Looking Ahead to Chapter 7

7

Focus In Chapter 7, you will learn to write, graph, and solve systems of equations both graphically and algebraically. You will also learn how to write, graph, and solve systems of linear inequalities, as well as identify solutions to linear inequalities.

Chapter Warm-up

Answer these questions to help you review skills that you will need in Chapter 7.

Use the distributive property to simplify each expression.

1. $3(5x + 7)$

2. $-2(10 + 4y)$

3. $-\frac{1}{6}(3x - 12)$

Write each linear equation in standard form.

4. $y = -4x + 7$

5. $2y - 4 = 3x + 23$

6. $-8y = \frac{3}{5}x + \frac{1}{5}$

Read the problem scenario below.

A bicycle company is trying to determine the number of bikes that they have sold. The company began in 1995. In the year 1997, the company sold a total of 285 bikes, and in the year 2000, the company sold a total of 684 bikes. Assume that the number of bikes sold is a linear function of the time in years since 1995.

7. Find the linear function that describes the total number of bikes sold as a function of the time in years since 1995.

8. Use the linear function to find the total number of bikes sold in 2010.

Key Terms

income ■ p. 299
profit ■ p. 299
point of intersection ■ p. 306, 309
break-even point ■ p. 306
system of linear equations ■ p. 309
linear system ■ p. 309
solution ■ p. 309
parallel lines ■ p. 311
perpendicular lines ■ p. 314
reciprocals ■ p. 314
standard form of a linear equation ■ p. 315
substitution method ■ p. 317
linear combinations method ■ p. 326
linear combination ■ p. 326
inequality ■ p. 332, 345
linear inequality in two variables ■ p. 345
inequality symbol ■ p. 345
linear equation ■ p. 347
coordinate plane ■ p. 347
half-plane ■ p. 347
system of linear inequalities ■ p. 353

CHAPTER

7

Systems of Equations and Inequalities

The earliest known bricks were made of mud. Today, most bricks are made of clay or ground shale. In Lesson 7.2, you will compare the number of bricks that can be laid by a novice bricklayer and an experienced bricklayer.

7.1 Making and Selling Markers and T-Shirts
Using a Graph to Solve a Linear System ■ p. 299

7.2 Time Study
Graphs and Solutions of Linear Systems ■ p. 307

7.3 Hiking Trip
Using Substitution to Solve a Linear System ■ p. 315

7.4 Basketball Tournament
Using Linear Combinations to Solve a Linear System ■ p. 323

7.5 Finding the Better Paying Job
Using the Best Method to Solve a Linear System, Part 1 ■ p. 329

7.6 World Oil: Supply and Demand
Using the Best Method to Solve a Linear System, Part 2 ■ p. 333

7.7 Picking the Better Option
Solving Linear Systems ■ p. 339

7.8 Video Arcade
Writing and Graphing an Inequality in Two Variables ■ p. 345

7.9 Making a Mosaic
Solving Systems of Linear Inequalities ■ p. 351

7

7.1 Making and Selling Markers and T-Shirts

Using a Graph to Solve a Linear System

Objectives

In this lesson, you will:

- Analyze cost and income equations.
- Graph cost and income equations on the same graph.
- Find the break-even point graphically.

Key Terms

- income
- profit
- point of intersection
- break-even point

SCENARIO You have a part-time job at a company that makes and sells color art markers. As part of your job, you are studying the company's production costs. The markers are made one color at a time. It costs $2 to manufacture each marker and there is a $100 set-up cost for each color. You are also studying the **income,** or the amount of money that the company earns, from the sales of the markers. The company sells the markers to office and art supply stores for $3 per marker.

Problem 1 Making and Selling Markers

A. Write an equation that gives the production cost in dollars to make one color of marker in terms of the number of markers produced. Be sure to describe what your variables represent. Use a complete sentence in your answer.

B. Write an equation that gives the income in dollars in terms of the number of markers sold. Be sure to describe what your variables represent. Use a complete sentence in your answer.

C. Find the production cost to make 80 markers of the same color. Show all your work and use a complete sentence in your answer.

Find the income from selling the 80 markers that you made. Show all your work and use a complete sentence in your answer.

Find the profit from the sale of the 80 markers that you made. Show all your work and use a complete sentence in your answer.

Take Note

Remember that the **profit** is the amount of money that is left from sales (income) after the production costs are subtracted.

Problem 1 Making and Selling Markers

D. Find the production cost to make 100 markers of the same color. Show all your work and use a complete sentence in your answer.

Find the income from selling the 100 markers that you made. Show all your work and use a complete sentence in your answer.

Find the profit if 100 markers are made and sold. Show all your work and use a complete sentence in your answer.

Investigate Problem 1

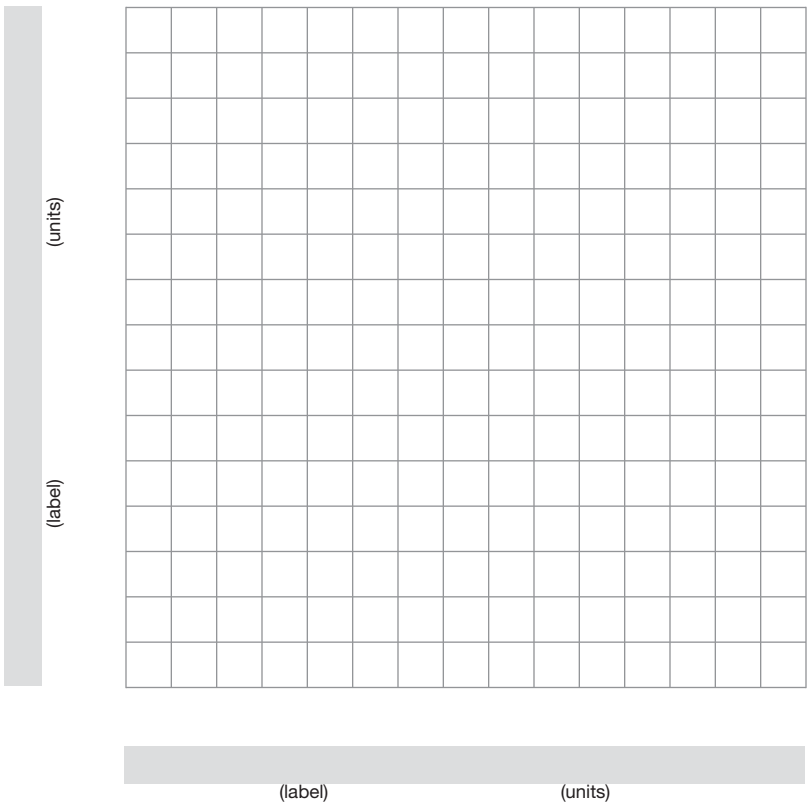
1. Complete the table of values that shows the production cost and income for different numbers of markers of the same color.

Quantity Name	Number of markers	Product cost	Income
Unit	markers	dollars	dollars
Expression	x		
	0		
	20		
	30		
	35		
	55		
	125		
	200		
	400		

Investigate Problem 1

2. Create a graph of both the production cost and income equations on the grid below. Use the bounds and intervals below. Be sure to label your graph clearly.

Variable quantity	Lower bound	Upper bound	Interval
Markers	0	150	10
Money	0	450	30



3. Use your graph to determine the numbers of markers for which the production cost is greater than the income. Use a complete sentence in your answer.

Use complete sentences to explain how you found your answer.

Investigate Problem 1

4. Use your graph to determine the numbers of markers for which the income is greater than the production cost. Use a complete sentence in your answer.

Use complete sentences to explain how you found your answer.

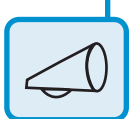
5. Use your graph to determine the number of markers for which the income is equal to the production cost. Use a complete sentence in your answer.

Use complete sentences to explain how you found your answer.

6. Describe the numbers of markers that must be sold in order for your profit to be at least $0. Use complete sentences to explain how you found your answer.

Take Note

Whenever you see the share with the class icon, your group should prepare a short presentation to share with the class that describes how you solved the problem. Be prepared to ask questions during other groups' presentations and to answer questions during your presentation.



Problem 2 Making and Selling T-Shirts

Your work at the marker company has inspired you to start your own business. You decide to design and sell customized T-shirts. The company that supplies your T-shirts charges you $7.50 for each T-shirt and a set-up cost of $22.50 for a new design. You decide to sell the T-shirts for $8.25 each.

A. Write an equation that gives the production cost in dollars to make one design of T-shirt in terms of the number of T-shirts made. Be sure to describe what your variables represent. Use a complete sentence in your answer.

B. Write an equation that gives the income (the amount of money that you earn) in dollars in terms of the number of T-shirts sold. Be sure to describe what your variables represent. Use a complete sentence in your answer.

C. Find the production cost to make 15 T-shirts in the same design. Show all your work and use a complete sentence in your answer.

Find the income from selling the 15 T-shirts that you made. Show all your work and use a complete sentence in your answer.

Find the profit from the sale of the 15 T-shirts that you made. Show all your work and use a complete sentence in your answer.

D. Find the production cost to make 30 T-shirts in the same design. Show all your work and use a complete sentence in your answer.

Find the income from selling the 30 T-shirts that you made. Show all your work and use a complete sentence in your answer.

