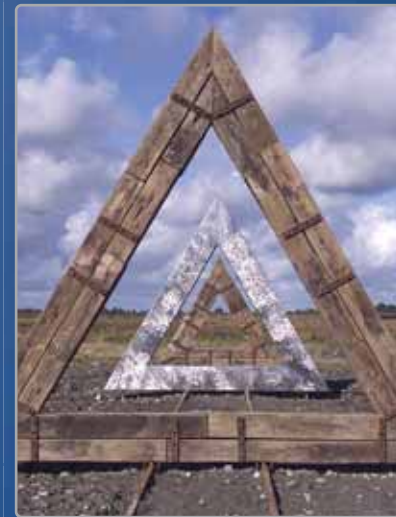


2 Reasoning and Proofs

- 2.1 Conditional Statements
- 2.2 Inductive and Deductive Reasoning
- 2.3 Postulates and Diagrams
- 2.4 Algebraic Reasoning
- 2.5 Proving Statements about Segments and Angles
- 2.6 Proving Geometric Relationships



Airport Runway (p. 108)



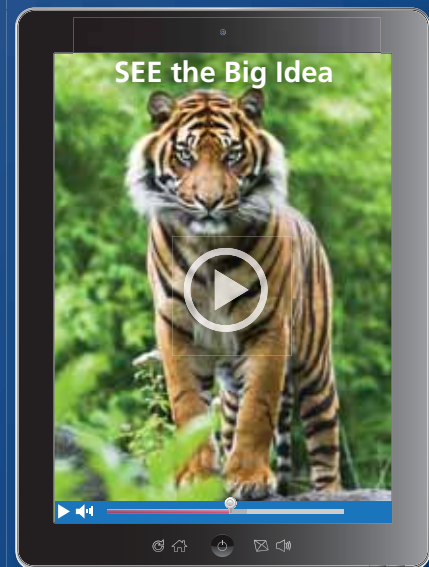
Sculpture (p. 104)



City Street (p. 95)



Guitar (p. 67)



Tiger (p. 81)

Maintaining Mathematical Proficiency

Finding the n th Term of an Arithmetic Sequence

Example 1 Write an equation for the n th term of the arithmetic sequence 2, 5, 8, 11, ...
Then find a_{20} .

The first term is 2, and the common difference is 3.

$$a_n = a_1 + (n - 1)d \quad \text{Equation for an arithmetic sequence}$$

$$a_n = 2 + (n - 1)3 \quad \text{Substitute 2 for } a_1 \text{ and 3 for } d.$$

$$a_n = 3n - 1 \quad \text{Simplify.}$$

Use the equation to find the 20th term.

$$a_n = 3n - 1 \quad \text{Write the equation.}$$

$$a_{20} = 3(20) - 1 \quad \text{Substitute 20 for } n.$$

$$= 59 \quad \text{Simplify.}$$

► The 20th term of the arithmetic sequence is 59.

Write an equation for the n th term of the arithmetic sequence.
Then find a_{50} .

1. 3, 9, 15, 21, ...

2. -29, -12, 5, 22, ...

3. 2.8, 3.4, 4.0, 4.6, ...

4. $\frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{5}{6}, \dots$

5. 26, 22, 18, 14, ...

6. 8, 2, -4, -10, ...

Rewriting Literal Equations

Example 2 Solve the literal equation $3x + 6y = 24$ for y .

$$3x + 6y = 24 \quad \text{Write the equation.}$$

$$3x - 3x + 6y = 24 - 3x \quad \text{Subtract } 3x \text{ from each side.}$$

$$6y = 24 - 3x \quad \text{Simplify.}$$

$$\frac{6y}{6} = \frac{24 - 3x}{6} \quad \text{Divide each side by 6.}$$

$$y = 4 - \frac{1}{2}x \quad \text{Simplify.}$$

► The rewritten literal equation is $y = 4 - \frac{1}{2}x$.

Solve the literal equation for x .

7. $2y - 2x = 10$

8. $20y + 5x = 15$

9. $4y - 5 = 4x + 7$

10. $y = 8x - x$

11. $y = 4x + zx + 6$

12. $z = 2x + 6xy$

13. **ABSTRACT REASONING** Can you use the equation for an arithmetic sequence to write an equation for the sequence 3, 9, 27, 81, ...? Explain your reasoning.

Using Correct Reasoning

Core Concept

Deductive Reasoning

When you use *deductive reasoning*, you start with two or more true statements and *deduce* or *infer* the truth of another statement. Here is an example.

1. **Premise:** If a polygon is a triangle, then the sum of its angle measures is 180° .
2. **Premise:** Polygon ABC is a triangle.
3. **Conclusion:** The sum of the angle measures of polygon ABC is 180° .

This pattern for deductive reasoning is called a *sylllogism*.

EXAMPLE 1 Recognizing Flawed Reasoning

The syllogisms below represent common types of *flawed reasoning*. Explain why each conclusion is not valid.

- | | |
|---|---|
| a. When it rains, the ground gets wet.
The ground is wet.
Therefore, it must have rained. | b. If $\triangle ABC$ is equilateral, then it is isosceles.
$\triangle ABC$ is not equilateral.
Therefore, it must not be isosceles. |
| c. All squares are polygons.
All trapezoids are quadrilaterals.
Therefore, all squares are quadrilaterals. | d. No triangles are quadrilaterals.
Some quadrilaterals are not squares.
Therefore, some squares are not triangles. |

SOLUTION

- The ground may be wet for another reason.
- A triangle can be isosceles but not equilateral.
- All squares are quadrilaterals, but not because all trapezoids are quadrilaterals.
- No squares are triangles.

Monitoring Progress

Decide whether the syllogism represents correct or flawed reasoning. If flawed, explain why the conclusion is not valid.

- | | |
|--|--|
| 1. All triangles are polygons.
Figure ABC is a triangle.
Therefore, figure ABC is a polygon. | 2. No trapezoids are rectangles.
Some rectangles are not squares.
Therefore, some squares are not trapezoids. |
| 3. If polygon $ABCD$ is a square, then it is a rectangle.
Polygon $ABCD$ is a rectangle.
Therefore, polygon $ABCD$ is a square. | 4. If polygon $ABCD$ is a square, then it is a rectangle.
Polygon $ABCD$ is not a square.
Therefore, polygon $ABCD$ is not a rectangle. |

2.1 Conditional Statements

Essential Question When is a conditional statement true or false?

A *conditional statement*, symbolized by $p \rightarrow q$, can be written as an “if-then statement” in which p is the *hypothesis* and q is the *conclusion*. Here is an example.

If a polygon is a triangle, then the sum of its angle measures is 180° .

hypothesis, p

conclusion, q

EXPLORATION 1 Determining Whether a Statement Is True or False

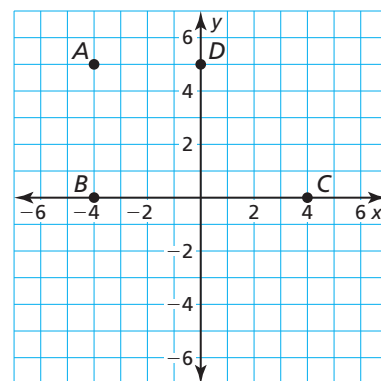
Work with a partner. A hypothesis can either be true or false. The same is true of a conclusion. For a conditional statement to be true, the hypothesis and conclusion do not necessarily both have to be true. Determine whether each conditional statement is true or false. Justify your answer.

- If yesterday was Wednesday, then today is Thursday.
- If an angle is acute, then it has a measure of 30° .
- If a month has 30 days, then it is June.
- If an even number is not divisible by 2, then 9 is a perfect cube.

EXPLORATION 2 Determining Whether a Statement Is True or False

Work with a partner. Use the points in the coordinate plane to determine whether each statement is true or false. Justify your answer.

