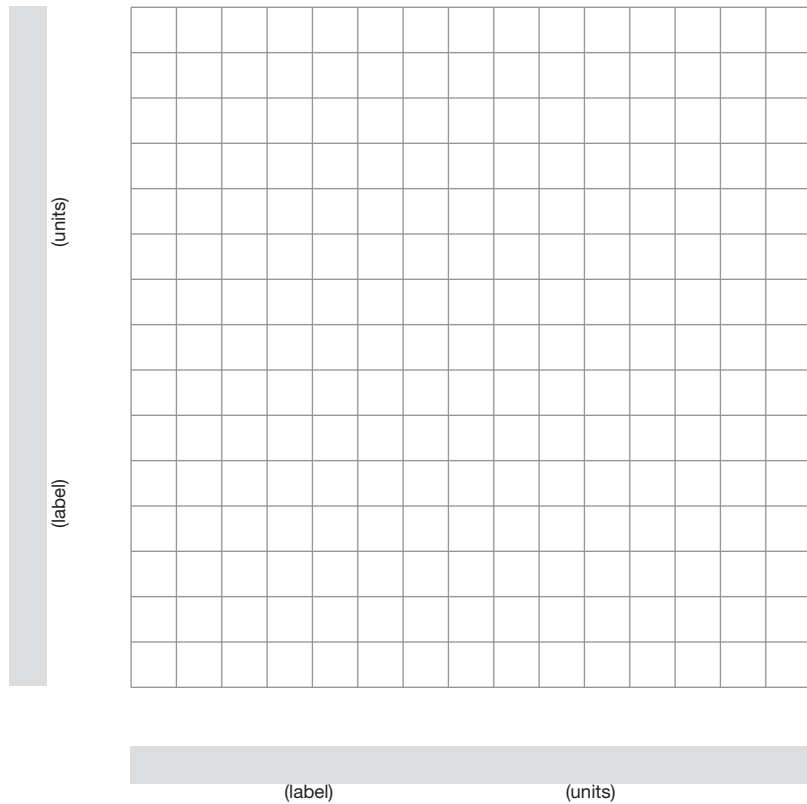
Use your equation to find the value of x when y is -16 . Show all your work and use a complete sentence in your answer.

Use your equation to find the value of x when y is -10 . Show all your work and use a complete sentence in your answer.

3. Write the ordered pairs that you created in Question 2.
4. Write three more sets of ordered pairs that satisfy your equation.
5. Use the grid on the next page to graph the ordered pairs in Questions 3 and 4. Then create a graph of your equation. Use the bounds and intervals given below. Be sure to label your graph clearly.

Variable quantity	Lower bound	Upper bound	Interval
Balloons sold	-5	10	1
Profit	-16	14	2

Investigate Problem 1



6. Use complete sentences to explain which point on your graph represents in the problem situation.

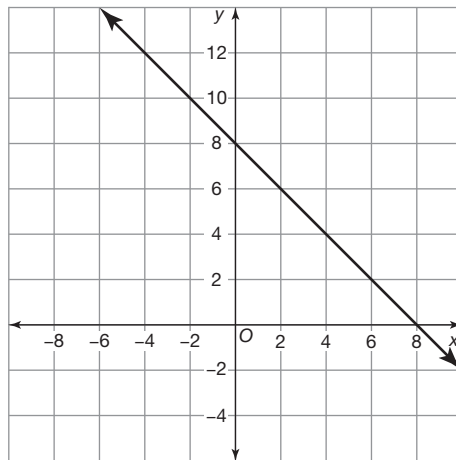
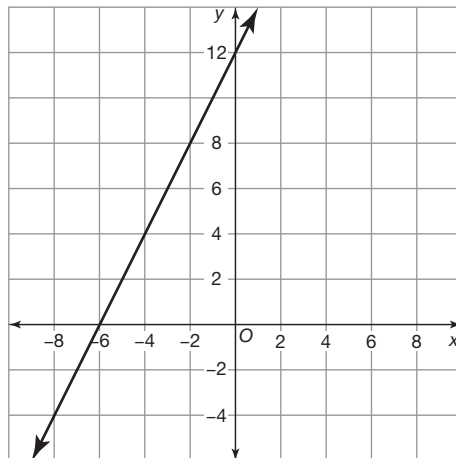
7. Name the point where your graph crosses the x -axis. What does this point tell you about the relationship between the profit and the number of balloons sold? Use complete sentences in your answer.

Name the point where your graph crosses the y -axis. What does this point tell you about the relationship between the profit and the number of balloons sold? Use complete sentences in your answer.

Investigate Problem 1

- 8. Just the Math: Intercepts** In Question 7, you found the *intercepts* of the graph. The **x-intercept** of a graph is the x -coordinate of the point where the graph crosses the x -axis. The **y-intercept** of a graph is the y -coordinate of the point where the graph crosses the y -axis. Name the x - and y -intercepts of the graph in Question 5. Use a complete sentence in your answer.

Find the x - and y -intercepts of each graph below.



- 9.** What do you notice about a point that contains the x -intercept? What do you notice about a point that contains the y -intercept? Use complete sentences in your answer.

Investigate Problem 1

- 10. Just the Math: Finding Intercepts Algebraically** If you know the equation of a graph, you can *algebraically* find the intercepts. To *algebraically* find the x -intercept, find the value of x when y is 0. For instance, the x -intercept of the graph of $y = 2x + 4$ is -2 because:

$$0 = 2x + 4 \quad \text{Substitute 0 for } y.$$

$$-4 = 2x \quad \text{Subtract 4 from each side.}$$

$$-2 = x \quad \text{Divide each side by 2.}$$

To *algebraically* find the y -intercept, find the value of y when x is 0. For instance, the y -intercept of the graph of $y = 2x + 4$ is 4 because:

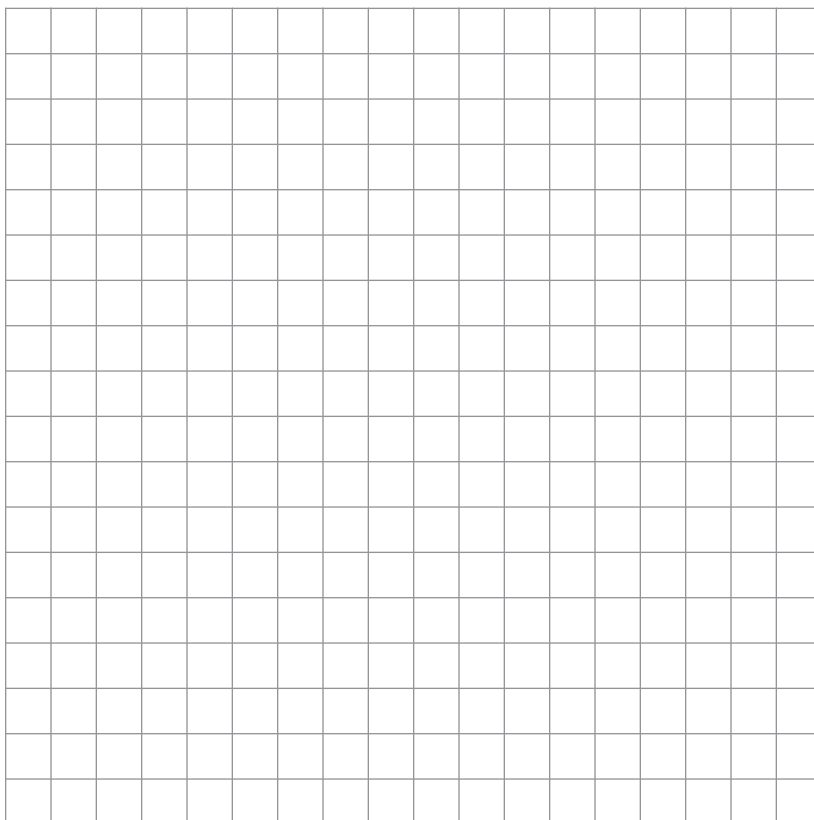
$$y = 2(0) + 4 \quad \text{Substitute 0 for } x.$$

$$= 0 + 4 \quad \text{Multiply.}$$

$$= 4 \quad \text{Add.}$$

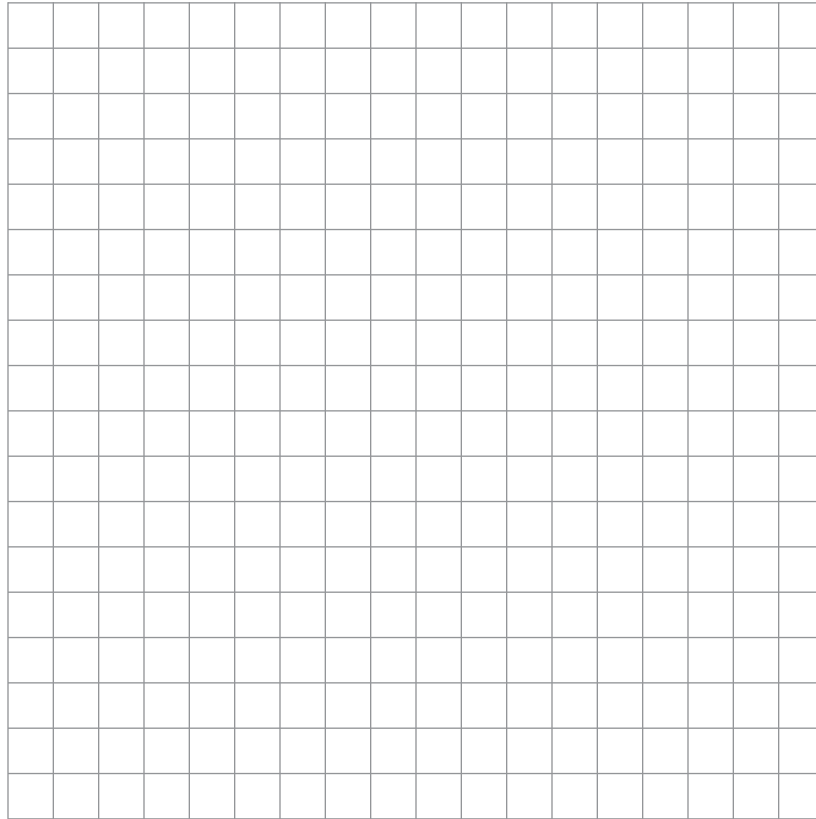
Algebraically find the intercepts of the graph of the equation. Show all your work. Then graph the equation to check your answer.