Find the profit if 30 T-shirts are made and sold. Show all your work and use a complete sentence in your answer.

Take Note

Remember that the profit is the amount of money that is left from sales (income) after the production costs are subtracted.

Investigate Problem 2

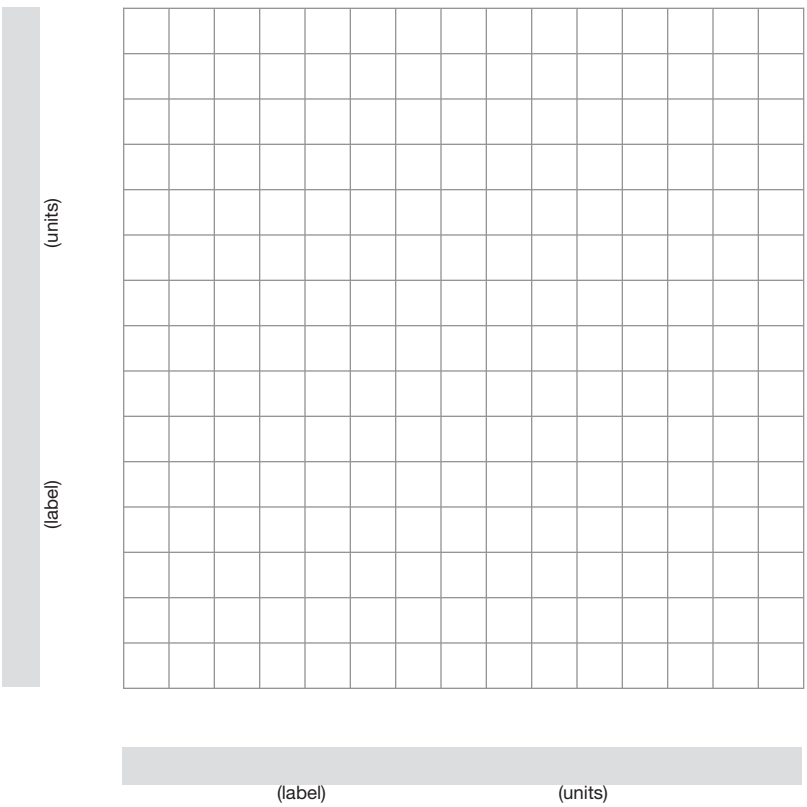
1. Complete the table of values on the next page that shows the production cost and income for different numbers of T-shirts in the same design.

Investigate Problem 2

Quantity Name	Number of T-shirts	Product cost	Income
Unit	T-shirts	dollars	dollars
Expression	x		
	0		
	20		
	25		
	30		
	100		
	200		
	400		

2. Create a graph of both the production cost and income equations on the grid below. Use the bounds and intervals below. Be sure to label your graph clearly.

Variable quantity	Lower bound	Upper bound	Interval
T-shirts	0	45	3
Money	0	375	25



Investigate Problem 2

3. Use your graph to determine the numbers of T-shirts for which the production cost is greater than the income. Use a complete sentence in your answer.

Use complete sentences to explain how you found your answer.

4. Use your graph to determine the numbers of T-shirts for which the income is greater than the production cost. Use a complete sentence in your answer.

Use complete sentences to explain how you found your answer.

5. Use your graph to determine the number of T-shirts for which the income is equal to the production cost. Use a complete sentence in your answer.

Use complete sentences to explain how you found your answer.

Investigate Problem 2

6. Describe the numbers of T-shirts that must be sold in order for your profit to be at least $0. Use complete sentences to explain how you found your answer.

7. **Just the Math: Break-Even Point** When two graphs cross (or intersect) each other, the point where they cross is called a **point of intersection.** When one line represents the production cost of an item and the other line represents the income from selling the item, the *x*-coordinate of this point is called the break-even point. What is the **break-even point** for making and selling markers? Use a complete sentence in your answer.

What is the company's profit at the break-even point? Show all your work and use a complete sentence in your answer.

What is the break-even point for making and selling T-shirts? Use a complete sentence in your answer.

What is your profit from the T-shirts at the break-even point? Show your work and use a complete sentence in your answer.

7.2 Time Study

Graphs and Solutions of Linear Systems

Objectives

In this lesson, you will:

- Determine the number of solutions of a linear system.
- Identify parallel and perpendicular lines.

Key Terms

- system of linear equations
- linear system
- solution
- point of intersection
- parallel lines
- perpendicular lines
- reciprocals

SCENARIO A process engineer is performing a time study on a construction site. As part of the study, the work rates of a novice (beginner) bricklayer and a more experienced bricklayer are being recorded. At the beginning of the study, the novice had put 1510 bricks into place and was setting the bricks in place at a rate of thirty eight bricks per hour. The experienced worker started the job after the novice and had put 960 bricks into place so far and was setting the bricks in place at a rate of sixty bricks per hour.

Problem 1 The Novice and the Pro

A. For each worker, write an equation that gives the total number of bricks y set in place in terms of the time x in hours after the beginning of the time study.

B. After eight hours of the time study, how many bricks in all will each worker have set into place? Show all your work and use complete sentences in your answer.

Which worker has set more bricks into place after eight hours of the time study? Use a complete sentence in your answer.

C. After forty hours of the time study, how many bricks in all will each worker have set into place? Show all your work and use complete sentences in your answer.

Which worker has set more bricks into place after forty hours of the time study? Use a complete sentence in your answer.

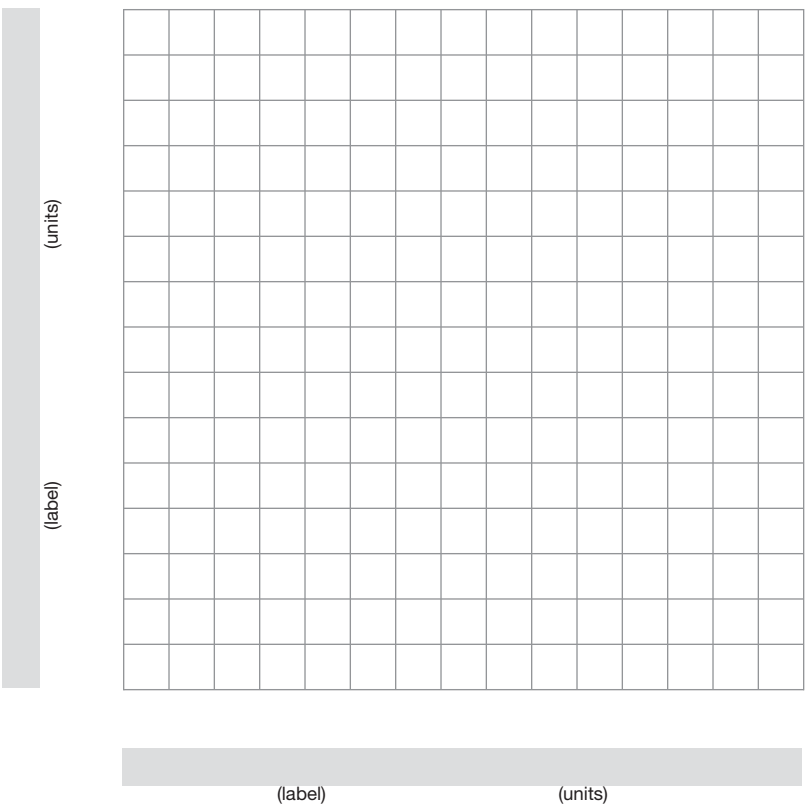
Problem 1 The Novice and the Pro

D. Find the number of hours that the time study would need to run in order for each worker to set a total of 2460 bricks. Show all your work and use complete sentences in your answer.

Investigate Problem 1

1. Create a graph of both equations on the grid below. First, choose your bounds and intervals. Be sure to label your graph clearly.

Variable quantity	Lower bound	Upper bound	Interval



2. Find the amount of time that it will take in the time study for the number of bricks set by each worker to be the same. Use a complete sentence to explain how you found your answer.

Investigate Problem 1

3. What does the slope of each line represent in this problem situation? Use a complete sentence in your answer.

Which worker sets bricks faster? How do you know? Use a complete sentence in your answer.

4. What does the *y*-intercept of each line represent in this problem situation? Use a complete sentence in your answer.

How do the *y*-intercepts of the lines compare? What does this mean in the problem situation? Use complete sentences in your answer.

5. Just the Math: Systems of Linear Equations

In this lesson and in Lesson 7.1, you considered the graphs of two linear equations together. When you do this, you form a **system of linear equations** or a **linear system.** Write the linear system represented by the graph in Problem 1.

6. Just the Math: Solution of a Linear System

The **solution** of a linear system is an ordered pair (*x*, *y*) that is a solution to *both* equations in the system. Graphically, the solution is the **point of intersection** of the system. What is the solution of the linear system in this problem situation? Use your graph to help you. Write your answer using a complete sentence.

Algebraically, verify that the ordered pair is a solution of your system. Remember that the ordered pair needs to be a solution of both equations.

Take Note

Recall that to algebraically verify that an ordered pair is a solution of an equation, substitute the values given by the ordered pair for *x* and *y* in the equation. These values should give you a true statement.