- $\triangle ABC$ is a right triangle.
- $\triangle BDC$ is an equilateral triangle.
- $\triangle BDC$ is an isosceles triangle.
- Quadrilateral $ABCD$ is a trapezoid.
- Quadrilateral $ABCD$ is a parallelogram.



CONSTRUCTING VIABLE ARGUMENTS

To be proficient in math, you need to distinguish correct logic or reasoning from that which is flawed.

EXPLORATION 3 Determining Whether a Statement Is True or False

Work with a partner. Determine whether each conditional statement is true or false. Justify your answer.

- If $\triangle ADC$ is a right triangle, then the Pythagorean Theorem is valid for $\triangle ADC$.
- If $\angle A$ and $\angle B$ are complementary, then the sum of their measures is 180° .
- If figure $ABCD$ is a quadrilateral, then the sum of its angle measures is 180° .
- If points A , B , and C are collinear, then they lie on the same line.
- If $\overleftrightarrow{AB}$ and $\overleftrightarrow{BD}$ intersect at a point, then they form two pairs of vertical angles.

Communicate Your Answer

- When is a conditional statement true or false?
- Write one true conditional statement and one false conditional statement that are different from those given in Exploration 3. Justify your answer.

2.1 Lesson

Core Vocabulary

conditional statement, p. 66
if-then form, p. 66
hypothesis, p. 66
conclusion, p. 66
negation, p. 66
converse, p. 67
inverse, p. 67
contrapositive, p. 67
equivalent statements, p. 67
perpendicular lines, p. 68
biconditional statement, p. 69
truth value, p. 70
truth table, p. 70

What You Will Learn

- ▶ Write conditional statements.
- ▶ Use definitions written as conditional statements.
- ▶ Write biconditional statements.
- ▶ Make truth tables.

Writing Conditional Statements

Core Concept

Conditional Statement

A **conditional statement** is a logical statement that has two parts, a *hypothesis* p and a *conclusion* q . When a conditional statement is written in **if-then form**, the “if” part contains the **hypothesis** and the “then” part contains the **conclusion**.

Words If p , then q .

Symbols $p \rightarrow q$ (read as “ p implies q ”)

EXAMPLE 1 Rewriting a Statement in If-Then Form

Use red to identify the hypothesis and blue to identify the conclusion. Then rewrite the conditional statement in if-then form.

- a. All birds have feathers. b. You are in Texas if you are in Houston.

SOLUTION

- a. All **birds** have **feathers**. b. **You are in Texas** if **you are in Houston**.

▶ If **an animal is a bird**,
then **it has feathers**.

▶ If **you are in Houston**,
then **you are in Texas**.

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Use red to identify the hypothesis and blue to identify the conclusion. Then rewrite the conditional statement in if-then form.

1. All 30° angles are acute angles. 2. $2x + 7 = 1$, because $x = -3$.

Core Concept

Negation

The **negation** of a statement is the *opposite* of the original statement. To write the negation of a statement p , you write the symbol for negation ($\sim$) before the letter. So, “not p ” is written $\sim p$.

Words not p

Symbols $\sim p$

EXAMPLE 2 Writing a Negation

Write the negation of each statement.

- a. The ball is red. b. The cat is *not* black.

SOLUTION

- a. The ball is *not* red. b. The cat is black.

Core Concept

Related Conditionals

Consider the conditional statement below.

Words If p , then q . **Symbols** $p \rightarrow q$

Converse To write the **converse** of a conditional statement, exchange the hypothesis and the conclusion.

Words If q , then p . **Symbols** $q \rightarrow p$

Inverse To write the **inverse** of a conditional statement, negate both the hypothesis and the conclusion.

Words If not p , then not q . **Symbols** $\sim p \rightarrow \sim q$

Contrapositive To write the **contrapositive** of a conditional statement, first write the converse. Then negate both the hypothesis and the conclusion.

Words If not q , then not p . **Symbols** $\sim q \rightarrow \sim p$

A conditional statement and its contrapositive are either both true or both false. Similarly, the converse and inverse of a conditional statement are either both true or both false. In general, when two statements are both true or both false, they are called **equivalent statements**.

COMMON ERROR

Just because a conditional statement and its contrapositive are both true does not mean that its converse and inverse are both false. The converse and inverse could also both be true.

EXAMPLE 3

Writing Related Conditional Statements



Let p be “you are a guitar player” and let q be “you are a musician.” Write each statement in words. Then decide whether it is *true* or *false*.

- the conditional statement $p \rightarrow q$
- the converse $q \rightarrow p$
- the inverse $\sim p \rightarrow \sim q$
- the contrapositive $\sim q \rightarrow \sim p$

SOLUTION

- Conditional: If you are a guitar player, then you are a musician.
true; Guitar players are musicians.
- Converse: If you are a musician, then you are a guitar player.
false; Not all musicians play the guitar.
- Inverse: If you are not a guitar player, then you are not a musician.
false; Even if you do not play a guitar, you can still be a musician.
- Contrapositive: If you are not a musician, then you are not a guitar player.
true; A person who is not a musician cannot be a guitar player.

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In Exercises 3 and 4, write the negation of the statement.

- The shirt is green.
- The shoes are *not* red.
- Repeat Example 3. Let p be “the stars are visible” and let q be “it is night.”

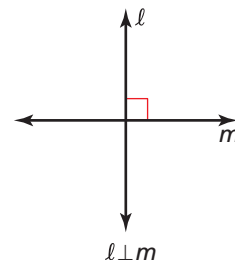
Using Definitions

You can write a definition as a conditional statement in if-then form or as its converse. Both the conditional statement and its converse are true for definitions. For example, consider the definition of *perpendicular lines*.

If two lines intersect to form a right angle, then they are **perpendicular lines**.

You can also write the definition using the converse: If two lines are perpendicular lines, then they intersect to form a right angle.

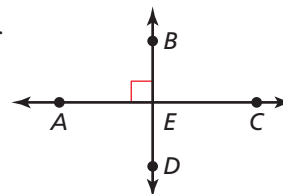
You can write “line ℓ is perpendicular to line m ” as $\ell \perp m$.



EXAMPLE 4 Using Definitions

Decide whether each statement about the diagram is true. Explain your answer using the definitions you have learned.

- $\overleftrightarrow{AC} \perp \overleftrightarrow{BD}$
- $\angle AEB$ and $\angle CEB$ are a linear pair.
- $\overrightarrow{EA}$ and $\overrightarrow{EB}$ are opposite rays.



SOLUTION

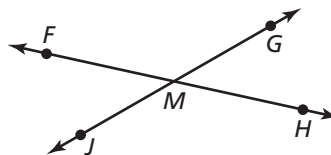
- This statement is *true*. The right angle symbol in the diagram indicates that the lines intersect to form a right angle. So, you can say the lines are perpendicular.
- This statement is *true*. By definition, if the noncommon sides of adjacent angles are opposite rays, then the angles are a linear pair. Because $\overrightarrow{EA}$ and $\overrightarrow{EC}$ are opposite rays, $\angle AEB$ and $\angle CEB$ are a linear pair.
- This statement is *false*. Point E does not lie on the same line as A and B , so the rays are not opposite rays.

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Use the diagram. Decide whether the statement is true. Explain your answer using the definitions you have learned.



- $\angle JMF$ and $\angle FMG$ are supplementary.
- Point M is the midpoint of $\overline{FH}$.
- $\angle JMF$ and $\angle HMG$ are vertical angles.
- $\overleftrightarrow{FH} \perp \overleftrightarrow{JG}$

Writing Biconditional Statements

Core Concept

Biconditional Statement

When a conditional statement and its converse are both true, you can write them as a single *biconditional statement*. A **biconditional statement** is a statement that contains the phrase “if and only if.”

Words p if and only if q **Symbols** $p \leftrightarrow q$

Any definition can be written as a biconditional statement.

EXAMPLE 5 Writing a Biconditional Statement

Rewrite the definition of perpendicular lines as a single biconditional statement.

Definition If two lines intersect to form a right angle, then they are perpendicular lines.