$$y = 3x - 9$$



Investigate Problem 1

$$y = -4x + 8$$



11. Use what you know about x-intercepts and y-intercepts to complete each statement.

In the ordered pair (6, 0) the _____ is the x-intercept.

In the ordered pair (0, -5) the _____ is the y-intercept.

A vertical line that does not lie on an axis has one _____ and no _____.

A horizontal line that does not lie on an axis has one _____ and no _____.

A straight line that is neither vertical nor horizontal has _____ x-intercept(s) and _____ y-intercept(s).



5.3

Recycling and Saving

Finding the Slope of a Line

Objectives

In this lesson, you will:

- Find unit rates.
- Describe slopes of lines.
- Find rates of change.
- Find slopes of lines through two points.

Key Terms

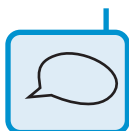
- unit rate
- slope
- vertical change
- horizontal change
- rate of change

Glass recycled

Lower Bound: 0
Upper Bound: 15
Interval: 1

Earnings

Lower Bound: 0
Upper Bound: 0.75
Interval: 0.05



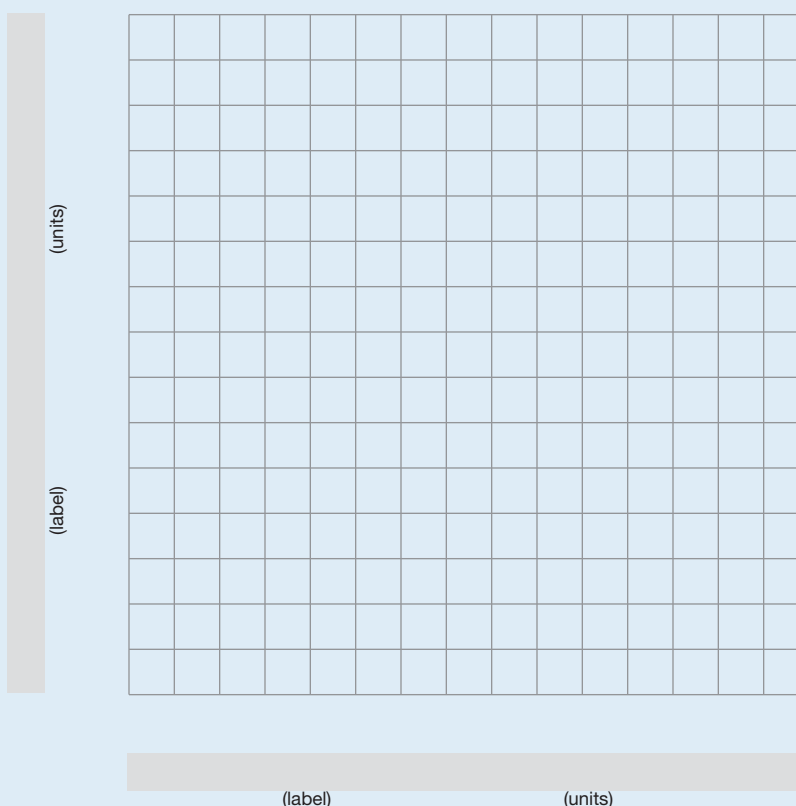
SCENARIO Your class is raising money for a new wheelchair ramp at a community center. Your class plans to collect and recycle materials such as aluminum and glass to earn money.



Problem 1 Recycling Glass

The scrap yard where you recycle the materials pays \$.05 per pound of glass.

- Write an equation that relates the weight of the recycled glass in pounds to the amount of money in dollars earned from recycling. Use x to represent the weight of the recycled glass and use y to represent the amount of money earned.
- Write a sentence that predicts what the graph of the equation will look like. Then use the grid below to create a graph of your equation. In your graph, use the bounds and intervals given at the left. Be sure to label your graph clearly.



Take Note

Recall that a *unit rate* is a rate per one given unit.

Investigate Problem 1

1. Use your graph to find the increase in earnings when the amount of recycled glass increases by 1 pound. Use a complete sentence in your answer. Then write a unit rate that compares the increase in earnings to the increase in the amount of recycled glass.
2. Use your graph to find the increase in earnings when the amount of recycled glass increases by 5 pounds. Use a complete sentence in your answer. Then write a unit rate that compares the increase in earnings to the increase in the amount of recycled glass.
3. Use your graph to find the increase in earnings when the amount of recycled glass increases by 10 pounds. Use a complete sentence in your answer. Then write a unit rate that compares the increase in earnings to the increase in the amount of recycled glass.
4. Explain how you found your answers in Questions 1 through 3. What do you notice about the unit rates? Use complete sentences in your answer.



Problem 2 Recycling Plastic

The scrap yard pays \$.40 for each pound of plastic that you recycle.

- A. Write an equation that relates the weight of the recycled plastic in pounds to the amount of money in dollars earned from recycling. Use x to represent the weight of the recycled plastic and use y to represent the amount of money earned.
- B. What will the graph of the equation look like? Use a complete sentence to explain your reasoning.



Problem 2**Recycling Plastic**

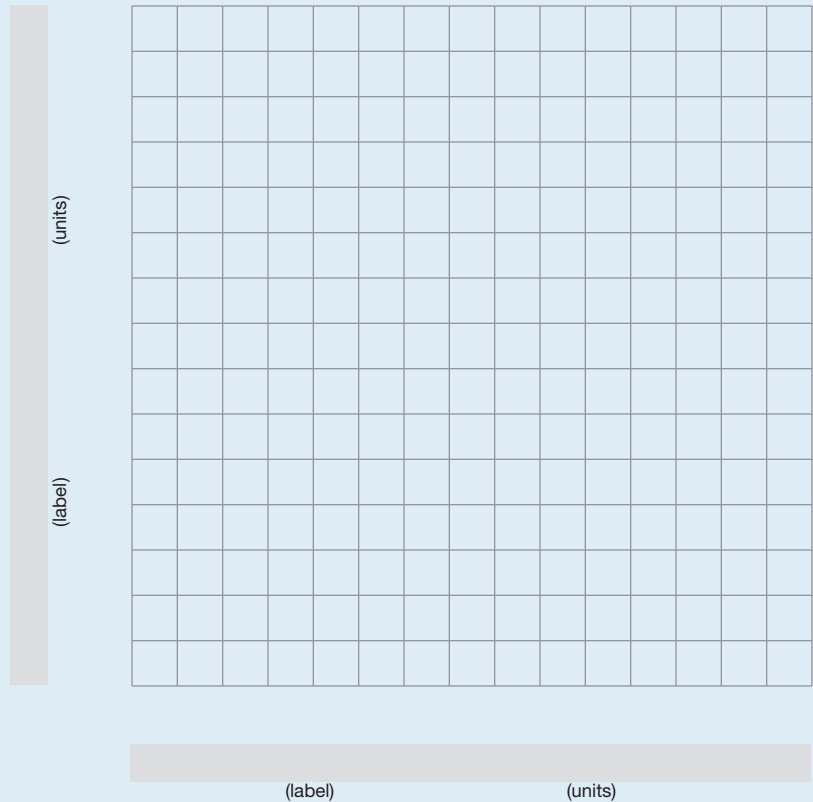
- C. Use the grid below to create a graph of your equation. Use the bounds and intervals given at the left. Be sure to label your graph clearly.