Problem 2 The Pros

A. Another experienced bricklayer is having her time recorded as a part of the time study. At the beginning of the study, this worker had set 600 bricks so far and can set 60 bricks in one hour. Write an equation that gives the total number of bricks y set in place in terms of the time x in hours after the beginning of the time study.

B. Write a linear system that shows the total number of bricks set in terms of time for both experienced workers.

C. Create a graph of the linear system on the grid below. First, choose your bounds and intervals. Be sure to label your graph clearly.

Variable quantity	Lower bound	Upper bound	Interval

(units)

(label)

(label) (units)

Investigate Problem 2

1. What does the slope of each line represent in this problem situation? Use a complete sentence in your answer.

   Which worker sets bricks faster? How do you know? Use a complete sentence in your answer.

2. What does the *y*-intercept of each line represent in this problem situation? Use a complete sentence in your answer.

   How do the *y*-intercepts of the lines compare? What does this mean in the problem situation? Use complete sentences in your answer.

3. Does there appear to be any point of intersection of the lines?

4. Use complete sentences to describe how the lines are related to each other.

5. **Just the Math: Parallel Lines** The lines that you graphed in part (C) are *parallel lines*. Two lines in the same plane are **parallel** to each other if they do not intersect. What can you conclude about the slopes of parallel lines? Use a complete sentence in your answer.

6. Does the linear system for the two experienced workers have a solution? Use complete sentences to explain your reasoning.

7. Will the two experienced workers ever set the same number of bricks during the time study? Use complete sentences to explain your reasoning.

Investigate Problem 2

8. Just the Math: Number of Solutions of a Linear System So far in this lesson, we have seen a linear system with one solution and a linear system with no solution. Use complete sentences to describe the graphs of these kinds of linear systems.

Consider the following linear system:

$y = 2x - 4$ and $y = -2(2 - x)$.

Complete the table of values for this linear system.

Expression

x	$2x - 4$	$-2(2 - x)$
–5		
0		
5		
10		
12		
15		

What can you conclude about the number of solutions of this linear system? Use a complete sentence in your answer.

Because every point on the graph of $y = 2x - 4$ is on the graph of $y = -2(2 - x)$, we can say that this system has an *infinite number* of solutions. Use a complete sentence to explain why you think this is true.

Problem 3 When Is the Job Done?

A. The experienced bricklayer who sets bricks at a rate of 60 bricks per hour and has set 960 bricks so far must set approximately 20,000 additional bricks before the job is done. Write an equation that gives the total number of bricks y left to set in terms of the time x in hours after the beginning of the time study.

Problem 3 When Is the Job Done?

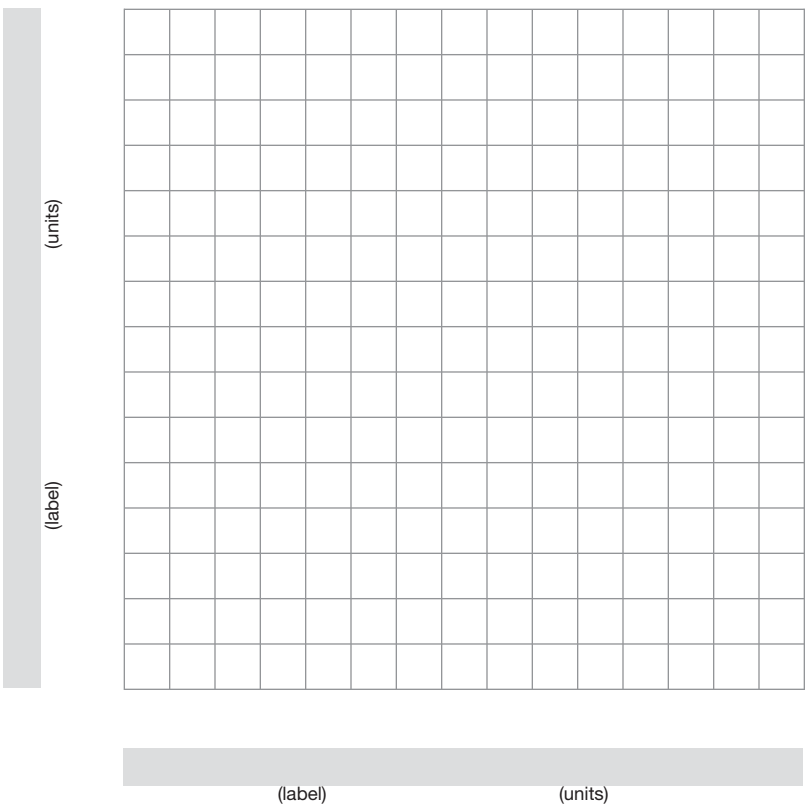
B. Form a linear system with the equation in part (A) and the equation from Problem 1, part (A) that gives the total number of bricks set by this worker in terms of the time after the beginning of the time study.

7

Investigate Problem 3

1. Create a graph of the linear system on the grid below. First, choose your bounds and intervals. Be sure to label your graph clearly

Variable quantity	Lower bound	Upper bound	Interval



2. What does the point of intersection of the lines represent?

3. Compare the slopes of the lines. Use a complete sentence in your answer.

Investigate Problem 3

4. Are the lines *perpendicular*? That is, do they intersect at a right angle?

5. Consider the graph of your linear system and the equations of the lines. What do you notice about the slopes of perpendicular lines?

Take Note

Two numbers are **reciprocals** if their product is 1.

6. Just the Math: Perpendicular Lines A property of **perpendicular lines** is that the product of their slopes must be –1. So, this means that the slopes must have opposite signs and must be **reciprocals** of each other. For instance, the lines $y = -3x + 4$ and $y = \frac{1}{3}x + 1$ are perpendicular because $-3\left(\frac{1}{3}\right) = -1$. Algebraically show that the lines in your graph in Question 1 are *not* perpendicular. Show your work.

7. Determine whether the graphs of each pair of equations are parallel, perpendicular, or neither. Show your work and use a complete sentence to explain your reasoning.

$y = \frac{2}{3}x + 4$ and $y = -\frac{3}{2}x + 1$

$y = 5x - 4$ and $y = -5x + 4$

$y = 4x$ and $y = \frac{1}{4}x - 2$

$y = -1.8x + 15$ and $y = 6 - 1.8x$

7.3 Hiking Trip

Using Substitution to Solve a Linear System

Objective

In this lesson, you will:

- Solve linear systems by using substitution.

Key Terms

- standard form of a linear equation
- substitution method

Take Note

A linear equation is in **standard form** if it is written as $Ax + By = C$, where A, B, and C are constants and A and B are not both zero.

SCENARIO The Outdoor Club at school is going on a hiking trip and is making trail mix as part of the food that they will take. The trail mix will be made up of nuts and dried fruits, such as raisins, dried cherries, and banana chips. The nuts cost $4.50 per pound and the dried fruits cost $3.25 per pound. The group can spend $15 on the trail mix.

Problem 1 Making Trail Mix

A. Write an equation in standard form that relates the numbers of pounds of nuts and dried fruits that can be bought for $15. Use x to represent the number of pounds of nuts and y to represent the number of pounds of dried fruits that can be bought.

B. The group agreed to have one and a half times as much dried fruits as nuts in the mix. Write an equation in x and y as defined in part (A) that represents this situation.

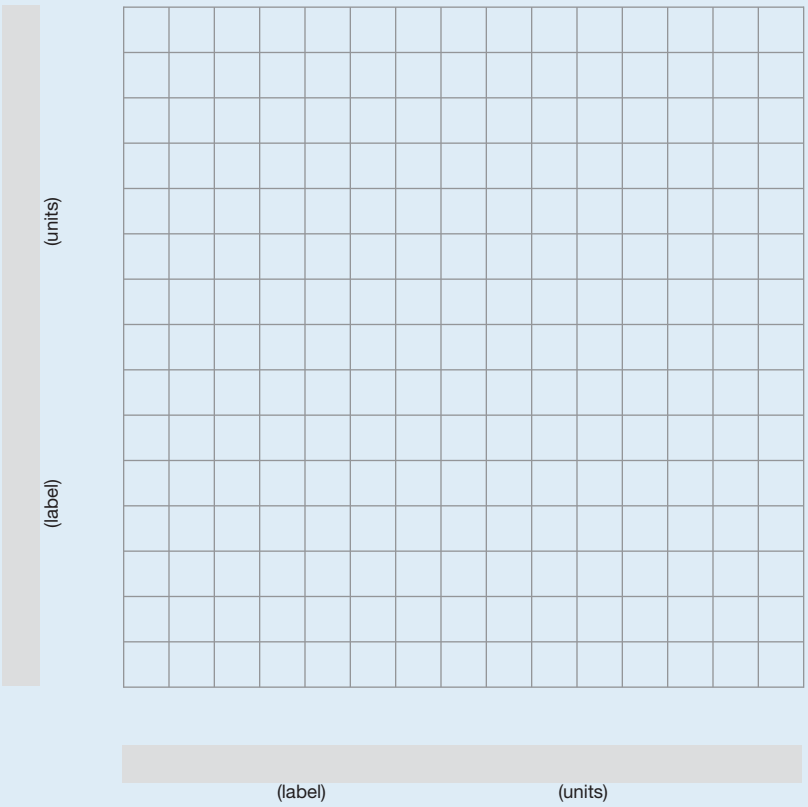
C. Will two pounds of nuts and three pounds of dried fruits satisfy both of your equations? Show all your work.