SOLUTION

Let p be “two lines intersect to form a right angle” and let q be “they are perpendicular lines.”

Use red to identify p and blue to identify q .

Write the definition $p \rightarrow q$.

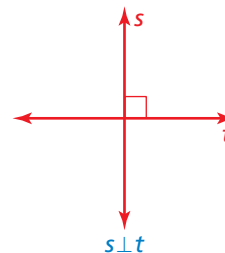
Definition If **two lines intersect to form a right angle**, then **they are perpendicular lines**.

Write the converse $q \rightarrow p$.

Converse If **two lines are perpendicular lines**, then **they intersect to form a right angle**.

Use the definition and its converse to write the biconditional statement $p \leftrightarrow q$.

► **Biconditional** **Two lines intersect to form a right angle** if and only if **they are perpendicular lines**.



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10. Rewrite the definition of a right angle as a single biconditional statement.

Definition If an angle is a right angle, then its measure is 90° .

11. Rewrite the definition of congruent segments as a single biconditional statement.

Definition If two line segments have the same length, then they are congruent segments.

12. Rewrite the statements as a single biconditional statement.

If Mary is in theater class, then she will be in the fall play. If Mary is in the fall play, then she must be taking theater class.

13. Rewrite the statements as a single biconditional statement.

If you can run for President, then you are at least 35 years old. If you are at least 35 years old, then you can run for President.

Making Truth Tables

The **truth value** of a statement is either true (T) or false (F). You can determine the conditions under which a conditional statement is true by using a **truth table**. The truth table below shows the truth values for hypothesis p and conclusion q .

Conditional		
p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

The conditional statement $p \rightarrow q$ is only false when a true hypothesis produces a false conclusion.

Two statements are *logically equivalent* when they have the same truth table.

EXAMPLE 6 Making a Truth Table

Use the truth table above to make truth tables for the converse, inverse, and contrapositive of a conditional statement $p \rightarrow q$.

SOLUTION

The truth tables for the converse and the inverse are shown below. Notice that the converse and the inverse are logically equivalent because they have the same truth table.

Converse		
p	q	$q \rightarrow p$
T	T	T
T	F	T
F	T	F
F	F	T

Inverse				
p	q	$\sim p$	$\sim q$	$\sim p \rightarrow \sim q$
T	T	F	F	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

The truth table for the contrapositive is shown below. Notice that a conditional statement and its contrapositive are logically equivalent because they have the same truth table.

Contrapositive				
p	q	$\sim q$	$\sim p$	$\sim q \rightarrow \sim p$
T	T	F	F	T
T	F	T	F	F
F	T	F	T	T
F	F	T	T	T

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14. Make a truth table for the conditional statement $p \rightarrow \sim q$.

15. Make a truth table for the conditional statement $\sim(p \rightarrow q)$.

2.1 Exercises

Dynamic Solutions available at BigIdeasMath.com

Vocabulary and Core Concept Check

- VOCABULARY** What type of statements are either both true or both false?
- WHICH ONE DOESN'T BELONG?** Which statement does *not* belong with the other three? Explain your reasoning.

If today is Tuesday, then tomorrow is Wednesday.

If it is Independence Day, then it is July.

If an angle is acute, then its measure is less than 90° .

If you are an athlete, then you play soccer.

Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, copy the conditional statement. Underline the hypothesis and circle the conclusion.

- If a polygon is a pentagon, then it has five sides.
- If two lines form vertical angles, then they intersect.
- If you run, then you are fast.
- If you like math, then you like science.

In Exercises 7–12, rewrite the conditional statement in if-then form. (See Example 1.)

- $9x + 5 = 23$, because $x = 2$.
- Today is Friday, and tomorrow is the weekend.
- You are in a band, and you play the drums.
- Two right angles are supplementary angles.
- Only people who are registered are allowed to vote.
- The measures of complementary angles sum to 90° .

In Exercises 13–16, write the negation of the statement. (See Example 2.)

- The sky is blue.
- The lake is cold.
- The ball is *not* pink.
- The dog is *not* a Lab.

In Exercises 17–24, write the conditional statement $p \rightarrow q$, the converse $q \rightarrow p$, the inverse $\sim p \rightarrow \sim q$, and the contrapositive $\sim q \rightarrow \sim p$ in words. Then decide whether each statement is true or false. (See Example 3.)

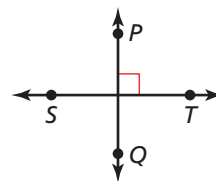
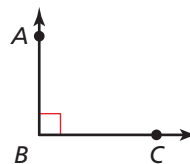
- Let p be “two angles are supplementary” and let q be “the measures of the angles sum to 180° .”

- Let p be “you are in math class” and let q be “you are in Geometry.”
- Let p be “you do your math homework” and let q be “you will do well on the test.”
- Let p be “you are not an only child” and let q be “you have a sibling.”
- Let p be “it does not snow” and let q be “I will run outside.”
- Let p be “the Sun is out” and let q be “it is daytime.”
- Let p be “ $3x - 7 = 20$ ” and let q be “ $x = 9$.”
- Let p be “it is Valentine’s Day” and let q be “it is February.”

In Exercises 25–28, decide whether the statement about the diagram is true. Explain your answer using the definitions you have learned. (See Example 4.)

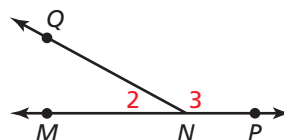
25. $m\angle ABC = 90^\circ$

26. $\overrightarrow{PQ} \perp \overrightarrow{ST}$



27. $m\angle 2 + m\angle 3 = 180^\circ$

28. M is the midpoint of $\overline{AB}$.



In Exercises 29–32, rewrite the definition of the term as a biconditional statement. (See Example 5.)

29. The *midpoint* of a segment is the point that divides the segment into two congruent segments.
30. Two angles are *vertical angles* when their sides form two pairs of opposite rays.
31. *Adjacent angles* are two angles that share a common vertex and side but have no common interior points.
32. Two angles are *supplementary angles* when the sum of their measures is 180° .

In Exercises 33–36, rewrite the statements as a single biconditional statement. (See Example 5.)

33. If a polygon has three sides, then it is a triangle.
If a polygon is a triangle, then it has three sides.
34. If a polygon has four sides, then it is a quadrilateral.
If a polygon is a quadrilateral, then it has four sides.
35. If an angle is a right angle, then it measures 90° .
If an angle measures 90° , then it is a right angle.
36. If an angle is obtuse, then it has a measure between 90° and 180° .
If an angle has a measure between 90° and 180° , then it is obtuse.
37. **ERROR ANALYSIS** Describe and correct the error in rewriting the conditional statement in if-then form.



Conditional statement
All high school students take four English courses.

If-then form
If a high school student takes four courses, then all four are English courses.

38. **ERROR ANALYSIS** Describe and correct the error in writing the converse of the conditional statement.



Conditional statement
If it is raining, then I will bring an umbrella.

Converse
If it is not raining, then I will not bring an umbrella.

In Exercises 39–44, create a truth table for the logical statement. (See Example 6.)

39. $\sim p \rightarrow q$
40. $\sim q \rightarrow p$
41. $\sim(\sim p \rightarrow \sim q)$
42. $\sim(p \rightarrow \sim q)$
43. $q \rightarrow \sim p$
44. $\sim(q \rightarrow p)$

45. **USING STRUCTURE** The statements below describe three ways that rocks are formed.



Igneous rock is formed from the cooling of molten rock.



Sedimentary rock is formed from pieces of other rocks.



Metamorphic rock is formed by changing temperature, pressure, or chemistry.

- a. Write each statement in if-then form.
- b. Write the converse of each of the statements in part (a). Is the converse of each statement true? Explain your reasoning.
- c. Write a true if-then statement about rocks that is different from the ones in parts (a) and (b). Is the converse of your statement true or false? Explain your reasoning.
46. **MAKING AN ARGUMENT** Your friend claims the statement “If I bought a shirt, then I went to the mall” can be written as a true biconditional statement. Your sister says you cannot write it as a biconditional. Who is correct? Explain your reasoning.
47. **REASONING** You are told that the contrapositive of a statement is true. Will that help you determine whether the statement can be written as a true biconditional statement? Explain your reasoning.