Plastic recycled

Lower Bound: 0
 Upper Bound: 15
 Interval: 1

Earnings

Lower Bound: 0
 Upper Bound: 6
 Interval: 0.40

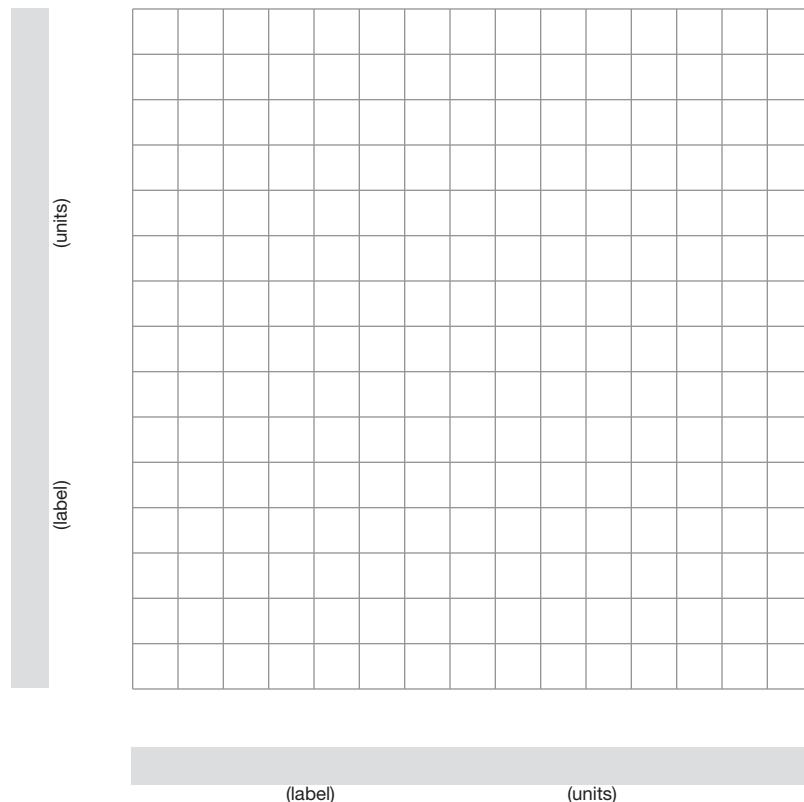
**Investigate Problem 2**

1. Use your graph to find the increase in earnings when the amount of recycled plastic increases by 1 pound. Use a complete sentence in your answer. Then write a unit rate that compares the increase in earnings to the increase in the amount of recycled plastic.
2. Use your graph to find the increase in earnings when the amount of recycled plastic increases by 5 pounds. Use a complete sentence in your answer. Then write a unit rate that compares the increase in earnings to the increase in the amount of recycled plastic.

Investigate Problem 2

3. Use your graph to find the increase in earnings when the amount of recycled plastic increases by 10 pounds. Use a complete sentence in your answer. Then write a unit rate that compares the increase in earnings to the increase in the amount of recycled plastic.
4. Explain how you found your answers in Questions 1 through 3. What do you notice about the unit rates? Use complete sentences in your answer.
5. Use the grid below to graph both equations together. Use the bounds and intervals given below. Be sure to label your graph clearly.

Variable quantity	Lower bound	Upper bound	Interval
Material recycled	0	15	1
Earnings	0	6	0.40



Investigate Problem 2



6. How are the graphs for glass and plastic recyclables similar? How are they different? Use complete sentences to explain your reasoning.



7. Just the Math: Slope as a Rate of Change

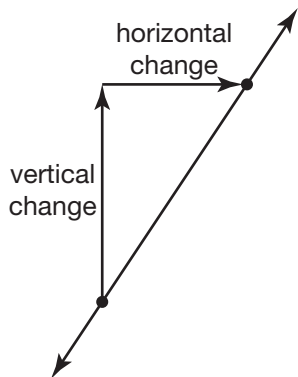
In Problems 1 and 2, you were using your graphs to determine the change in earnings given the change in the amount of recycled material. The change in earnings was represented by a vertical change in the graph, and the change in the amount of recycled material was represented by a horizontal change in the graph. The **slope** of a line is the ratio of the vertical change to the horizontal change.

$$\text{Slope} = \frac{\text{vertical change}}{\text{horizontal change}}$$

When the slope is used to describe a rate of increase (or decrease) in a real-life situation, the slope represents a **rate of change**. You found rates of change in Investigate Problem 1, Questions 1 through 3 and Investigate Problem 2, Questions 1 through 3.

What is the slope of your graph in Problem 1? Write your answer as a unit rate.

What is the slope of your first graph in Problem 2? Write your answer as a unit rate.



8. **Just the Math: Slope as a Ratio** Slope can also be used to represent the ratio a vertical distance to horizontal distance. Suppose that the wheelchair ramp for the community center will have a slope of $\frac{1}{20}$. Draw a picture of the ramp and label the vertical and horizontal distances. Be sure to include units.

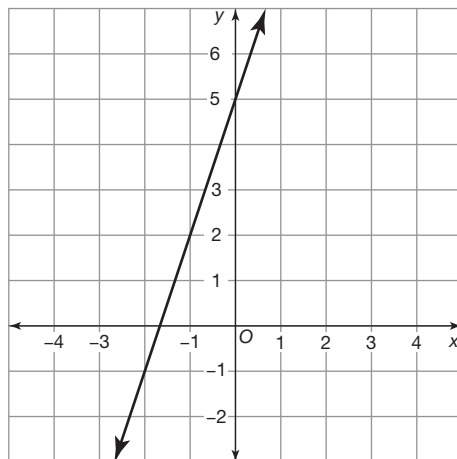
Investigate Problem 2



Take Note

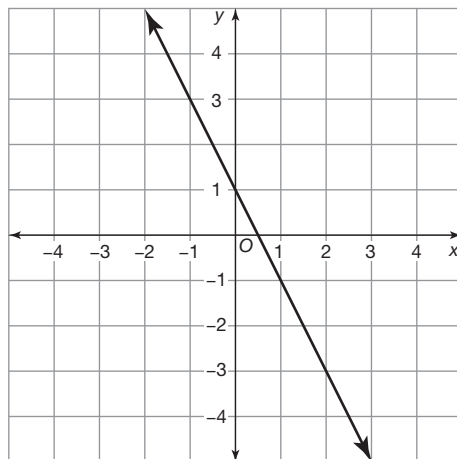
Ratios of the form $\frac{?}{0}$ or $\frac{0}{0}$ are considered to be *undefined* because you cannot divide a number by 0.

9. The slope of a line is either *positive*, *negative*, *zero*, or *undefined*. Determine whether the slope of the line in each graph is positive, negative, zero, or undefined. Complete each sentence to explain.



The vertical change of the line is a _____ number, and the horizontal change is a _____ number.

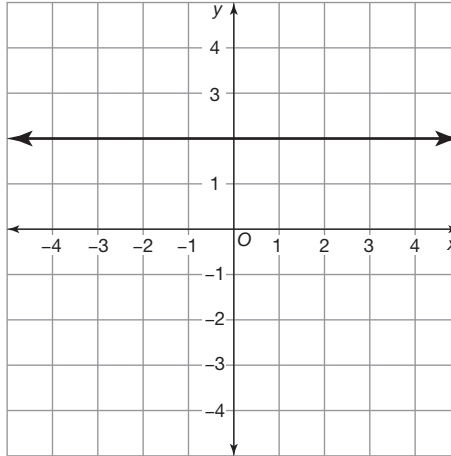
So, the slope, the ratio of the vertical change to the horizontal change, is _____.



The vertical change of the line is a _____ number, and the horizontal change is a _____ number.

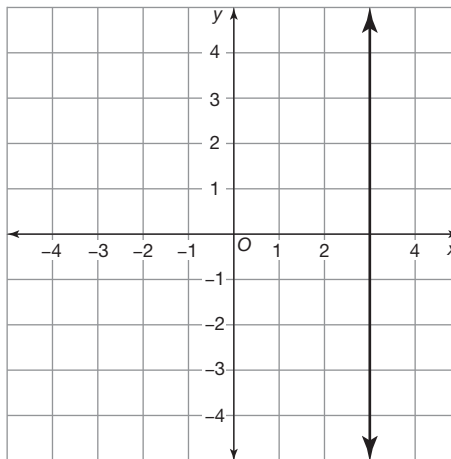
So, the slope, the ratio of the vertical change to the horizontal change, is _____.

Investigate Problem 2



The vertical change of the line is _____, and the horizontal change is a _____ number.

So, the slope, the ratio of the vertical change to the horizontal change, is _____.



The vertical change of the line is a _____ number, and the horizontal change is _____.

So, the slope, the ratio of the vertical change to the horizontal change, is _____.