D. Will two and one quarter pounds of nuts and one and a half pounds of dried fruits satisfy both of your equations? Show all your work.

Problem 1 Making Trail Mix

E. Create a graph of both equations on the grid below. First, choose your bounds and intervals. Be sure to label your graph clearly.

Variable quantity	Lower bound	Upper bound	Interval



F. Can you determine the solution of this linear system exactly from your graph? Use a complete sentence to explain your answer.

Investigate Problem 1

1. Estimate the point of intersection from your graph.

Check your point in each of the equations. Is your point the solution of the linear system?

2. Just the Math: Substitution Method In many systems it is difficult to determine the solution from the graph, so there is an algebraic method for finding the solution. Consider the linear system for this problem situation:

$4.50x + 3.25y = 15$

$y = 1.5x.$

Because y is equal to $1.5x$, we can substitute $1.5x$ for y in the first equation.

$4.50x + 3.25y = 15$

$4.50x + 3.25(1.5x) = 15$

You now have an equation in x only. Solve this equation for x. Show all your work.

Take Note

It does not matter which equation from the linear system that you use to find the value of y. You could have used the equation $4.50x + 3.25y = 15$ to find the value of y.

Now that you have the x-value of the solution, find the y-value by substituting your result for x into the equation $y = 1.5x$. Show all your work.

So, the solution to the linear system is (1.6, 2.4). Is this solution confirmed by your graph?

3. Interpret the solution of the linear system in the problem situation. Use a complete sentence in your answer.

Investigate Problem 1

4. How many pounds of trail mix will the club have? Use a complete sentence to explain how you found your answer.

Problem 2 Hiking the Trail

The Outdoor Club splits up into two smaller groups to hike the trail. The first group leaves the beginning of the trail and hikes at a rate of 2.5 miles per hour. The second group leaves 30 minutes later and hikes at a rate of 2.5 miles per hour.

A. Write an equation for the first group that gives the distance hiked y in miles in terms of the amount of time x in hours that the group has been hiking.

B. How far will the first group have traveled after 30 minutes of hiking? Show your work and use a complete sentence in your answer.

C. Write an equation for the second group that gives the distance hiked y in miles in terms of the amount of time since the first group started hiking x.

D. How far will each group have traveled 45 minutes after the first group started hiking? Show all your work and use a complete sentence in your answer.

How far will each group have traveled after 2 hours? Show all your work and use a complete sentence in your answer.

E. Will the second group catch up to the first group? Use complete sentences to explain your reasoning.

Investigate Problem 2

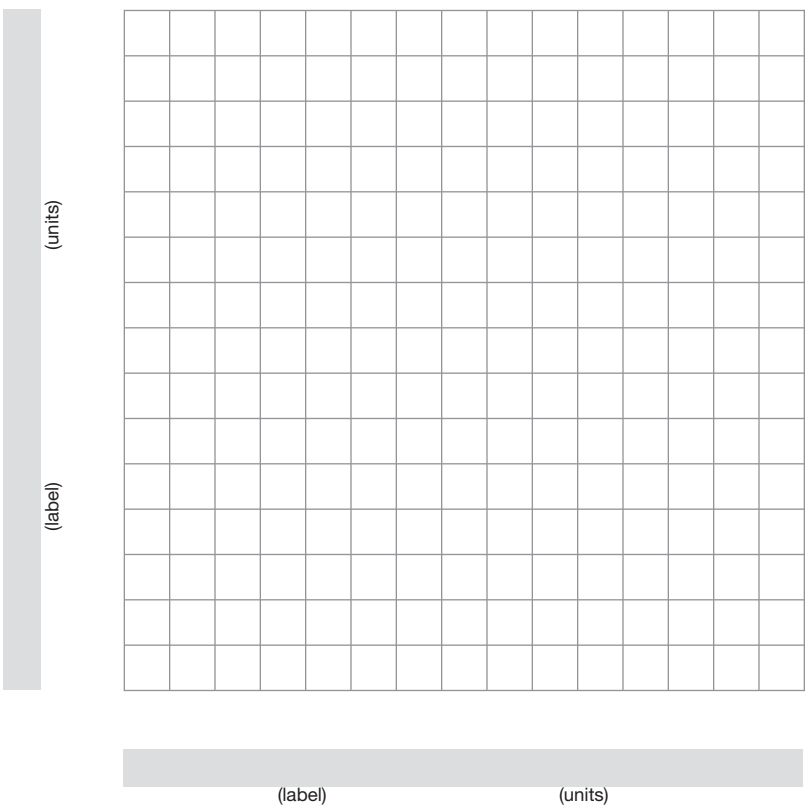
1. Solve the linear system by using the substitution method. First, write your system below.

Next, because you have an expression for y in terms of x, substitute your expression for y from the first equation into the second equation.

Now solve the equation for x. What is the result? Use a complete sentence in your answer.

2. Create a graph of your linear system on the grid below. First, choose your bounds and intervals. Be sure to label your graph clearly.

Variable quantity	Lower bound	Upper bound	Interval



Investigate Problem 2

3. What is the relationship between the lines in the graph? Use a complete sentence in your answer.

4. What is the solution of the linear system? Use a complete sentence in your answer.

5. What is the result when you try to algebraically solve a linear system that has no solution? Use a complete sentence in your answer.

Problem 3 Camping

Another community group joins the Outdoor Club at the campsite. The new group has rented six tents and twenty four sleeping bags for $186. The Outdoor Club rented from the same place and rented eight tents and thirty sleeping bags for $236. Each tent costs the same, and each sleeping bag costs the same.

A. For each group, write an equation in standard form for this problem situation. Use x to represent the cost of one tent in dollars and use y to represent the cost of one sleeping bag in dollars.

B. Without solving the linear system, interpret the solution of the linear system in part (A). Use a complete sentence in your answer.

C. Can you tell from looking at the equations whether the linear system has a solution? Use a complete sentence to explain your reasoning.

Investigate Problem 3

1. How does this linear system differ from the linear systems that you wrote in Problems 1 and 2? Use complete sentences in your answer.

2. To solve this linear system by using the substitution method, what do you think you would have to do first? Use a complete sentence in your answer.

3. Write the equation for the community group in slope-intercept form. Show all your work.

4. Now, use the substitution method to solve the linear system. Begin by substituting your expression from Question 3 for y in terms of x into the equation for the Outdoor Club.

   $8x + 30(\square) = 236$

   Now solve this equation for x. Show all your work.

   Finally, find the value for y. Show all your work.

5. Check your answer algebraically. Show all your work.

6. Interpret the solution of the linear system in the problem situation. Use complete sentences in your answer.

Take Note

Whenever a product involves a sum, such as $4(x + 3)$, you must use the distributive property to simplify: $4(x + 3) = 4(x) + 4(3)$.

Investigate Problem 3

7. If possible, solve each linear system by using the substitution method. Show all your work and use a complete sentence in your answer. Then check your answer algebraically.

$4x + 3y = 10$
$y = 2x$

$y = 2x - 1$
$y = 3x + 1$

$8x - 2y = 7$
$2x + y = 4$

$6x + 3y = 5$
$y = -2x + 1$

7.4 Basketball Tournament

Using Linear Combinations to Solve a Linear System

Objective

In this lesson, you will:

- Solve a linear system by using linear combinations.

Key Terms

- standard form of a linear equation
- linear combinations method
- linear combination

SCENARIO Your school hosted a basketball tournament. Tickets were sold before the tournament and at the door. More tickets were bought before the tournament than were bought at the door. In fact, there was a difference of 84 tickets between the two kinds of tickets sold. A total of 628 tickets were sold.

Problem 1 Ticket Sales

A. Write an equation in standard form that represents the total number of tickets sold. Use x to represent the number of tickets sold before the tournament and use y to represent the number of tickets sold at the door.

B. Write an equation in standard form that represents the difference in the numbers of tickets sold.

C. How are these equations different? How are they the same? Use complete sentences in your answer.

Investigate Problem 1

1. Write the linear system for this problem situation below.

Now, add the equations together.

Solve the resulting equation. Use a complete sentence in your answer.