48. **PROBLEM SOLVING** Use the conditional statement to identify the if-then statement as the converse, inverse, or contrapositive of the conditional statement. Then use the symbols to represent both statements.

Conditional statement

If I rode my bike to school, then I did not walk to school.

If-then statement

If I did not ride my bike to school, then I walked to school.

p

q

$\sim$

$\rightarrow$

$\leftrightarrow$

USING STRUCTURE In Exercises 49–52, rewrite the conditional statement in if-then form. Then underline the hypothesis and circle the conclusion.

49.

*If you tell the truth,
you don't have to
remember anything.*
Mark Twain

50.

You have to expect
things of yourself
before you can
do them.
Michael Jordan

51.

If one is lucky, a solitary
fantasy can totally
transform one million
realities.
Maya Angelou

52.

Whoever is happy
will make others
happy too.
Anne Frank

53. **MATHEMATICAL CONNECTIONS** Can the statement “If $x^2 - 10 = x + 2$, then $x = 4$ ” be combined with its converse to form a true biconditional statement?

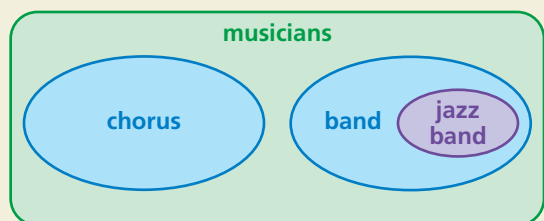
54. **CRITICAL THINKING** The largest natural arch in the United States is Landscape Arch, located in Thompson, Utah. It spans 290 feet.



- a. Use the information to write at least two true conditional statements.
- b. Which type of related conditional statement must also be true? Write the related conditional statements.
- c. What are the other two types of related conditional statements? Write the related conditional statements. Then determine their truth values. Explain your reasoning.
55. **REASONING** Which statement has the same meaning as the given statement?
- Given statement**
You can watch a movie after you do your homework.
- (A) If you do your homework, then you can watch a movie afterward.
- (B) If you do not do your homework, then you can watch a movie afterward.
- (C) If you cannot watch a movie afterward, then do your homework.
- (D) If you can watch a movie afterward, then do not do your homework.
56. **THOUGHT PROVOKING** Write three conditional statements, where one is always true, one is always false, and one depends on the person interpreting the statement.

57. **CRITICAL THINKING** One example of a conditional statement involving dates is “If today is August 31, then tomorrow is September 1.” Write a conditional statement using dates from two different months so that the truth value depends on when the statement is read.

58. **HOW DO YOU SEE IT?** The Venn diagram represents all the musicians at a high school. Write three conditional statements in if-then form describing the relationships between the various groups of musicians.



59. **MULTIPLE REPRESENTATIONS** Create a Venn diagram representing each conditional statement. Write the converse of each conditional statement. Then determine whether each conditional statement and its converse are true or false. Explain your reasoning.

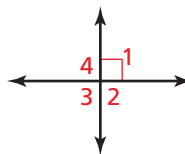
- If you go to the zoo to see a lion, then you will see a cat.
- If you play a sport, then you wear a helmet.
- If this month has 31 days, then it is not February.

60. **DRAWING CONCLUSIONS** You measure the heights of your classmates to get a data set.

- Tell whether this statement is true: If x and y are the least and greatest values in your data set, then the mean of the data is between x and y .
- Write the converse of the statement in part (a). Is the converse true? Explain your reasoning.
- Copy and complete the statement below using *mean*, *median*, or *mode* to make a conditional statement that is true for any data set. Explain your reasoning.

If a data set has a mean, median, and a mode, then the _____ of the data set will always be a data value.

61. **WRITING** Write a conditional statement that is true, but its converse is false.
62. **CRITICAL THINKING** Write a series of if-then statements that allow you to find the measure of each angle, given that $m\angle 1 = 90^\circ$. Use the definition of linear pairs.



63. **WRITING** Advertising slogans such as “Buy these shoes! They will make you a better athlete!” often imply conditional statements. Find an advertisement or write your own slogan. Then write it as a conditional statement.

Maintaining Mathematical Proficiency

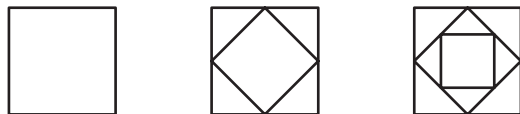
Reviewing what you learned in previous grades and lessons

Find the pattern. Then draw the next two figures in the sequence. (*Skills Review Handbook*)

64.



65.



Find the pattern. Then write the next two numbers. (*Skills Review Handbook*)

66. 1, 3, 5, 7, ...

67. 12, 23, 34, 45, ...

68. $2, \frac{4}{3}, \frac{8}{9}, \frac{16}{27}, \dots$

69. 1, 4, 9, 16, ...

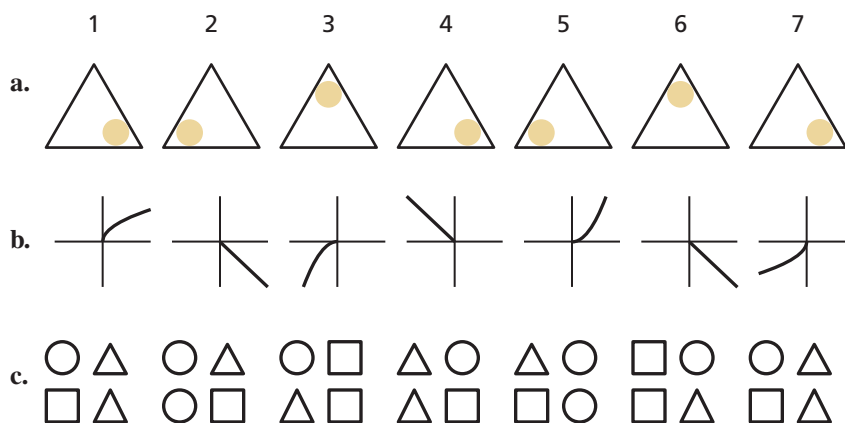
2.2 Inductive and Deductive Reasoning

Essential Question How can you use reasoning to solve problems?

A **conjecture** is an unproven statement based on observations.

EXPLORATION 1 Writing a Conjecture

Work with a partner. Write a conjecture about the pattern. Then use your conjecture to draw the 10th object in the pattern.



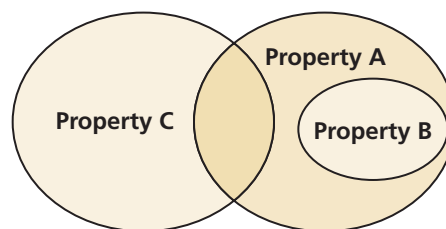
CONSTRUCTING VIABLE ARGUMENTS

To be proficient in math, you need to justify your conclusions and communicate them to others.

EXPLORATION 2 Using a Venn Diagram

Work with a partner. Use the Venn diagram to determine whether the statement is true or false. Justify your answer. Assume that no region of the Venn diagram is empty.

- If an item has Property B, then it has Property A.
- If an item has Property A, then it has Property B.
- If an item has Property A, then it has Property C.
- Some items that have Property A do not have Property B.
- If an item has Property C, then it does not have Property B.
- Some items have both Properties A and C.
- Some items have both Properties B and C.



EXPLORATION 3 Reasoning and Venn Diagrams

Work with a partner. Draw a Venn diagram that shows the relationship between different types of quadrilaterals: squares, rectangles, parallelograms, trapezoids, rhombuses, and kites. Then write several conditional statements that are shown in your diagram, such as “If a quadrilateral is a square, then it is a rectangle.”