10. Use complete sentences to describe lines that have positive slope, negative slope, zero slope, or undefined slope.

Investigate Problem 2



- 11. Just the Math: Finding the Slope of a Line Through Two Points** You can use the coordinates of two points to find the slope of the line through the points. The slope m of the line through (x_1, y_1) and (x_2, y_2) is

$$m = \frac{y_2 - y_1}{x_2 - x_1}.$$

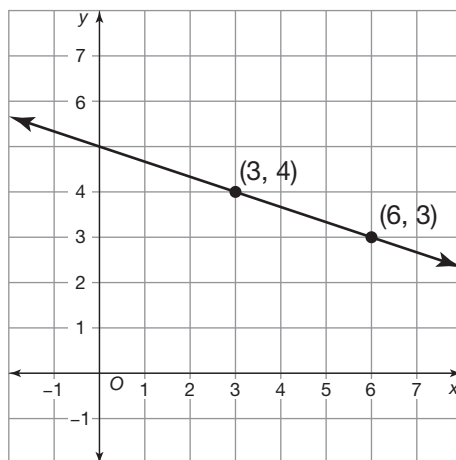
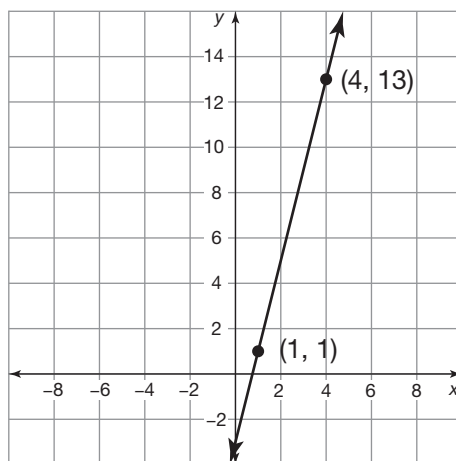
For instance, to find the slope of the line through the points $(2, 6)$ and $(5, 10)$, let the point (x_1, y_1) be the point $(2, 6)$ and let the point (x_2, y_2) be the point $(5, 10)$. Complete the ratio below to find the slope of the line through the points.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\boxed{} - \boxed{}}{\boxed{} - \boxed{}} = \frac{\boxed{}}{\boxed{}}$$

Take Note

Because the letter m is commonly used to represent the slope of a line, you should avoid using this letter as a variable.

- 12.** Use the coordinates of the points to find the slope of each line. Show all your work.



Objectives

In this lesson,
you will:

- Compare slopes and y -intercepts of lines.
- Identify slopes and y -intercepts from equations.
- Write equations in slope-intercept form.
- Graph equations in slope-intercept form.



SCENARIO

Three friends have been training together for months to run in their first marathon. Their program has consisted of walking, jogging, and running, gradually building up to be able to run the 26 miles of the marathon. In the race, the starting times for runners are often different because of the number of people running in the race. The winner is the person with the fastest time.



Problem 1

Going the Distance

The marathon has started, and the first friend is now 8 miles into the race and is currently running at a constant rate of 4.5 miles per hour.

- A. How far will the first friend be into the race one hour after the first eight miles are completed? Show all your work and use a complete sentence in your answer.
- B. How far will the first friend be into the race two hours after the first eight miles are completed? Show all your work and use a complete sentence in your answer.
- C. How far will the first friend be into the race thirty minutes after the first eight miles are completed? Show all your work and use a complete sentence in your answer.
- D. Use complete sentences to explain how to find the distance the friend has run for any amount of time after the first eight miles are completed.

Key Term

- slope-intercept form

Investigate Problem 1

1. Write an equation that gives the total distance that the first friend has run in terms of the number of hours that have passed after the first eight miles were completed. Use x to represent the number of hours that have passed and y to represent the total distance run in miles.

Investigate Problem 1

2. Use your equation to find the number of miles into the race that the first friend will be in 2.5 hours after the first eight miles are completed. Show all your work and use a complete sentence in your answer.

Use your equation to find the number of hours that it will take for the first friend to complete 21.5 miles of the race after the first eight miles are completed. Show all your work and use a complete sentence in your answer.

Problem 2 Where's the Finish Line?



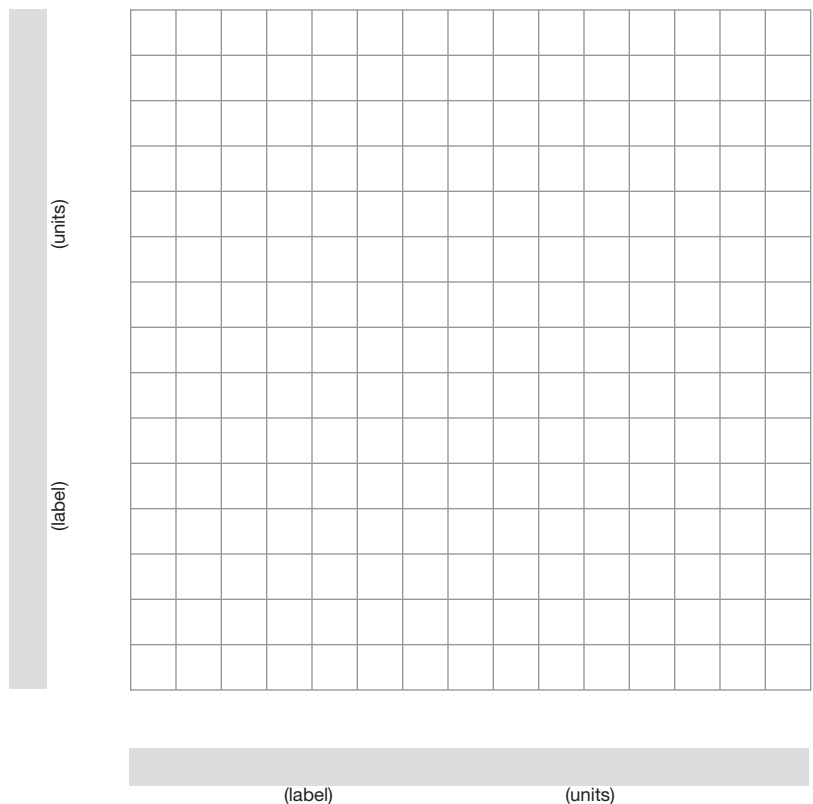
The second friend has completed 6 miles of the marathon and is currently running at a constant rate of 2.5 miles in 0.5 hour.

- A. How many miles is the second friend now running in one hour? Show all your work.
- B. How far will the second friend be into the race 1.5 hours after the first 6 miles are completed? Show all your work and use a complete sentence in your answer.
- C. How far will the second friend be into the race 120 minutes after the first 6 miles are completed? Show all your work and use a complete sentence in your answer.
- D. How far will the second friend be into the race 45 minutes after the first 6 miles are completed? Show all your work and use a complete sentence in your answer.
- E. Write an equation that gives the total distance that the second friend has run in terms of the number of hours that have passed after the first 6 miles were completed. Use x to represent the number of hours that have passed and y to represent the total distance run in miles.

Investigate Problem 2

1. The third friend has completed 5 miles and is currently running at a constant rate of 12 miles in 3 hours. Write an equation that gives the total distance in terms of the number of hours that have passed after the first 5 miles were completed. Use x to represent the number of hours that have passed and y to represent the total distance run in miles. Show all your work and use complete sentences to explain.
2. How are the three equations that you have written the same? How are they different? Use complete sentences in your answer.
3. Use the grid below to create graphs of all three equations. First, choose your bounds and intervals. Be sure that you can see the y -intercepts and the results at the end of the race. Label your graph clearly.

Variable quantity	Lower bound	Upper bound	Interval



Investigate Problem 2

4. For each line, identify the slope and y -intercept. Show all your work.

First friend: slope: _____ y -intercept: _____

Second friend: slope: _____ y -intercept: _____

Third friend: slope: _____ y -intercept: _____

5. Compare your equation for the first friend to the slope and intercepts of the equation's graph. What do you notice? Use a complete sentence in your answer.

Compare your equation for the second friend to the slope and intercepts of the equation's graph. What do you notice? Use a complete sentence in your answer.

Compare your equation for the third friend to the slope and intercepts of the equation's graph. What do you notice? Use a complete sentence in your answer.



6. **Just the Math: Slope-Intercept Form** The linear equations you have been writing so far have been in *slope-intercept form*. A linear equation in **slope-intercept form** is $y = mx + b$ where m is the slope of the line and b is the y -intercept.

Write each equation in slope-intercept form, if necessary. Then identify the slope and y -intercept.

$$y = 3x + 2$$

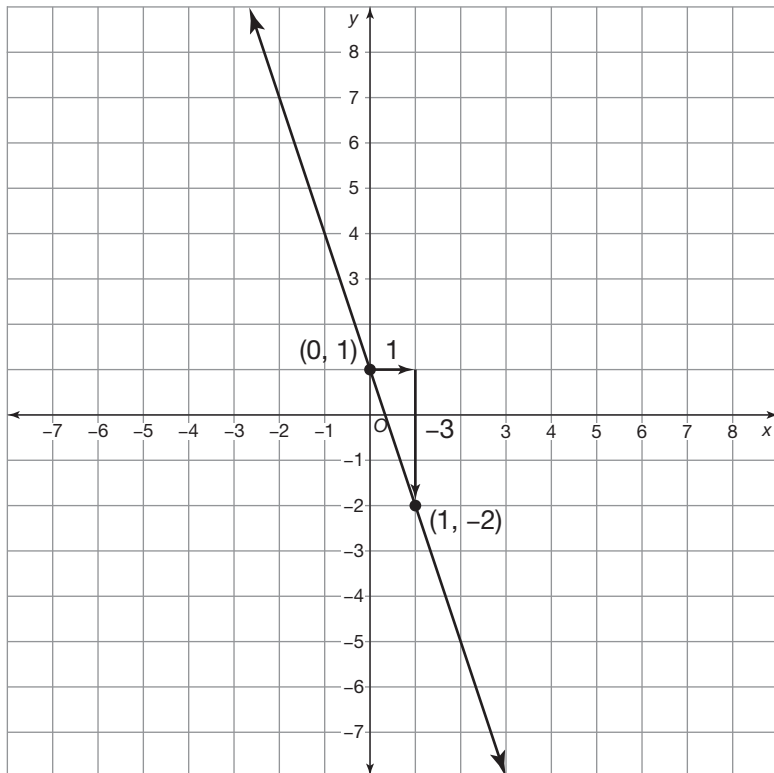
$$y = -0.8x + 1.4$$

$$y = 6x - 10$$

$$y = 4(x + 3)$$

Investigate Problem 2

7. Once you can identify the slope-intercept form of a linear equation, you can quickly draw a graph of the equation. For instance, to graph the equation $y = -3x + 1$, begin by plotting the point that contains the y -intercept, $(0, 1)$. Then write the slope -3 as $\frac{-3}{1}$, which tells you to move down three units for every one unit you move to the right. You can use this slope to plot another point on the line, $(1, -2)$. Then draw a straight line through the points.