Investigate Problem 1

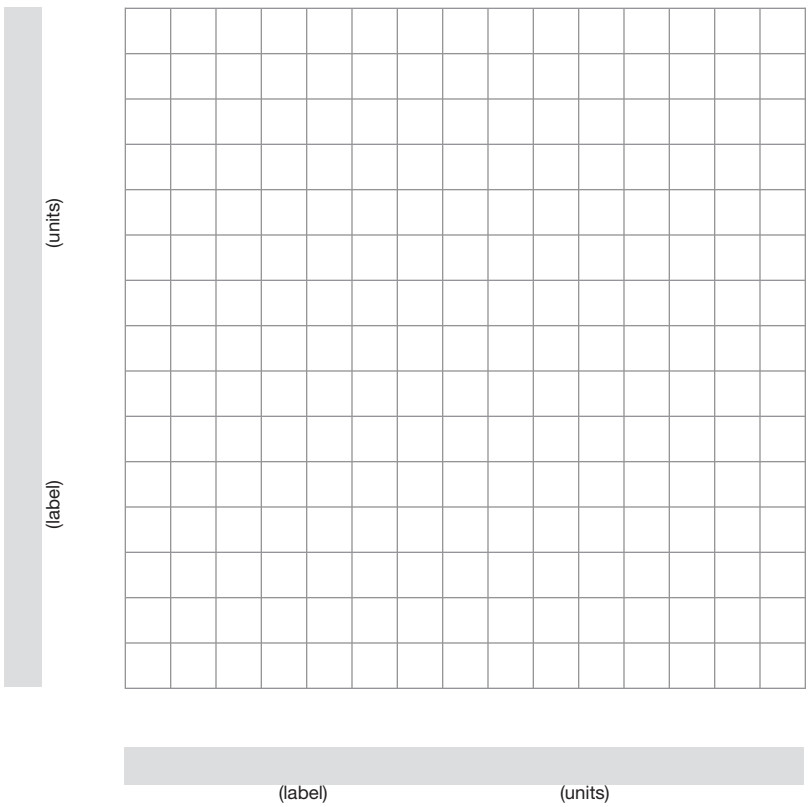
Now find the value for y by substituting your value for x into one of the original equations.

What is the solution of your linear system? Use a complete sentence in your answer.

2. Check your solution algebraically.

3. Check your solution by creating a graph of your linear system on the grid below. First, choose your bounds and intervals. Be sure to label your graph clearly.

Variable quantity	Lower bound	Upper bound	Interval



Investigate Problem 1

4. Interpret the solution of the linear system in the problem situation. Use a complete sentence in your answer.

5. What effect did adding the equations together have? Use complete sentences in your answer.

6. Describe how the coefficients of y in the original system are related. Use a complete sentence in your answer.

Problem 2 Traveling to the Tournament

A team that entered the tournament sold popcorn and mixed nuts to raise enough money to travel to the tournament. They made \$1.50 from each tin of popcorn and \$2 from each tin of mixed nuts. They raised a total of \$655 and sold 390 tins.

A. Write an equation in standard form that represents the total amount of money raised. Use x to represent the number of tins of popcorn sold and use y to represent the number of tins of nuts sold.

B. Write an equation in standard form that represents the total number of tins sold.

C. How are these equations different? How are they the same? Use complete sentences in your answer.

Investigate Problem 2

1. Multiply each side of the equation that represents the total number of tins sold by –2. Show your work.

Take Note

When a variable is multiplied by a number, the number is called the **coefficient.**

Investigate Problem 2

2. Write a linear system from the equation in part (A) and the equation in Question 1.

3. How do the coefficients of the equations in your linear system compare? Use complete sentences in your answer.

4. Add the equations in your linear system together. Then simplify the result. Show your work.

5. What does the result in Question 4 represent? Use a complete sentence in your answer.

6. Find the value for y by substituting your value for x into the original equation from part (B). Show your work.

7. What is the solution of the linear system? Interpret the solution of the linear system in the problem situation. Use complete sentences in your answer.

8. Check your solution algebraically. Show all your work.

9. Just the Math: Linear Combinations Method

The method you used to solve the linear systems in Problems 1 and 2 is called the **linear combinations method.** A **linear combination** is an equation that is the result of adding two equations to each other. The goal of adding the equations together is to get an equation in one variable. Then you can find the value of one variable and use it to find the value of the other variable.

Investigate Problem 2

In many cases, one (or both) of the equations in the system must be multiplied by a constant so that when the equations are added together, the result is an equation in one variable. For instance, consider the system

$4x + 2y = 3$
$5x - 3y = 1.$

What is the least common multiple of 2 and 3?

What do you have to multiply 2 by to get 6? What do you have to multiply 3 by to get 6?

So, multiply the first equation by 3 and multiply the second equation by 2. Complete the steps below.

$3(4x + 2y) = 3(3)$ ⇨ $\square x + \square y = \square$

$2(5x - 3y) = 2(1)$ ⇨ $\square x - \square y = \square$

Now, solve the new linear system. Show all your work and use a complete sentence in your answer.

10. For each linear system below, describe the first step you would take to solve the system by using the linear combinations method. Identify the variable that will be solved for when you add equations. Use complete sentences in your answer.

$4x - 3y = 8$ and $2x - 3y = 1$

$3x + 4y = 2$ and $2x - y = 4$

$6x + 5y = 1$ and $3x + 4y = 2$

$8x + 3y = 2$ and $-7x + 4y = 5$

Investigate Problem 2

11. Solve each linear system using linear combinations. Show all your work.

$-5x + 2y = -10$
$3x - 6y = -18$

$7x - 4y = -3$
$2x + 5y = -7$

12. Describe the kinds of linear systems for which you would use the substitution method you learned in the last lesson to solve the system. Describe the kinds of linear systems for which you would use the linear combinations method to solve the system. Use complete sentences in your answer.

7.5 Finding the Better Paying Job

Using the Best Method to Solve a Linear System, Part 1

Objective

In this lesson, you will:

- Solve a linear system by using an algebraic method.

Key Terms

- linear system
- inequality

SCENARIO A friend of yours interviewed for two different sales positions at competing companies. One of the companies, Stellar, pays $500 per week plus a 10% commission on the total sales per week in dollars. The other company, Lunar, pays $200 per week plus a 20% commission on the total sales per week in dollars.

Problem 1 Comparing Salaries

A. Write an equation that gives the weekly salary from Stellar in dollars in terms of the weekly total sales in dollars. Be sure to define your variables. Use a complete sentence in your answer.

B. Write an equation that gives the weekly salary from Lunar in dollars in terms of the weekly total sales in dollars. Be sure to define your variables. Use a complete sentence in your answer.

C. Find the salary from Stellar if the total sales are $1200 in one week. Show your work and use a complete sentence in your answer.

Find the salary from Lunar if the total sales are $1200 in one week. Show your work and use a complete sentence in your answer.

D. Find the total sales from Lunar if the weekly salary was $1200. Show your work and use a complete sentence in your answer.

Problem 1 Comparing Salaries

E. The salary from Stellar for one week is $540. Find the salary at Lunar if the total sales at Lunar are the same as the total sales at Stellar for this week. Show all your work and use a complete sentence in your answer.

Investigate Problem 1

1. Use an algebraic method to determine whether the salary from Lunar will ever be the same as the salary at Stellar. Show all your work and use a complete sentence in your answer.

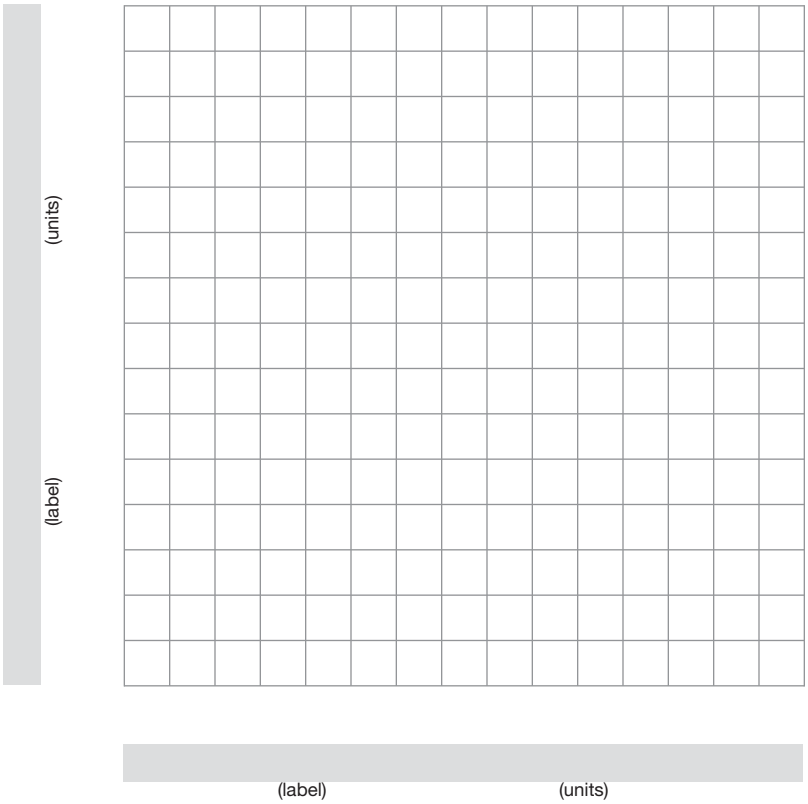
If the salaries will be the same, what will the salaries be? Show all your work and use a complete sentence in your answer.

2. Which method did you use to find the answer to Question 1? Use a complete sentence to explain your choice.

3. Check your solution by creating a graph of your linear system on the grid on the next page. First, choose your bounds and intervals. Be sure to label your graph clearly.

Variable quantity	Lower bound	Upper bound	Interval

Investigate Problem 1



Is your solution confirmed by your graph?

4. Complete the table of values that shows the salaries from both companies for different sales amounts.

Quantity Name	Total sales	Stellar salary	Lunar salary
Unit	dollars	dollars	dollars
Expression	x		
	0		
	100		
	500		
	2500		
	10,000		

Investigate Problem 1

5. Which company would you recommend to your friend? Why? Use complete sentences in your answer.

6. Your friend interviews at a third company, Solar. Solar pays a salary of $750 per week with no commissions. Write an equation that gives the salary in dollars in terms of the total sales in dollars. Then add the graph of this equation to your graph in Question 3.