Communicate Your Answer

- How can you use reasoning to solve problems?
- Give an example of how you used reasoning to solve a real-life problem.

2.2 Lesson

Core Vocabulary

conjecture, p. 76
inductive reasoning, p. 76
counterexample, p. 77
deductive reasoning, p. 78

What You Will Learn

- ▶ Use inductive reasoning.
- ▶ Use deductive reasoning.

Using Inductive Reasoning

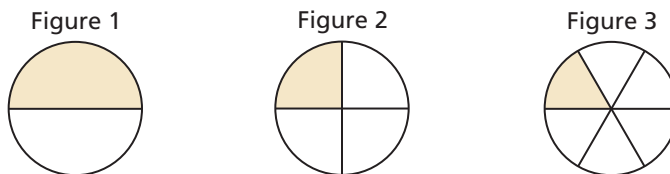
Core Concept

Inductive Reasoning

A **conjecture** is an unproven statement that is based on observations. You use **inductive reasoning** when you find a pattern in specific cases and then write a conjecture for the general case.

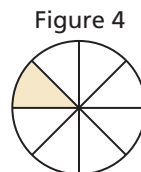
EXAMPLE 1 Describing a Visual Pattern

Describe how to sketch the fourth figure in the pattern. Then sketch the fourth figure.



SOLUTION

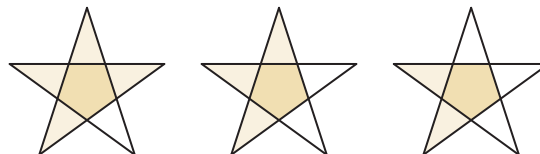
Each circle is divided into twice as many equal regions as the figure number. Sketch the fourth figure by dividing a circle into eighths. Shade the section just above the horizontal segment at the left.



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1. Sketch the fifth figure in the pattern in Example 1.

Sketch the next figure in the pattern.

2. 
3. 

EXAMPLE 2**Making and Testing a Conjecture**

Numbers such as 3, 4, and 5 are called *consecutive integers*. Make and test a conjecture about the sum of any three consecutive integers.

SOLUTION

Step 1 Find a pattern using a few groups of small numbers.

$$3 + 4 + 5 = 12 = 4 \cdot 3$$

$$7 + 8 + 9 = 24 = 8 \cdot 3$$

$$10 + 11 + 12 = 33 = 11 \cdot 3$$

$$16 + 17 + 18 = 51 = 17 \cdot 3$$

Step 2 Make a conjecture.

Conjecture The sum of any three consecutive integers is three times the second number.

Step 3 Test your conjecture using other numbers. For example, test that it works with the groups $-1, 0, 1$ and $100, 101, 102$.

$$-1 + 0 + 1 = 0 = 0 \cdot 3 \quad \checkmark$$

$$100 + 101 + 102 = 303 = 101 \cdot 3 \quad \checkmark$$

Core Concept

Counterexample

To show that a conjecture is true, you must show that it is true for all cases. You can show that a conjecture is false, however, by finding just one *counterexample*. A **counterexample** is a specific case for which the conjecture is false.

EXAMPLE 3**Finding a Counterexample**

A student makes the following conjecture about the sum of two numbers. Find a counterexample to disprove the student's conjecture.

Conjecture The sum of two numbers is always more than the greater number.

SOLUTION

To find a counterexample, you need to find a sum that is less than the greater number.

$$-2 + (-3) = -5$$

$$-5 \not> -2$$

► Because a counterexample exists, the conjecture is false.

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4. Make and test a conjecture about the sign of the product of any three negative integers.
5. Make and test a conjecture about the sum of any five consecutive integers.

Find a counterexample to show that the conjecture is false.

6. The value of x^2 is always greater than the value of x .
7. The sum of two numbers is always greater than their difference.

Using Deductive Reasoning

Core Concept

Deductive Reasoning

Deductive reasoning uses facts, definitions, accepted properties, and the laws of logic to form a logical argument. This is different from *inductive reasoning*, which uses specific examples and patterns to form a conjecture.

Laws of Logic

Law of Detachment

If the hypothesis of a true conditional statement is true, then the conclusion is also true.

Law of Syllogism

If hypothesis p , then conclusion q .
If hypothesis q , then conclusion r .
If hypothesis p , then conclusion r .

$\searrow \quad \swarrow$ If these statements are true,
 $\leftarrow$ then this statement is true.

EXAMPLE 4

Using the Law of Detachment

If two segments have the same length, then they are congruent. You know that $BC = XY$. Using the Law of Detachment, what statement can you make?

SOLUTION

Because $BC = XY$ satisfies the hypothesis of a true conditional statement, the conclusion is also true.

► So, $\overline{BC} \cong \overline{XY}$.

EXAMPLE 5

Using the Law of Syllogism

If possible, use the Law of Syllogism to write a new conditional statement that follows from the pair of true statements.

- a. If $x^2 > 25$, then $x^2 > 20$.
If $x > 5$, then $x^2 > 25$.
- b. If a polygon is regular, then all angles in the interior of the polygon are congruent.
If a polygon is regular, then all its sides are congruent.

SOLUTION

- a. Notice that the conclusion of the second statement is the hypothesis of the first statement. The order in which the statements are given does not affect whether you can use the Law of Syllogism. So, you can write the following new statement.
► If $x > 5$, then $x^2 > 20$.
- b. Neither statement's conclusion is the same as the other statement's hypothesis.
► You cannot use the Law of Syllogism to write a new conditional statement.

EXAMPLE 6**Using Inductive and Deductive Reasoning**

What conclusion can you make about the product of an even integer and any other integer?

SOLUTION**MAKING SENSE OF PROBLEMS**

In geometry, you will frequently use inductive reasoning to make conjectures. You will also use deductive reasoning to show that conjectures are true or false. You will need to know which type of reasoning to use.

Step 1 Look for a pattern in several examples. Use inductive reasoning to make a conjecture.

$$\begin{array}{llll} (-2)(2) = -4 & (-1)(2) = -2 & 2(2) = 4 & 3(2) = 6 \\ (-2)(-4) = 8 & (-1)(-4) = 4 & 2(-4) = -8 & 3(-4) = -12 \end{array}$$

Conjecture Even integer • Any integer = Even integer

Step 2 Let n and m each be any integer. Use deductive reasoning to show that the conjecture is true.

$2n$ is an even integer because any integer multiplied by 2 is even.

$2nm$ represents the product of an even integer $2n$ and any integer m .

$2nm$ is the product of 2 and an integer nm . So, $2nm$ is an even integer.

► The product of an even integer and any integer is an even integer.

EXAMPLE 7**Comparing Inductive and Deductive Reasoning**

Decide whether inductive reasoning or deductive reasoning is used to reach the conclusion. Explain your reasoning.

- Each time Monica kicks a ball up in the air, it returns to the ground. So, the next time Monica kicks a ball up in the air, it will return to the ground.
- All reptiles are cold-blooded. Parrots are not cold-blooded. Sue's pet parrot is not a reptile.

SOLUTION

- Inductive reasoning, because a pattern is used to reach the conclusion.
- Deductive reasoning, because facts about animals and the laws of logic are used to reach the conclusion.

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- If $90^\circ < m\angle R < 180^\circ$, then $\angle R$ is obtuse. The measure of $\angle R$ is 155° . Using the Law of Detachment, what statement can you make?
- Use the Law of Syllogism to write a new conditional statement that follows from the pair of true statements.
If you get an A on your math test, then you can go to the movies.
If you go to the movies, then you can watch your favorite actor.
- Use inductive reasoning to make a conjecture about the sum of a number and itself. Then use deductive reasoning to show that the conjecture is true.
- Decide whether inductive reasoning or deductive reasoning is used to reach the conclusion. Explain your reasoning.
All multiples of 8 are divisible by 4.
64 is a multiple of 8.
So, 64 is divisible by 4.