Suppose we had interpreted the slope -3 as $\frac{3}{-1}$. What would be the second point on the line?

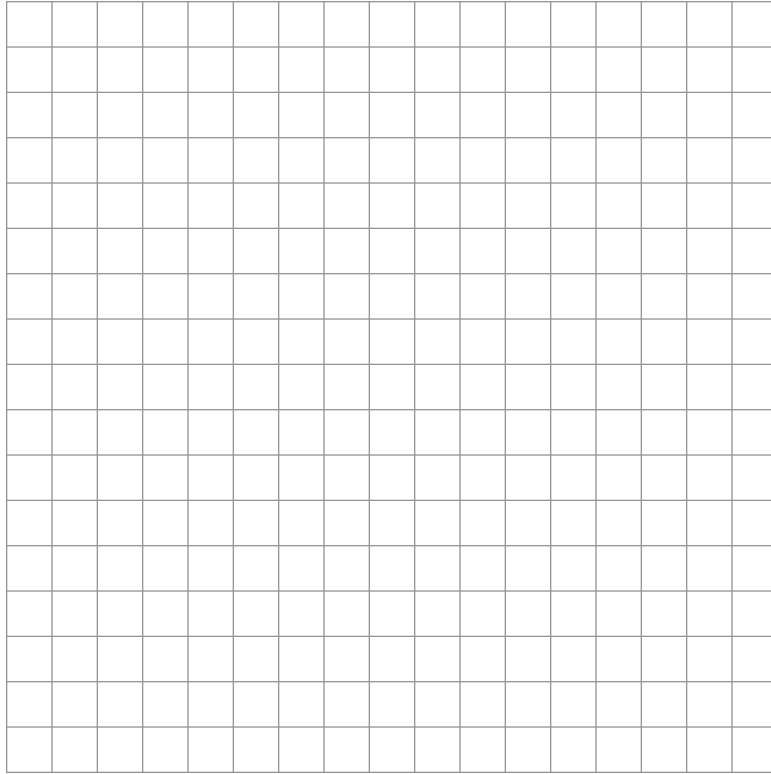
Show that the point $(1, -2)$ and the point you found above are both solutions of the equation $y = -3x + 1$. What does this tell you about the slope of the line? Use complete sentences in your answer.

Investigate Problem 2

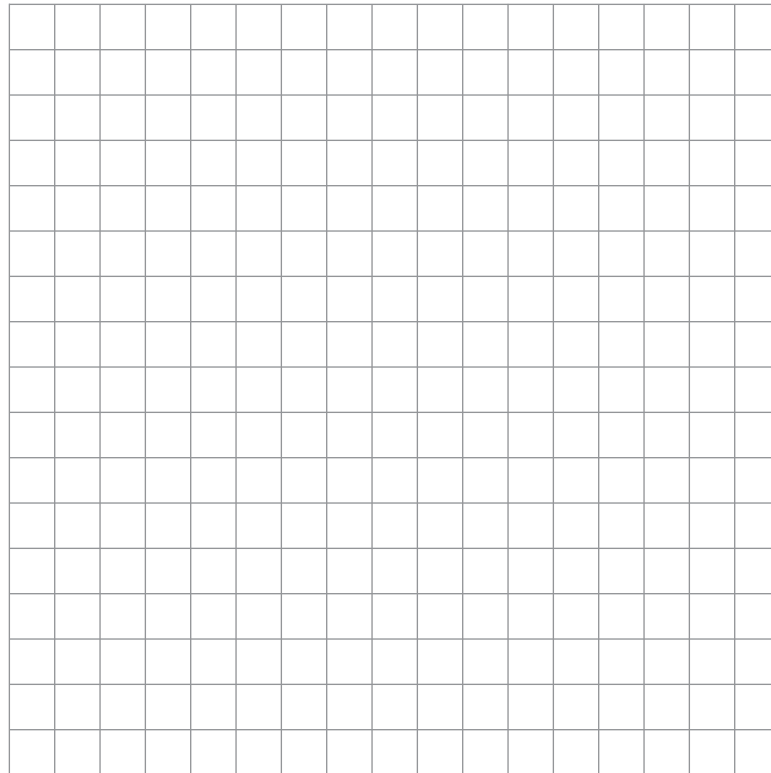


8. Draw a graph of each equation by using the slope and y-intercept of the equation.

$$y = -2x + 3$$

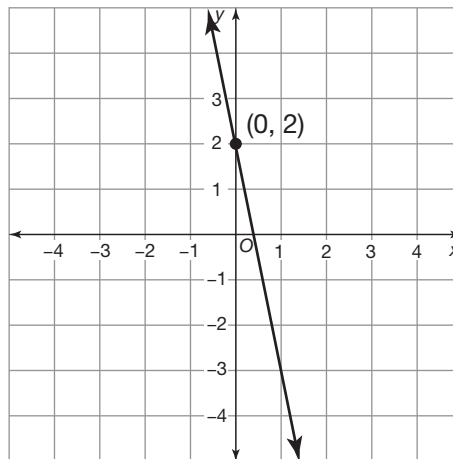
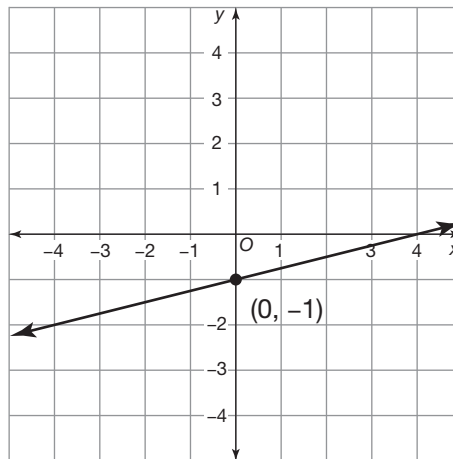


$$y = \frac{2}{3}x - 1$$



Investigate Problem 2

9. Write the equation of the line from its graph.



10. Consider all lines with a positive slope. How does the number that represents the slope of a steep line compare to the number that represents the slope of a line that is less steep? Use a complete sentence to explain.

Consider all lines with a negative slope. How does the number that represents the slope of a steep line compare to the number that represents the slope of a line that is less steep? Use a complete sentence to explain.



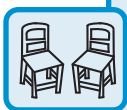
Objectives

In this lesson,
you will:

- Use a point and a slope to write an equation of a line in point-slope form.
- Use a point and a slope to write an equation of a line in slope-intercept form.
- Use two points to write an equation of a line in point-slope form and in slope-intercept form.



SCENARIO Your local community center recently offered a class to young adults about saving money. You took the class and applied what you learned in the class by saving money regularly.



Problem 1

Saving for the Future

You started saving money regularly 6 months ago, and you have \$410 in the account today. You have been meeting your goal of saving an additional \$15 each week.

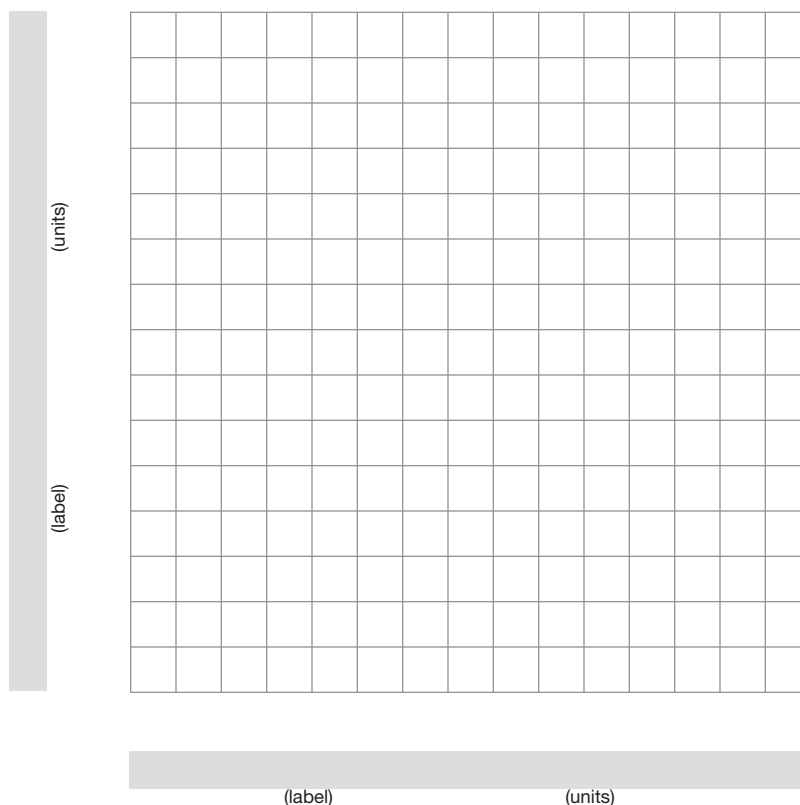
- A. How much money had you saved one week ago? Show your work and use a complete sentence in your answer.
- B. How much money had you saved one month ago? (Assume 4 weeks in a month.) Show your work and use a complete sentence in your answer.
- C. How much money had you saved two months ago? Show your work and use a complete sentence in your answer.
- D. Complete the table below that shows the amount of money that you have saved for different numbers of weeks.