7. Describe the conditions for which the salary from Solar is better than the salaries at Stellar and Lunar. Show all your work and use complete sentences in your answer.

8. Your friend takes the job with Stellar and wants to earn at least $975 each week. Write an **inequality** that represents this situation.

Solve the inequality. Then use a complete sentence to explain what the solution means in the problem situation.

7.6 World Oil: Supply and Demand

Using the Best Method to Solve a Linear System, Part 2

Objective

In this lesson, you will:

- Solve a linear system by using an algebraic method.

Key Term

- linear system

SCENARIO In 2003, there were approximately 28,179.4 million barrels of oil being produced in the world. In 1965, there were approximately 15,856 million barrels of oil being produced in the world.

Problem 1 Supply and Demand

A. What is the rate of change in the amount of oil being produced in millions of barrels per year from 1965 to 2003? Show all your work and use a complete sentence in your answer. Round your answer to the nearest tenth, if necessary.

B. The amount of oil being produced is called the *supply* of oil. Write an equation that gives the supply in millions of barrels in terms of the number of years since 1965. Assume that the rate of change in the supply is the same as the rate of change from 1965 to 2003. Be sure to define your variables. Show all your work and use complete sentences in your answer.

C. The amount of oil that the world uses is called the *demand* for oil. In 1965, the demand was approximately 15,179 million barrels per year and was increasing at a rate of 360.1 million barrels per year. Write an equation that gives the demand in millions of barrels in terms of the number of years since 1965. Be sure to define your variables and use a complete sentence in your answer.

Investigate Problem 1

1. In which year was the supply 18,000 million barrels? Show all your work and use a complete sentence in your answer.

2. In which year will the supply be 30,000 million barrels? Show all your work and use a complete sentence in your answer.

3. Find the demand in 1975. Show all your work and use a complete sentence in your answer.

4. Find the demand in 2010. Show all your work and use a complete sentence in your answer.

5. In which year was the demand 18,000 million barrels of oil? Show all your work and use a complete sentence in your answer.

Investigate Problem 1

6. In which year will the demand be 40,000 million barrels of oil? Show all your work and use a complete sentence in your answer.

7. Find the supply 25 years after 1965. Show all your work and use a complete sentence in your answer.

8. In which year was the demand 10,000 million barrels of oil? Show all your work and use a complete sentence in your answer.

9. Find the year in which the supply was 10,000 million barrels. Show all your work and use a complete sentence in your answer.

10. Write the linear system that represents the supply and the demand since 1965.

11. Do you think that the supply was ever the same as the demand? Use what you know about the equations of a linear system to explain your answer.

Investigate Problem 1

12. Complete the table of values below that shows the supply and the demand for different numbers of years.

Quantity Name	Years since 1965	Supply	Demand
Unit	years	million barrels	million barrels
Expression	x		
	1		
	10		
	40		
	50		

13. Use the table to decide whether the supply was ever the same as the demand. Use a complete sentence to explain your reasoning.

If so, determine the number of years that will pass before the supply and demand will be equal. Show all your work and use a complete sentence in your answer.

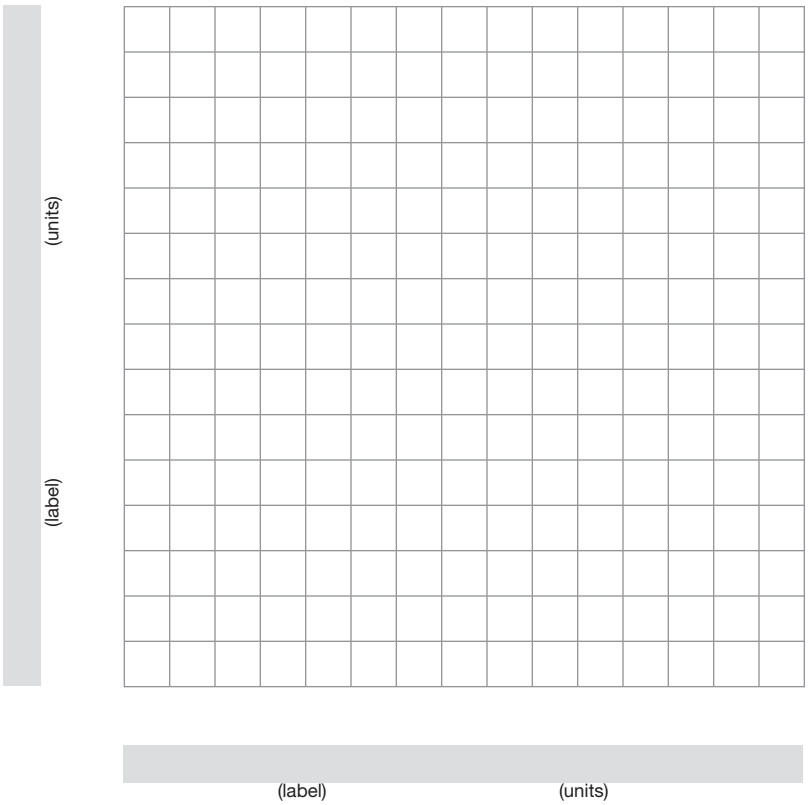
How did you find your answer? Use a complete sentence to explain.

What was the amount of oil when the supply and demand were equal? Show all your work and use a complete sentence in your answer.

Investigate Problem 1

14. Check your estimate by creating a graph of your linear system on the grid below. First, choose your bounds and intervals. Be sure to label your graph clearly.

Variable quantity	Lower bound	Upper bound	Interval



Is your estimate confirmed by your graph?

15. Do you think that your equations are accurate models for your data? Use complete sentences to explain your reasoning.

7

7.7 Picking the Better Option

Solving Linear Systems

Objective

In this lesson, you will:

- Use a system of linear equations to solve a problem.

Key Term

- break-even point

SCENARIO The Bici Bicycle Company is planning on making a low price ultra-light bicycle. There are two different plans being considered for building this bicycle. The first plan includes a cost of $125,000 to design and build a prototype bicycle. The materials and labor costs for each bike made under the first plan will be $225. The second plan includes a cost of $100,000 to design and build the prototype. The materials and labor costs for each bike made under the second plan will be $275.

Problem 1 Which Plan Is the Better Plan?

A. Before you begin comparing the plans, what factors do you think are important to consider? Use complete sentences in your answer.

B. For each plan, write an equation that gives the total cost in dollars in terms of the total number of bicycles made. Be sure to define your variables.

C. For each plan, what do the slope and *y*-intercept of the graph of the equation represent in the problem situation? Use complete sentences in your answer.

D. Will there be a number of bicycles for which the total costs are the same? How do you know? Use a complete sentence to explain your reasoning.

E. Describe the different methods you can use to find the number of bicycles for which the total costs are the same. Use a complete sentence in your answer.

Investigate Problem 1

1. Complete the table of values that shows the total cost of both plans for different numbers of bicycles.

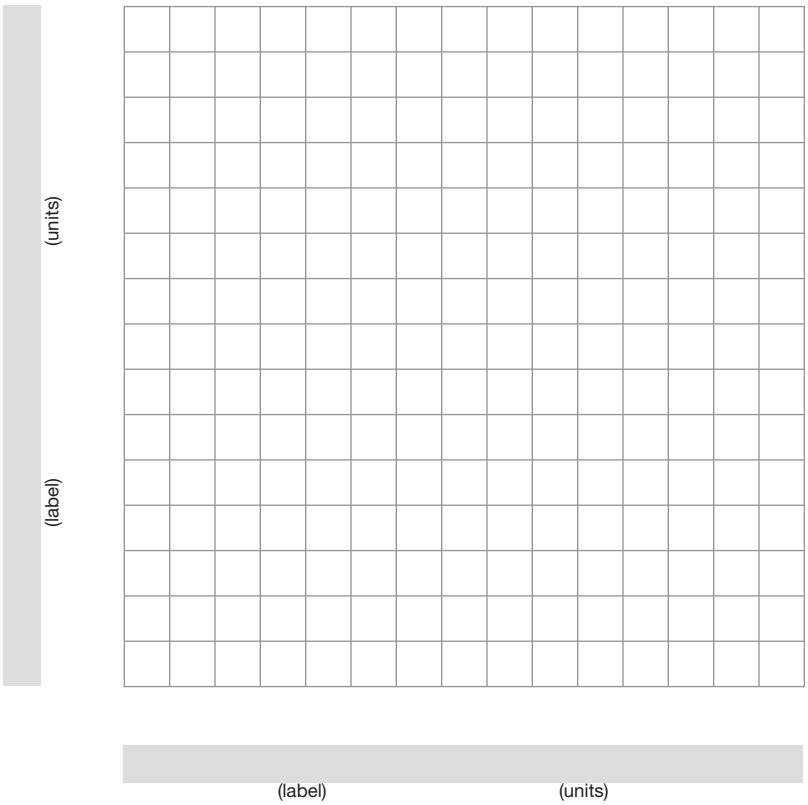
Quantity Name	Bicycles made	Plan 1 cost	Plan 2 cost
Unit	bicycles	dollars	dollars
Expression			

2. Can you determine from your table the number of bicycles for which the total costs are the same? If so, describe the numbers of bicycles for which the first plan is better and the numbers of bicycles for which the second plan is better. If not, use an algebraic method to answer the question. Then describe the numbers of bicycles for which each plan is the better plan.