2.2 Exercises

Dynamic Solutions available at BigIdeasMath.com

Vocabulary and Core Concept Check

- VOCABULARY** How does the prefix “counter-” help you understand the term counterexample?
- WRITING** Explain the difference between inductive reasoning and deductive reasoning.

Monitoring Progress and Modeling with Mathematics

In Exercises 3–8, describe the pattern. Then write or draw the next two numbers, letters, or figures.
(See Example 1.)

3. $1, -2, 3, -4, 5, \dots$ 4. $0, 2, 6, 12, 20, \dots$

5. $Z, Y, X, W, V, \dots$ 6. $J, F, M, A, M, \dots$



In Exercises 9–12, make and test a conjecture about the given quantity. (See Example 2.)

- the product of any two even integers
- the sum of an even integer and an odd integer
- the quotient of a number and its reciprocal
- the quotient of two negative integers

In Exercises 13–16, find a counterexample to show that the conjecture is false. (See Example 3.)

- The product of two positive numbers is always greater than either number.
- If n is a nonzero integer, then $\frac{n+1}{n}$ is always greater than 1.
- If two angles are supplements of each other, then one of the angles must be acute.
- A line s divides $\overline{MN}$ into two line segments. So, the line s is a segment bisector of $\overline{MN}$.

In Exercises 17–20, use the Law of Detachment to determine what you can conclude from the given information, if possible. (See Example 4.)

- If you pass the final, then you pass the class. You passed the final.
- If your parents let you borrow the car, then you will go to the movies with your friend. You will go to the movies with your friend.
- If a quadrilateral is a square, then it has four right angles. Quadrilateral $QRST$ has four right angles.
- If a point divides a line segment into two congruent line segments, then the point is a midpoint. Point P divides $\overline{LH}$ into two congruent line segments.

In Exercises 21–24, use the Law of Syllogism to write a new conditional statement that follows from the pair of true statements, if possible. (See Example 5.)

- If $x < -2$, then $|x| > 2$. If $x > 2$, then $|x| > 2$.
- If $a = 3$, then $5a = 15$. If $\frac{1}{2}a = 1\frac{1}{2}$, then $a = 3$.
- If a figure is a rhombus, then the figure is a parallelogram. If a figure is a parallelogram, then the figure has two pairs of opposite sides that are parallel.
- If a figure is a square, then the figure has four congruent sides. If a figure is a square, then the figure has four right angles.

In Exercises 25–28, state the law of logic that is illustrated.

- If you do your homework, then you can watch TV. If you watch TV, then you can watch your favorite show.
If you do your homework, then you can watch your favorite show.

26. If you miss practice the day before a game, then you will not be a starting player in the game.

You miss practice on Tuesday. You will not start the game Wednesday.

27. If $x > 12$, then $x + 9 > 20$. The value of x is 14.
So, $x + 9 > 20$.

28. If $\angle 1$ and $\angle 2$ are vertical angles, then $\angle 1 \cong \angle 2$.
If $\angle 1 \cong \angle 2$, then $m\angle 1 = m\angle 2$.
If $\angle 1$ and $\angle 2$ are vertical angles, then $m\angle 1 = m\angle 2$.

In Exercises 29 and 30, use inductive reasoning to make a conjecture about the given quantity. Then use deductive reasoning to show that the conjecture is true. (See Example 6.)

29. the sum of two odd integers
30. the product of two odd integers

In Exercises 31–34, decide whether inductive reasoning or deductive reasoning is used to reach the conclusion. Explain your reasoning. (See Example 7.)

31. Each time your mom goes to the store, she buys milk. So, the next time your mom goes to the store, she will buy milk.
32. Rational numbers can be written as fractions. Irrational numbers cannot be written as fractions. So, $\frac{1}{2}$ is a rational number.
33. All men are mortal. Mozart is a man, so Mozart is mortal.
34. Each time you clean your room, you are allowed to go out with your friends. So, the next time you clean your room, you will be allowed to go out with your friends.

ERROR ANALYSIS In Exercises 35 and 36, describe and correct the error in interpreting the statement.

35. If a figure is a rectangle, then the figure has four sides. A trapezoid has four sides.



Using the Law of Detachment, you can conclude that a trapezoid is a rectangle.

36. Each day, you get to school before your friend.



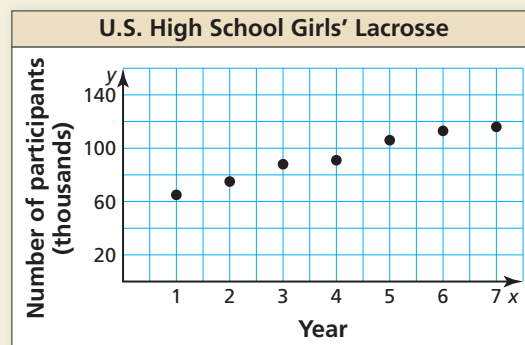
Using deductive reasoning, you can conclude that you will arrive at school before your friend tomorrow.

37. **REASONING** The table shows the average weights of several subspecies of tigers. What conjecture can you make about the relation between the weights of female tigers and the weights of male tigers? Explain your reasoning.



	Weight of female (pounds)	Weight of male (pounds)
Amur	370	660
Bengal	300	480
South China	240	330
Sumatran	200	270
Indo-Chinese	250	400

38. **HOW DO YOU SEE IT?** Determine whether you can make each conjecture from the graph. Explain your reasoning.



- a. More girls will participate in high school lacrosse in Year 8 than those who participated in Year 7.
b. The number of girls participating in high school lacrosse will exceed the number of boys participating in high school lacrosse in Year 9.

39. **MATHEMATICAL CONNECTIONS** Use inductive reasoning to write a formula for the sum of the first n positive even integers.
40. **FINDING A PATTERN** The following are the first nine Fibonacci numbers.

1, 1, 2, 3, 5, 8, 13, 21, 34, ...

- a. Make a conjecture about each of the Fibonacci numbers after the first two.
b. Write the next three numbers in the pattern.
c. Research to find a real-world example of this pattern.

41. **MAKING AN ARGUMENT** Which argument is correct? Explain your reasoning.

Argument 1: If two angles measure 30° and 60° , then the angles are complementary. $\angle 1$ and $\angle 2$ are complementary. So, $m\angle 1 = 30^\circ$ and $m\angle 2 = 60^\circ$.

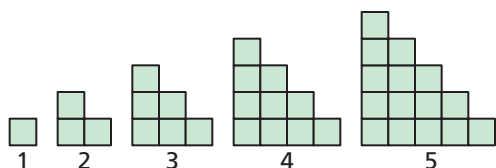
Argument 2: If two angles measure 30° and 60° , then the angles are complementary. The measure of $\angle 1$ is 30° and the measure of $\angle 2$ is 60° . So, $\angle 1$ and $\angle 2$ are complementary.

42. **THOUGHT PROVOKING** The first two terms of a sequence are $\frac{1}{4}$ and $\frac{1}{2}$. Describe three different possible patterns for the sequence. List the first five terms for each sequence.

43. **MATHEMATICAL CONNECTIONS** Use the table to make a conjecture about the relationship between x and y . Then write an equation for y in terms of x . Use the equation to test your conjecture for other values of x .

x	0	1	2	3	4
y	2	5	8	11	14

44. **REASONING** Use the pattern below. Each figure is made of squares that are 1 unit by 1 unit.



- Find the perimeter of each figure. Describe the pattern of the perimeters.
- Predict the perimeter of the 20th figure.

45. **DRAWING CONCLUSIONS** Decide whether each conclusion is valid. Explain your reasoning.

- Yellowstone is a national park in Wyoming.
 - You and your friend went camping at Yellowstone National Park.
 - When you go camping, you go canoeing.
 - If you go on a hike, your friend goes with you.
 - You go on a hike.
 - There is a 3-mile-long trail near your campsite.
- You went camping in Wyoming.
 - Your friend went canoeing.
 - Your friend went on a hike.
 - You and your friend went on a hike on a 3-mile-long trail.