Time since money started to be saved regularly	Amount saved
weeks	dollars
0	
4	
8	
12	
16	
20	
22	
23	
24	

Investigate Problem 1

1. Use the grid below to create a graph of the table in part (D). First, choose your bounds and intervals. Be sure to label your graph clearly.

Variable quantity	Lower bound	Upper bound	Interval



2. What is the slope of the line in your graph? Did you have to use the graph to determine the slope? Use complete sentences to explain your reasoning.
3. What is the y -intercept of your graph? Did you have to use the graph to determine the y -intercept? Use complete sentences to explain your reasoning.
4. Write an equation that gives the savings in terms of time. Use x to represent the time in weeks since you started saving money regularly and use y to represent your savings in dollars.

Investigate Problem 1

5. If you look at the original problem again, what information about the graph were you given? Use a complete sentence in your answer.



6. **Just the Math: Point-Slope Form** Instead of creating a table and a graph of the situation to use to write the equation, you can write the equation directly by using the **point-slope form** of a linear equation. The point-slope form of an equation of the line that passes through the point (x_1, y_1) and has slope m is

$$y - y_1 = m(x - x_1).$$

What point were you given in the problem statement?

What slope were you given in the problem statement?

Write the point-slope form of the equation.

Now write the equation in slope-intercept form. Show all your work.

What do you notice? Use a complete sentence in your answer.

7. What does the y -intercept mean in terms of the problem situation?
8. What are the advantages of each method of finding the equation? What are the disadvantages of each method? Use complete sentences in your answer.

Investigate Problem 1

9. Write the equation of a line that passes through the given point and has the given slope. Then write the equation in slope-intercept form. Show all your work.

Passes through $(-5, 6)$ and has slope of 3

Passes through $(4, 0)$ and has slope of $\frac{1}{2}$

Passes through $(3, -2)$ and has slope of -4

Take Note

You can check your answer by first checking that the given slope and the slope in your equation are the same. Then substitute the ordered pair into your equation and show that it is a solution.

Problem 2 Everybody's Saving!

A friend of yours also took the class and also started saving money regularly. She excitedly tells you that, after only 6 weeks she had \$132 saved, and after 12 weeks she had \$240 saved. She told you that she has been saving the same amount each week.

- A. Do you think that you can generalize this problem situation by using a linear equation? Use complete sentences to explain your reasoning.

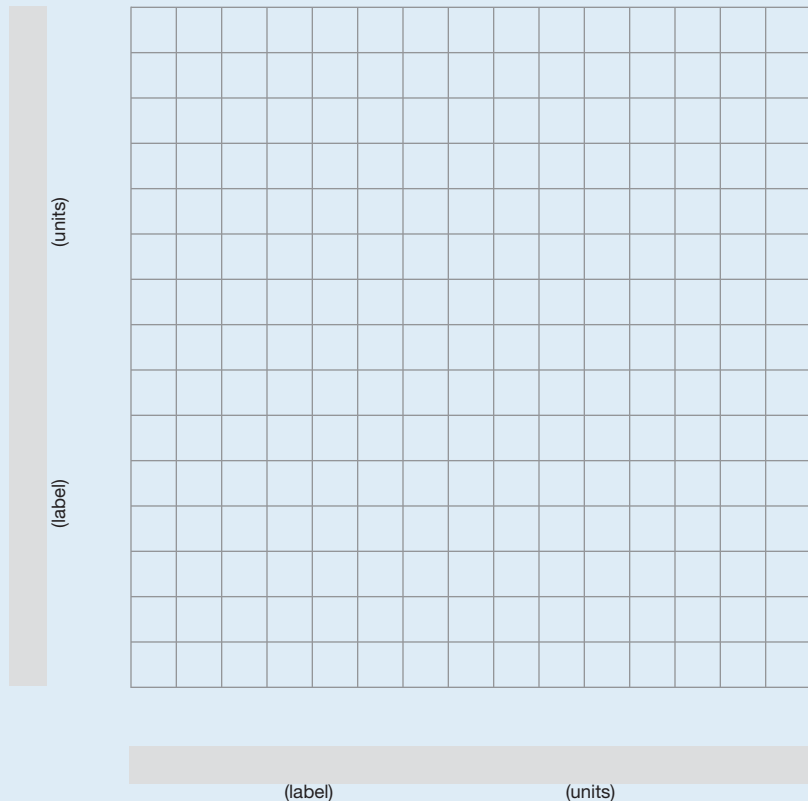
- B. Use the grid on the next page to create a graph with the information you have so far. First, choose your bounds and intervals. Be sure to label your graph clearly.

Variable quantity	Lower bound	Upper bound	Interval



Problem 2

Everybody's Saving!



- C.** How does this problem differ from the previous problem? Use complete sentences to explain.
- D.** What information do you need in order to write an equation that generalizes the problem situation? Use a complete sentence in your answer.

Investigate Problem 2

- 1.** Find the slope of the line that you graphed in part (B). Show all your work.

Investigate Problem 2

- Find an equation of the line in slope-intercept form. Show all your work.
- Could you have used either point to find the equation of the line? Use complete sentences to explain your reasoning.
- Find an equation of the line in slope-intercept form that passes through each given set of points. Show all your work.

$(-2, 2)$ and $(1, 5)$

$(1, 7)$ and $(3, 2)$

$(-3, 1)$ and $(8, 1)$

Take Note

You can check your answer by substituting each ordered pair into your equation. Both ordered pairs should be solutions of your equation.



Summary

Writing an Equation of a Line

To write the equation of a line, you need to know at least two pieces of information:

- The slope and the y -intercept,
- The slope and another point on the line, or
- Two points on the line.

Then, use the slope-intercept form of the equation of a line, $y = mx + b$, to write the equation of the line.

Objectives

In this lesson,
you will:

- Find the domain and range of a linear function.
- Graph a piecewise function.
- Write the equations for a piecewise function.

Key Terms

- domain
- range
- linear function
- piecewise function



SCENARIO As part of the class you took about saving money, you also learned about how to monitor your spending.



Problem 1 I Won?

You won \$48 in a ping-pong tournament. You figure that you will spend an average of \$3 of your winnings each day.

- A. How much money will you have left after three days? Show your work and use a complete sentence in your answer.
- B. How much money will you have left after five days? Show your work and use a complete sentence in your answer.
- C. How much money will you have left after ten days? Show your work and use a complete sentence in your answer.
- D. Complete the table below that shows the amount of money you have left for different numbers of days.

Time since you won	Amount of winnings left
days	dollars
0	
1	
2	
3	
4	
5	

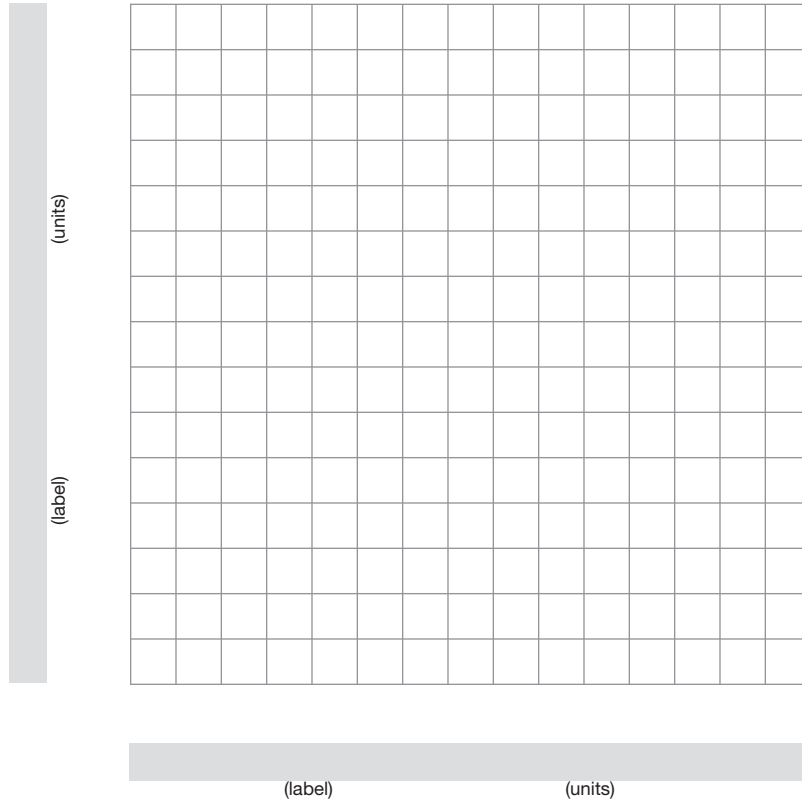
Investigate Problem 1

1. Write an equation that gives the amount of money you have left in terms of the number of days since you started spending your winnings. Be sure to tell what each variable in your equation represents.