3. Create a graph of your linear system on the grid on the next page to verify your answer. First, choose your bounds and intervals. Be sure to label your graph clearly.

Variable quantity	Lower bound	Upper bound	Interval

Investigate Problem 1



4. Now consider the selling price of the bikes. Suppose that the company wants to sell the bikes for $525 each. Write an equation that gives the total earnings in dollars in terms of the number of bicycles sold.

5. For each plan, determine the break-even point. Show all your work. Use complete sentences in your answer.

Take Note

Recall that the *break-even point* is the *x*-coordinate of the point where the graph of the cost intersects the graph of the income.

Investigate Problem 1

6. Use a complete sentence to explain what the break-even point means in this situation.

7. Use the results from Questions 2 and 5 to describe the numbers of bicycles for which each plan is better. Use complete sentences in your answer.

8. Suppose that the company wants to change the total costs of each plan. The company wants to reduce the material and labor costs under each plan by $22.50. For each plan, write the new equation for the total cost in terms of the number of bicycles made. Then use your equations to determine the numbers of bicycles for which each plan is better. Use complete sentences in your answers.

Investigate Problem 1

9. Does this answer surprise you? Why or why not? Use complete sentences to explain your answer.

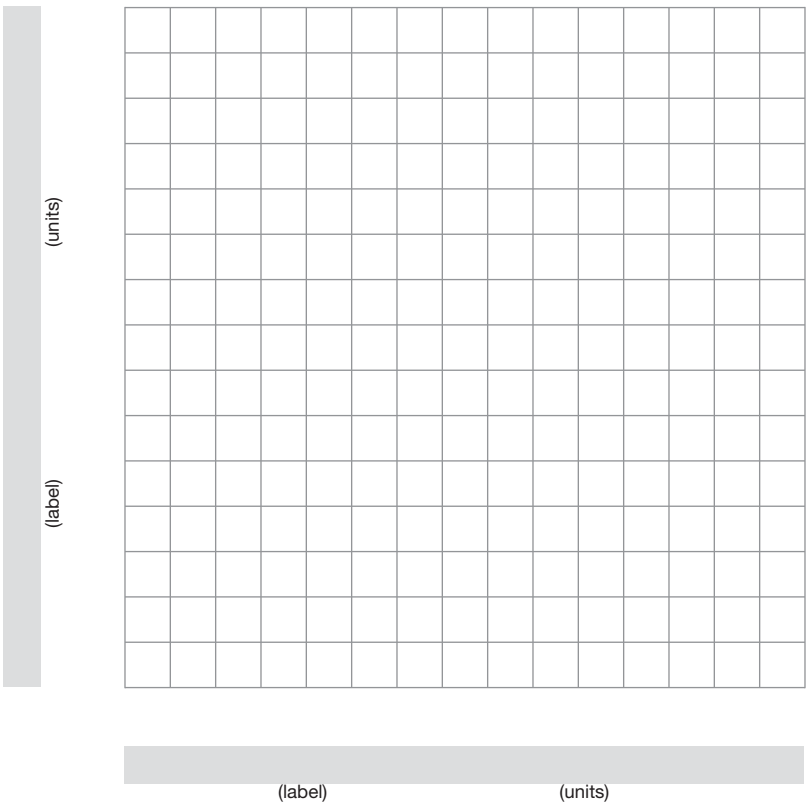
10. The company also decides to reduce the selling price under each plan to $450. For each new plan, determine the break-even point. Show all your work. Use complete sentences in your answer.

11. Use the results from Questions 8 and 10 to describe the numbers of bicycles for which each plan is better. Use complete sentences in your answer.

Investigate Problem 1

12. Create a graph of the linear system you found in Question 8 on the grid below. First, choose your bounds and intervals. Be sure to label your graph clearly.

Variable quantity	Lower bound	Upper bound	Interval



13. Estimate the point of intersection from your graph. Use complete sentences to compare the points of intersection that you found algebraically and graphically.

7.8 Video Arcade

Writing and Graphing an Inequality in Two Variables

Objectives

In this lesson, you will:

- Write an inequality in two variables.
- Graph an inequality in two variables.

Key Terms

- linear inequality in two variables
- inequality symbol
- linear equation
- coordinate plane
- half-plane

SCENARIO Your cousin's graduation party is at a restaurant that has a large video arcade. Each person at the party receives a card with fifty points on it to play the games in the arcade. One of your favorite games, a driving game, uses twelve card points per game. Another game that you like, a basketball game, uses eight points per game.

Problem 1 Playing Games

A. Can you play three driving games and two basketball games and not go over the number of points on the card? Show your work.

B. Can you play two driving games and three basketball games and not go over the number of points on the card? Show your work.

C. Can you play one driving game and four basketball games and not go over the number of points on the card? Show your work.

D. Write an expression that represents the total number of points used by playing x driving games and y basketball games.

E. What restrictions must be placed on this expression so that you do not go over the number of points on the card? Use a complete sentence in your answer.

F. One form of a **linear inequality in two variables** can be written as $Ax + By \leq C$. Write an inequality in two variables that represents this problem situation.

Take Note

Recall that an **inequality** is a statement that is formed by placing an **inequality symbol** ($<$, $>$, $\leq$, $\geq$) between two expressions.

Take Note

The forms of a linear inequality in two variables are:

$Ax + By < C$
$Ax + By > C$
$Ax + By \leq C$
$Ax + By \geq C$

Investigate Problem 1

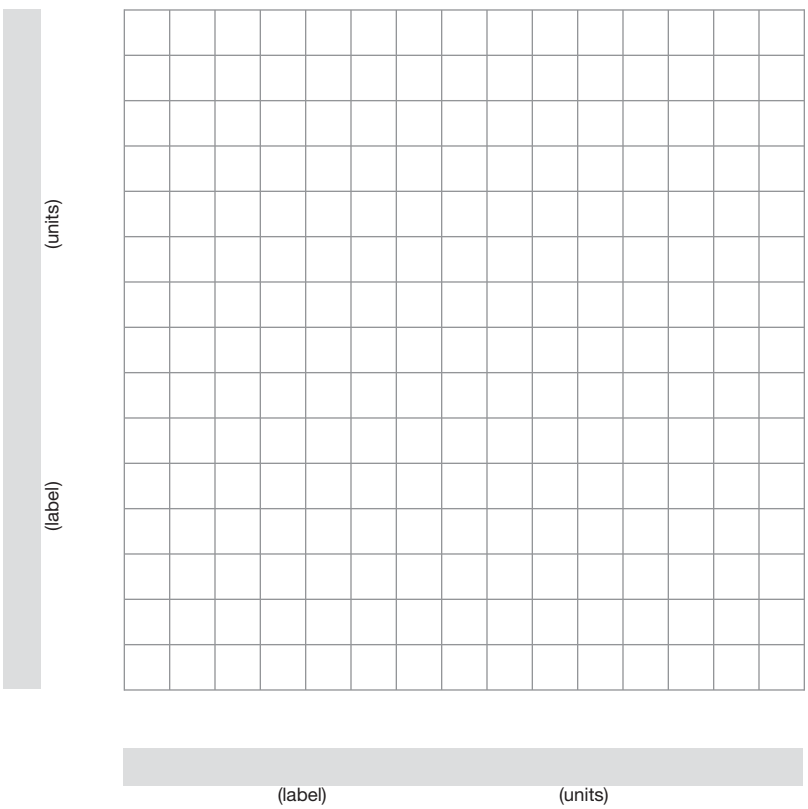
1. Complete the table on the next page that shows different numbers of driving and basketball games played and the numbers of points used.

Investigate Problem 1

Quantity Name	Driving games	Basketball games	Points used
Unit	games	games	points
	0	5	
	1	3	
	2	3	
	2	4	
	3	2	
	3	3	
	4	0	
	4	1	

2. Create a graph of the data in the table on the grid below. If the number of points used in a row does not exceed the card's points, draw a point for the numbers of games. If the number of points used does exceed the card's points, draw an "x" for the numbers of games. Use the bounds and intervals given below. Label your graph clearly.

Variable quantity	Lower bound	Upper bound	Interval
Driving game	0	7.5	0.5
Basketball game	0	7.5	0.5



Investigate Problem 1

3. Write an equation that represents the number of driving games x and the number of basketball games y that can be played for exactly 50 points. Then add the graph of this equation to your graph in Question 2.

4. What do you notice about your graph? Use a complete sentence in your answer.

5. Just the Math: Linear Inequality Shade the side of the graph that contains all of the points. This graph is the graph of the *linear inequality* $12x + 8y \leq 50$. A linear inequality is the same as a linear equation except that an inequality symbol ($<$, $>$, $\leq$, or $\geq$) is used instead of an equals sign. How do the solutions of the linear equation $12x + 8y = 50$ differ from the solutions of the linear inequality $12x + 8y \leq 50$? Use complete sentences in your answer.