46. **CRITICAL THINKING** Geologists use the Mohs' scale to determine a mineral's hardness. Using the scale, a mineral with a higher rating will leave a scratch on a mineral with a lower rating. Testing a mineral's hardness can help identify the mineral.

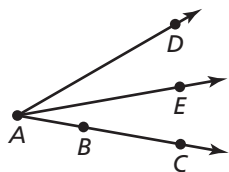
Mineral				
	Talc	Gypsum	Calcite	Fluorite
Mohs' rating	1	2	3	4

- The four minerals are randomly labeled A, B, C, and D. Mineral A is scratched by Mineral B. Mineral C is scratched by all three of the other minerals. What can you conclude? Explain your reasoning.
- What additional test(s) can you use to identify *all* the minerals in part (a)?

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Determine which postulate is illustrated by the statement. (Section 1.2 and Section 1.5)



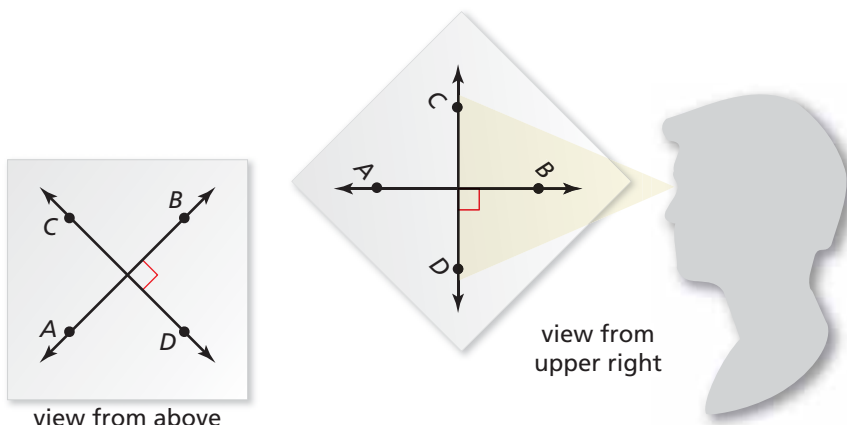
- $AB + BC = AC$
- $m\angle DAC = m\angle DAE + m\angle EAB$
- AD is the absolute value of the difference of the coordinates of A and D.
- $m\angle DAC$ is equal to the absolute value of the difference between the real numbers matched with $\overline{AD}$ and $\overline{AC}$ on a protractor.

2.3 Postulates and Diagrams

Essential Question In a diagram, what can be assumed and what needs to be labeled?

EXPLORATION 1 Looking at a Diagram

Work with a partner. On a piece of paper, draw two perpendicular lines. Label them $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$. Look at the diagram from different angles. Do the lines appear perpendicular regardless of the angle at which you look at them? Describe *all* the angles at which you can look at the lines and have them appear perpendicular.



ATTENDING TO PRECISION

To be proficient in math, you need to state the meanings of the symbols you choose.

EXPLORATION 2 Interpreting a Diagram

Work with a partner. When you draw a diagram, you are communicating with others. It is important that you include sufficient information in the diagram. Use the diagram to determine which of the following statements you can assume to be true. Explain your reasoning.

- All the points shown are coplanar.
- Points D , G , and I are collinear.
- Points A , C , and H are collinear.
- $\overleftrightarrow{EG}$ and $\overleftrightarrow{AH}$ are perpendicular.
- $\angle BCA$ and $\angle ACD$ are a linear pair.
- $\overleftrightarrow{AF}$ and $\overleftrightarrow{BD}$ are perpendicular.
- $\overleftrightarrow{AF}$ and $\overleftrightarrow{BD}$ are coplanar.
- $\overleftrightarrow{AF}$ and $\overleftrightarrow{BD}$ intersect.
- $\angle ACD$ and $\angle BCF$ are vertical angles.
- $\overleftrightarrow{EG}$ and $\overleftrightarrow{BD}$ are parallel.
- $\overleftrightarrow{EG}$ and $\overleftrightarrow{BD}$ do not intersect.
- $\overleftrightarrow{EG}$ and $\overleftrightarrow{BD}$ are perpendicular.
- $\overleftrightarrow{AC}$ and $\overleftrightarrow{FH}$ are the same line.

Communicate Your Answer

- In a diagram, what can be assumed and what needs to be labeled?
- Use the diagram in Exploration 2 to write two statements you can assume to be true and two statements you cannot assume to be true. Your statements should be different from those given in Exploration 2. Explain your reasoning.

2.3 Lesson

Core Vocabulary

line perpendicular to a plane,
p. 86

Previous

postulate
point
line
plane

What You Will Learn

- ▶ Identify postulates using diagrams.
- ▶ Sketch and interpret diagrams.

Identifying Postulates

Here are seven more postulates involving points, lines, and planes.

Postulates

Point, Line, and Plane Postulates

Postulate

Example

2.1 Two Point Postulate

Through any two points, there exists exactly one line.



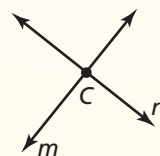
Through points A and B , there is exactly one line ℓ . Line ℓ contains at least two points.

2.2 Line-Point Postulate

A line contains at least two points.

2.3 Line Intersection Postulate

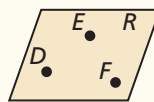
If two lines intersect, then their intersection is exactly one point.



The intersection of line m and line n is point C .

2.4 Three Point Postulate

Through any three noncollinear points, there exists exactly one plane.



Through points D , E , and F , there is exactly one plane, plane R . Plane R contains at least three noncollinear points.

2.5 Plane-Point Postulate

A plane contains at least three noncollinear points.

2.6 Plane-Line Postulate

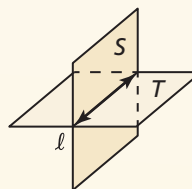
If two points lie in a plane, then the line containing them lies in the plane.



Points D and E lie in plane R , so $\overleftrightarrow{DE}$ lies in plane R .

2.7 Plane Intersection Postulate

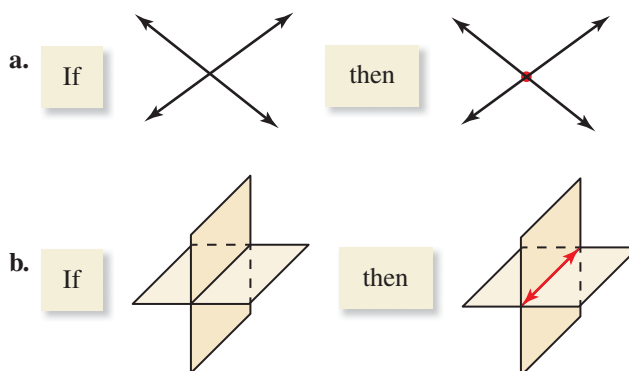
If two planes intersect, then their intersection is a line.



The intersection of plane S and plane T is line ℓ .

EXAMPLE 1**Identifying a Postulate Using a Diagram**

State the postulate illustrated by the diagram.

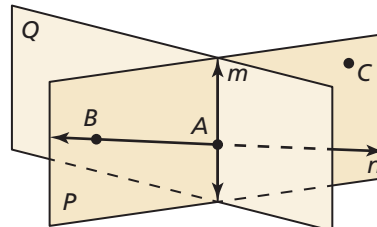
**SOLUTION**

a. **Line Intersection Postulate** If two lines intersect, then their intersection is exactly one point.

b. **Plane Intersection Postulate** If two planes intersect, then their intersection is a line.

EXAMPLE 2**Identifying Postulates from a Diagram**

Use the diagram to write examples of the Plane-Point Postulate and the Plane-Line Postulate.

**SOLUTION**

Plane-Point Postulate Plane P contains at least three noncollinear points, A , B , and C .

Plane-Line Postulate Point A and point B lie in plane P . So, line n containing points A and B also lies in plane P .