Investigate Problem 1

2. Use the grid below to create a graph of your equation. First, choose your bounds and intervals. Be sure to label your graph clearly.

Variable quantity	Lower bound	Upper bound	Interval



3. Find the x and y intercepts of the graph. Use complete sentences to explain what they mean in terms of the problem situation.
4. Is the slope of your line positive or negative? Does this make sense in terms of the problem situation? Use complete sentences to explain your reasoning.

Take Note

Remember that every linear equation, except those in the form $x = a$, is a linear function.

Take Note

Recall that the set of all possible input values of a function is the *domain* of the function and the set of all possible output values is the *range* of the function.

Investigate Problem 1

5. Consider your linear function without considering the problem situation. You can determine the domain of your linear function by using your graph. The x -axis gives you the potential x -values in the domain. For which x -values will you have only one y -value? What is the domain of the function? Use complete sentences to explain.

You can also determine the range by using your graph. The y -axis gives you the potential y -values for the given x -values in the domain. Which y -values are given by x -values from the domain? Use a complete sentence in your answer.

6. What do you think are the domain and range of any linear function of the form $f(x) = mx + b$? Use complete sentences to explain your reasoning.
7. Now consider your linear function again in terms of the problem situation. What is the domain of the linear function in the problem situation? Use complete sentences to explain your reasoning.

What is the range of the linear function in the problem situation? Use complete sentences to explain your reasoning.

Problem 2

Controlling Your Spending

Suppose that you did not spend the \$48 by spending \$3 each day. Instead, after five days of spending \$3 each day, you do not spend anything for five days. Then you reduce your spending to \$1.50 each day.



- A. Complete the table below that shows the amount of money you have left for different numbers of days.

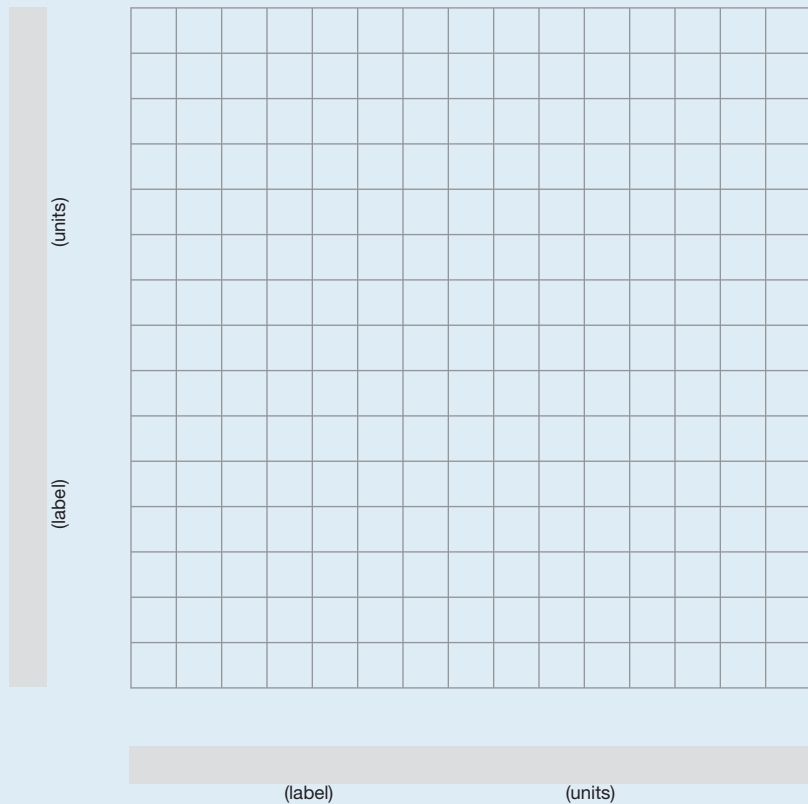
Time since money was won	Amount left
days	dollars
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	

- B. Use the grid on the next page to create a graph of the table in part (A). First, choose your bounds and intervals. Be sure that you are able to see the number of days when you have no money left. Label your graph clearly.

Variable quantity	Lower bound	Upper bound	Interval

Problem 2

Controlling Your Spending



Take Note

The function represented by the graph above can be called a *piecewise linear* function because each piece can be represented by a linear equation.

Investigate Problem 2

- Just the Math: Piecewise Function** The graph that you created in part (B) above represents a *piecewise function*. A **piecewise function** is a function that can be represented by more than one function, each of which corresponds to a part of the domain.

What is the domain of this function in the problem situation?

How many pieces make up this function? In other words, how many different equations are needed to describe this function? Use a complete sentence in your answer.

What is the domain of each piece?

Investigate Problem 2

2. Find the equation that represents the piece of the function from 0 days to 5 days. Show your work and use complete sentences to explain how you found your answer.
3. Find the equation that represents the piece of the function from 6 days to 10 days. Show your work and use complete sentences to explain your reasoning.
4. Find the equation that represents the piece of the function from 11 days to 32 days. Show your work and use complete sentences to explain your reasoning.

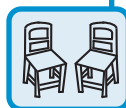


5. Just the Math: Writing a Piecewise Function

You can write your piecewise function in the following way. Use x to represent a number from the domain of your function f . Complete the definition of f below. For each piece of the domain, write the equation you found in Questions 2, 3, or 4.

$$f(x) = \begin{cases} \boxed{}, & 0 \leq x \leq 5 \\ \boxed{}, & 5 < x \leq 10 \\ \boxed{}, & 10 < x \leq 32 \end{cases}$$

6. Which piece should you use to find the x -intercept? Which piece should you use to find the y -intercept? Use complete sentences in your answer.



7. Find the intercepts of the graph. Show all your work.



8. When will you run out of money? How does this compare to the number of days it took you to run out of money in Problem 1?

Objectives

In this lesson,
you will:

- Write a linear equation in standard form.
- Convert an equation in slope-intercept form to standard form.
- Convert an equation in standard form to slope-intercept form.

Key Terms

- slope-intercept form
- standard form
- point-slope form



SCENARIO Your theater group at school is putting on a play next month. They want to advertise, so they have designed a flyer to distribute around the school and in local stores.

Problem 1

Sending Out the Flyers

One member of the group has found an office supply store that sells plain white paper for \$6 per ream (a package of 500 sheets) and blue paper for \$10 per ream. The group agrees that it would be good to get both white and blue paper.

- A. Write an expression that represents the total cost of the paper if the group buys x reams of white paper and y reams of blue paper.

- B. The theater group has a budget of \$30 for the paper. Complete the equation in terms of x and y that represents the amount that the group will spend if they spend the entire budgeted amount.

$$\underline{\hspace{2cm}} = 30$$

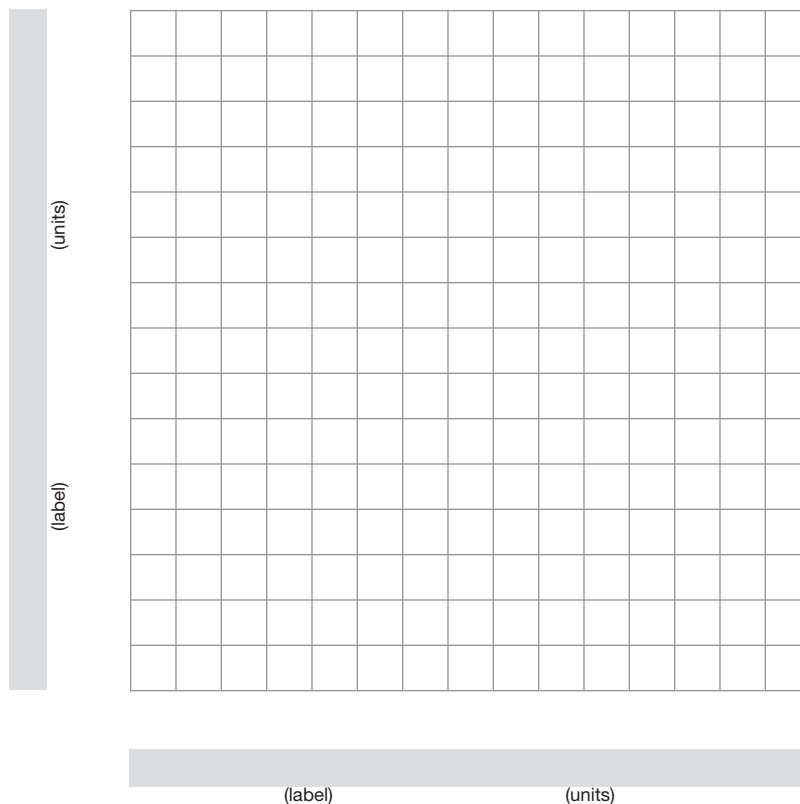
- C. Using the equation, write the intercepts of the equation's graph. Show all your work.

What do the intercepts mean in terms of the problem situation?
Use complete sentences in your answer.