Take Note

A **linear equation** in two variables is an equation in which each of the variables is raised to the first power (such as x, rather than x^2) and, when in simplest form, each variable only appears once.

6. Just the Math: Graphs of Linear Inequalities The graph of a linear inequality is a **half-plane,** or half of a **coordinate plane**. A line, given by the inequality, divides the plane into two half-planes and the inequality symbol tells you which half-plane contains all the solutions. If the symbol is $\leq$ or $\geq$, the graph includes the line. If the symbol is $<$ or $>$, the graph does not include the line and is represented by a dashed line. For which inequalities below would you include the line? Which inequalities below would you represent by using a dashed line? Write your answers using complete sentences.

$y > -6 - x$ $\qquad$ $2x + 3y \geq 4$ $\qquad$ $x + 5y \leq 10$

$3x + 12y > 5$ $\qquad$ $y \geq -x + 2$ $\qquad$ $x - y < 3$

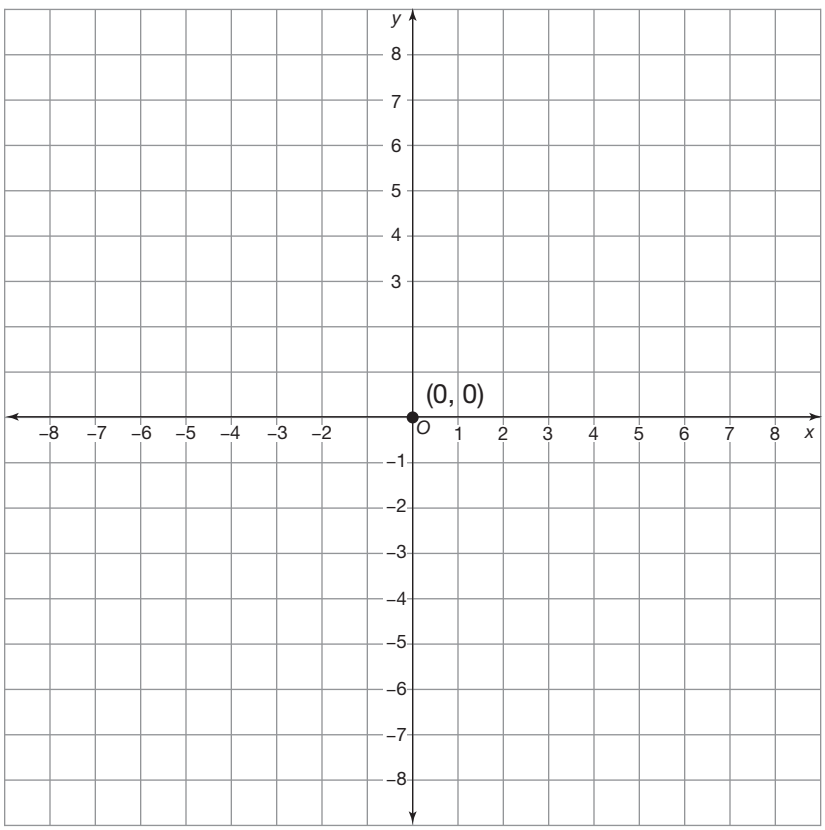
Investigate Problem 1

Consider the linear inequality $y < 4x + 3$. The line that divides the plane is given by $y = 4x + 3$. Should this line be a solid line or a dashed line? Use a complete sentence to explain. Then draw the correct type of line on the grid below.

After you draw the correct type of line, you need to decide which half-plane contains all the solutions, because this is the half-plane that you will shade. To make your decision, consider the point (0, 0). If (0, 0) is a solution, then the half-plane that contains (0, 0) contains all the solutions and should be shaded. If (0, 0) is not a solution, then the half-plane that does not contain (0, 0) contains all the solutions and should be shaded.

Is (0, 0) a solution? Show your work.

Now shade the correct half-plane on the grid below.



Investigate Problem 1

7. Graph each linear inequality.

$y > x + 2$

$y \leq -x + 3$

7

7.9 Making a Mosaic

Solving Systems of Linear Inequalities

Objectives

In this lesson, you will:

- Write a system of linear inequalities.
- Graph a system of linear inequalities.
- Identify solutions of a system of linear inequalities.

Key Terms

- system of linear equations
- linear inequality
- system of linear inequalities

SCENARIO A local arts group is donating a mural that will be placed at the entrance to your school. The mural will be 6 feet tall and 12 feet wide. The group has calculated that they will need approximately 110 bags of tiles to complete the project. The mural will be made of glass and metallic tiles. Each bag of glass tiles costs $10 and each bag of metallic tiles costs $18. Another group has donated $1500 for the purchase of the tiles.

Problem 1 Getting the Tiles

A. Write an equation that relates the numbers of bags of glass and metallic tiles that can be bought for $1500. Use x to represent the number of bags of glass tiles and use y to represent the number of bags of metallic tiles that can be bought for the mural.

B. Write an equation that relates the numbers of bags of glass and metallic tiles to the total number of bags of tiles needed for the project.

C. What does the solution of the linear system formed by the equations in part (A) and part (B) represent?

D. Solve the linear system. Show all your work and use a complete sentence in your answer.

E. What does the solution mean in the problem situation? Use a complete sentence in your answer.

Investigate Problem 1

1. Suppose that the group wants to buy 75 bags of glass tiles and 35 bags of metallic tiles. Is this enough tile?

   Can the group afford this assortment of tile? Show all your work and use a complete sentence to explain your reasoning.

2. Suppose that the group wants to buy 90 bags of glass tiles and 25 bags of metallic tile. Is this enough tile?

   Can the group afford this assortment of tile? Show all your work and use a complete sentence to explain your reasoning.

3. Suppose that the group wants to buy 80 bags of glass tiles and 38 bags of metallic tiles. Is this enough tile?

   Can the group afford this assortment of tile? Show all your work and use a complete sentence to explain your reasoning.

4. Does the group have to spend all of the money to get enough tile? Use a complete sentence to explain your reasoning.

   Write an inequality that represents the amounts of money the group can spend on x bags of glass tiles and y bags of metallic tiles.

Investigate Problem 1

5. Can the group buy more bags of tiles than is needed and not spend all the money? Use a complete sentence to explain your reasoning.

Write an inequality that represents the total numbers of bags of tiles they can use to complete the project.

6. **Just the Math: System of Linear Inequalities**

Together, the linear inequalities in Questions 4 and 5 form a **system of linear inequalities.** Write the system of linear inequalities below.

What do you think it means to be a solution of a system of linear inequalities? Use a complete sentence in your answer.

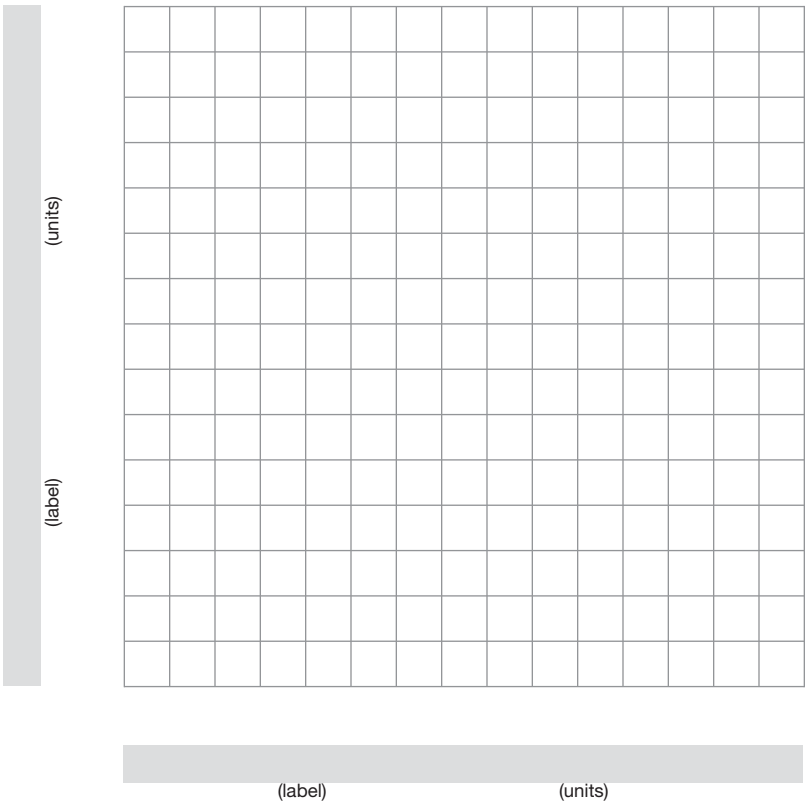
Determine whether the numbers of bags of tiles given in Questions 1 through 3 are solutions of your system of inequalities. Show all your work.

7. How many solutions do you think a system of linear inequalities can have? Use complete sentences to explain your reasoning.

Investigate Problem 1

8. Create a graph of your system of inequalities on the grid below. Use a different color pen or pencil for each inequality. First, choose your bounds and intervals. Be sure to label your graph clearly.

Variable quantity	Lower bound	Upper bound	Interval



9. What part of the graph do you think represents the solution of the system of linear inequalities? Use a complete sentence in your answer.

10. Identify three different solutions of the system of inequalities. What do these solutions represent in the problem situation? Use complete sentences in your answer.