Investigate Problem 1

1. Use the grid below to create a graph of the equation in part (B). First, choose your bounds and intervals. Be sure to label your graph clearly.

Variable quantity	Lower bound	Upper bound	Interval



2. How many reams of white paper can you buy if you buy two reams of blue paper? Show your work and explain your reasoning. Use complete sentences in your answer.
3. How many reams of blue paper can you buy if you buy one ream of white paper? Show your work and explain your reasoning. Use complete sentences in your answer.

Investigate Problem 1

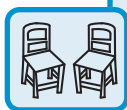


- 4. Just the Math: Standard Form** In part (B), you wrote a linear equation in *standard form*. A linear equation is in **standard form** if it is written as $Ax + By = C$, where A , B , and C are constants and A and B are not both zero. Identify A , B , and C in your equation in part (B).
- 5.** Are there any benefits in using an equation in standard form rather than using an equation in slope-intercept form?

Problem 2

Another Possibility

Another member of the group has found a different source for the paper, an art supply store. This member tells the group that the total cost and the number of reams of each kind of paper bought are related by the equation $y = -\frac{6}{9}x + \frac{30}{9}$, where x is the number of reams of white paper and y is the number of reams of blue paper.



- A.** How many reams of white paper can you buy if you buy two reams of blue paper? Show all your work. Use a complete sentence in your answer.
- B.** How many reams of blue paper can you buy if you buy three reams of white paper? Explain your reasoning. Use complete sentences in your answer.

Problem 2

Another Possibility

- C. Find both intercepts of the equation's graph by using the equation. Show all your work.

What do the intercepts mean in terms of the problem situation?
Use complete sentences in your answer.

5

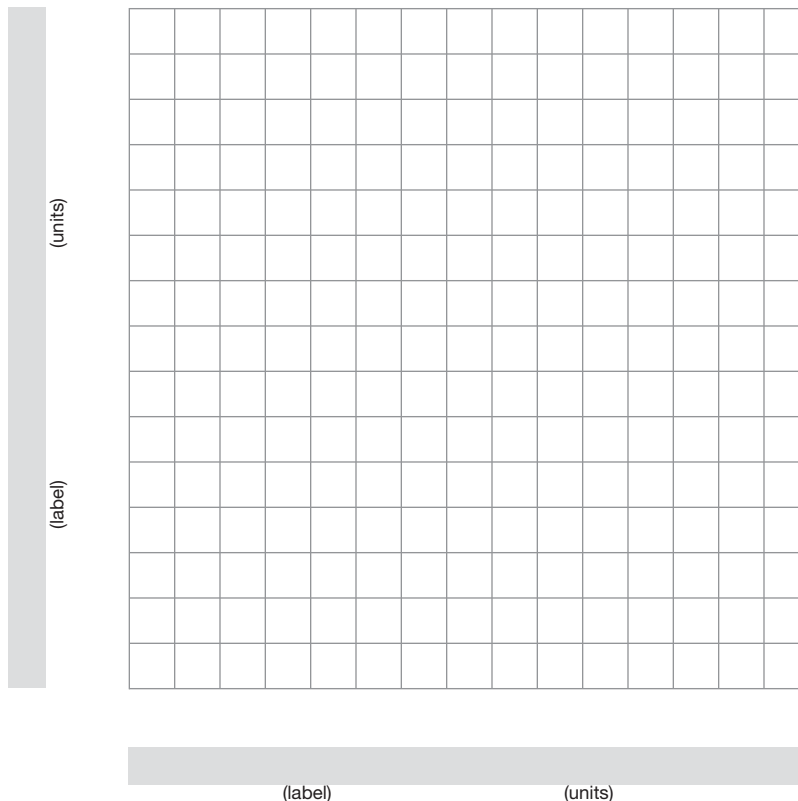
Investigate Problem 2

1. Use the grid on the next page to create a graph of the given equation. First, choose your bounds and intervals. Be sure to label your graph.

Variable quantity	Lower bound	Upper bound	Interval

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Investigate Problem 2



2. Can you tell the cost of one ream of each kind of paper from the information you have so far?

3. You can determine the cost of one ream of each kind of paper by writing your equation in standard form. To write $y = -\frac{6}{9}x + \frac{30}{9}$ in standard form, first clear the terms of fractional coefficients by multiplying each side of the equation by 9. Complete the steps below.

$$9y = 9\left(-\frac{6}{9}x + \frac{30}{9}\right)$$

$$9y = 9\left(\boxed{}\right) + 9\left(\boxed{}\right) \quad \text{Use the distributive property.}$$

$$9y = \boxed{} + \boxed{} \quad \text{Multiply.}$$

The final step is to get both variable expressions on one side of the equation. Write the final equation below.

Take Note

Recall that the coefficient of an expression is the number that is multiplied by a variable. For instance, in the expression $7x$, 7 is the coefficient of x .

Investigate Problem 2

How much does one ream of each kind of paper cost?
Use complete sentences to explain how you found your answer.



4. From which store should the group buy their paper?
Use complete sentences to explain your reasoning.

5. Write each equation in standard form. Be sure to clear the terms of fractional coefficients. Show all your work.

$$y = -6x + 4$$

$$y = \frac{1}{2}x + 5$$

Summary

Linear Equations

In this chapter, you have learned three different forms of a linear equation:

- Slope-intercept form:
 $y = mx + b$,
where m is the slope
and b is the y -intercept
- Point-slope form:
 $y - y_1 = m(x - x_1)$,
where m is the slope and
 (x_1, y_1) is a point on
the line
- Standard form:
 $Ax + By = C$, where A , B ,
and C are constants and
 A and B are not both zero

$$y = \frac{3}{8}x - \frac{1}{8}$$

$$y = -\frac{1}{2}x + \frac{3}{4}$$

6. Write each equation in slope-intercept form. Show all your work.

$$6x + 3y = 12$$

$$4x - 2y = 1$$

$$-3x + 5y = 10$$

$$8x - 10y = 15$$



Objective

In this lesson,
you will:

- Solve an equation for a specified variable.

**SCENARIO**

In Lesson 5.5, we talked about saving money. If you put your money into a savings account, the bank pays you money, called **interest**, for being able to use your money.

Key Terms

- interest
- principal
- simple interest
- interest rate
- literal equation

**Take Note**

Recall that to write a percent as a decimal, you divide the number in front of the percent symbol by 100.

Problem 1**How Interesting!**

The amount of money that you deposit into the account is called the **principal**. **Simple interest** is interest that is paid only as a percent of the principal. The formula for simple interest is $I = Prt$, where I is the amount of interest that you earn in dollars, P is the principal in dollars, r is the **interest rate** (the percent) in decimal form, and t is the amount of time in years that the money is in the account.

- Use a complete sentence to write the simple interest formula in words.
- Find the amount of interest earned in one year by depositing \$100 into an account that earns 2% interest for 1 year. Show all your work and use complete sentences to explain how you found your answer.
- Find the amount of time it will take to earn \$10 in simple interest if you deposit \$100 into an account that earns 2% interest. Show all your work and use complete sentences to explain how you found your answer.

Problem 1

How Interesting!

- D.** Suppose that you have \$200 to put into a savings account and your bank offers a 2.5% annual interest rate. Write a simple interest equation that represents this situation. Then simplify the equation.

If you want to easily determine how long you have to keep your money in the account to earn different amounts of interest, solve your equation for t . What is your new equation?

- E.** How long does the money need to be in the account if you want to earn \$10 in interest? Show your work and use a complete sentence in your answer.

How long does the money need to be in the account if you want to earn \$25 in interest? Show your work and use a complete sentence in your answer.

- F.** Is your equation a function? Use a complete sentence to explain.
- G.** Identify the independent variable and the dependent variable in your equation.

Investigate Problem 1

- 1. Just the Math: Literal Equations** The equation for simple interest, $I = Prt$, contains two or more variables to represent known quantities, so it is a **literal equation**. The formula for the area of a rectangle, $A = \ell w$, is also a literal equation. From the equation, upon what two quantities does the area of a rectangle depend? Write your answer using a complete sentence.

Suppose that you know that the area of a rectangle is 39 square feet and the length of the rectangle is 13 square feet. Use the formula for the area of a rectangle, $A = \ell w$, to find the width. Use a complete sentence to explain your reasoning.

Investigate Problem 1

In the same way that you solved the equation for w after you substituted the values, you can solve the equation $A = \ell w$ for w before you substitute. Solve the formula for the area of a rectangle, $A = \ell w$, for w . Show all your work.

Use a complete sentence to express the width of the rectangle in terms of the area and the length.

2. The formula to find the distance that you travel when you know the rate at which you are traveling and the time you spent traveling is $D = rt$, where D is the distance, r is the rate, and t is the time. Solve the equation for r . Show all your work. Then use a complete sentence to explain how you can find the rate when you know the distance and the time.
3. The formula that you can use to convert a temperature in degrees Celsius to degrees Fahrenheit is $F = \frac{9}{5}C + 32$. Solve the equation for C . Show all your work.
4. The formula that you can use to find the area of a trapezoid is $A = \frac{1}{2}h(b_1 + b_2)$ where h is the height of the trapezoid and b_1 and b_2 are the lengths of the two bases of the trapezoid. Solve the equation for b_2 . Show all your work.