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- Use the diagram in Example 2. Which postulate allows you to say that the intersection of plane P and plane Q is a line?
- Use the diagram in Example 2 to write an example of the postulate.
 - Two Point Postulate
 - Line-Point Postulate
 - Line Intersection Postulate

Sketching and Interpreting Diagrams

EXAMPLE 3 Sketching a Diagram

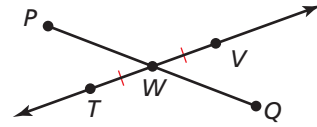
Sketch a diagram showing $\overleftrightarrow{TV}$ intersecting $\overline{PQ}$ at point W , so that $\overline{TW} \cong \overline{WV}$.

SOLUTION

Step 1 Draw $\overleftrightarrow{TV}$ and label points T and V .

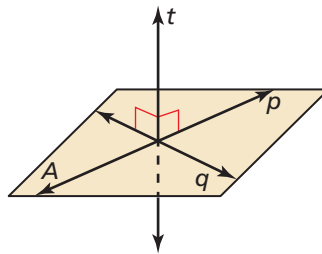
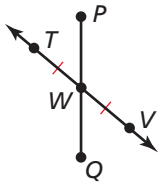
Step 2 Draw point W at the midpoint of $\overline{TV}$.
Mark the congruent segments.

Step 3 Draw $\overline{PQ}$ through W .



ANOTHER WAY

In Example 3, there are many ways you can sketch the diagram. Another way is shown below.



A line is a **line perpendicular to a plane** if and only if the line intersects the plane in a point and is perpendicular to every line in the plane that intersects it at that point.

In a diagram, a line perpendicular to a plane must be marked with a right angle symbol, as shown.

EXAMPLE 4 Interpreting a Diagram

Which of the following statements *cannot* be assumed from the diagram?

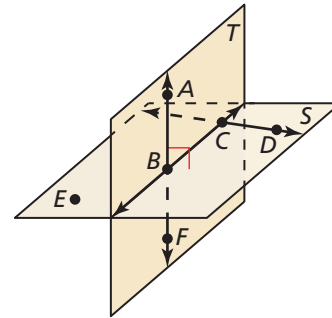
Points A , B , and F are collinear.

Points E , B , and D are collinear.

$\overleftrightarrow{AB} \perp \text{plane } S$

$\overleftrightarrow{CD} \perp \text{plane } T$

$\overleftrightarrow{AF}$ intersects $\overleftrightarrow{BC}$ at point B .



SOLUTION

No drawn line connects points E , B , and D . So, you cannot assume they are collinear. With no right angle marked, you cannot assume $\overleftrightarrow{CD} \perp \text{plane } T$.

Monitoring Progress



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Refer back to Example 3.

- If the given information states that $\overline{PW}$ and $\overline{QW}$ are congruent, how can you indicate that in the diagram?
- Name a pair of supplementary angles in the diagram. Explain.

Use the diagram in Example 4.

- Can you assume that plane S intersects plane T at $\overleftrightarrow{BC}$?
- Explain how you know that $\overleftrightarrow{AB} \perp \overleftrightarrow{BC}$.

2.3 Exercises

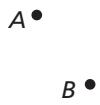
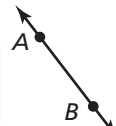
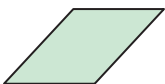
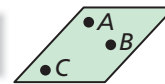
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Vocabulary and Core Concept Check

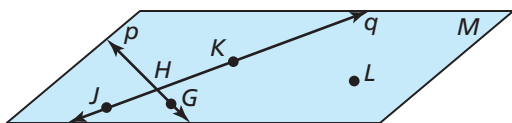
- COMPLETE THE SENTENCE** Through any _____ noncollinear points, there exists exactly one plane.
- WRITING** Explain why you need at least three noncollinear points to determine a plane.

Monitoring Progress and Modeling with Mathematics

In Exercises 3 and 4, state the postulate illustrated by the diagram. (See Example 1.)

- If  then 
- If  then 

In Exercises 5–8, use the diagram to write an example of the postulate. (See Example 2.)

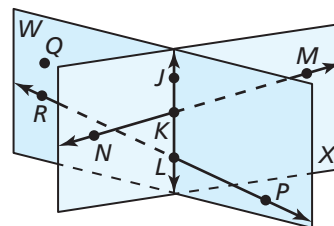


- Line-Point Postulate (Postulate 2.2)
- Line Intersection Postulate (Postulate 2.3)
- Three Point Postulate (Postulate 2.4)
- Plane-Line Postulate (Postulate 2.6)

In Exercises 9–12, sketch a diagram of the description. (See Example 3.)

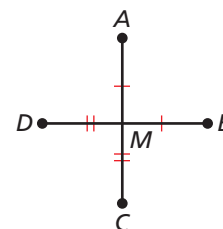
- plane P and line m intersecting plane P at a 90° angle
- $\overline{XY}$ in plane P , $\overline{XY}$ bisected by point A , and point C not on XY
- $\overline{XY}$ intersecting $\overline{WV}$ at point A , so that $XA = VA$
- $\overline{AB}$, $\overline{CD}$, and $\overline{EF}$ are all in plane P , and point X is the midpoint of all three segments.

In Exercises 13–20, use the diagram to determine whether you can assume the statement. (See Example 4.)



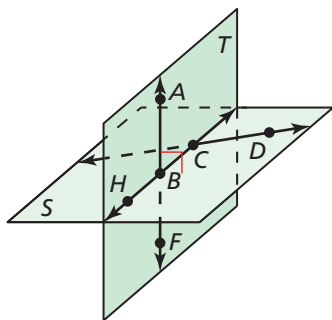
- Planes W and X intersect at $\overleftrightarrow{KL}$.
- Points K , L , M , and N are coplanar.
- Points Q , J , and M are collinear.
- $\overleftrightarrow{MN}$ and $\overleftrightarrow{RP}$ intersect.
- $\overleftrightarrow{JK}$ lies in plane X .
- $\angle PLK$ is a right angle.
- $\angle NKL$ and $\angle JKM$ are vertical angles.
- $\angle NKJ$ and $\angle JKM$ are supplementary angles.

ERROR ANALYSIS In Exercises 21 and 22, describe and correct the error in the statement made about the diagram.



-  M is the midpoint of $\overline{AC}$ and $\overline{BD}$.
-  $\overline{AC}$ intersects $\overline{BD}$ at a 90° angle, so $\overline{AC} \perp \overline{BD}$.

23. **ATTENDING TO PRECISION** Select all the statements about the diagram that you *cannot* conclude.



- (A) $A, B,$ and C are coplanar.
- (B) Plane T intersects plane S in $\overleftrightarrow{BC}$.
- (C) $\overleftrightarrow{AB}$ intersects $\overleftrightarrow{CD}$.
- (D) $H, F,$ and D are coplanar.
- (E) Plane $T \perp$ plane S .
- (F) Point B bisects $\overline{HC}$.
- (G) $\angle ABH$ and $\angle HBF$ are a linear pair.
- (H) $\overleftrightarrow{AF} \perp \overleftrightarrow{CD}$.

24. **HOW DO YOU SEE IT?** Use the diagram of line m and point C . Make a conjecture about how many planes can be drawn so that line m and point C lie in the same plane. Use postulates to justify your conjecture.



25. **MATHEMATICAL CONNECTIONS** One way to graph a linear equation is to plot two points whose coordinates satisfy the equation and then connect them with a line. Which postulate guarantees this process works for any linear equation?
26. **MATHEMATICAL CONNECTIONS** A way to solve a system of two linear equations that intersect is to graph the lines and find the coordinates of their intersection. Which postulate guarantees this process works for any two linear equations?

In Exercises 27 and 28, (a) rewrite the postulate in if-then form. Then (b) write the converse, inverse, and contrapositive and state which ones are true.

27. Two Point Postulate (Postulate 2.1)

28. Plane-Point Postulate (Postulate 2.5)

29. **REASONING** Choose the correct symbol to go between the statements.

number of points to
determine a line

number of points to
determine a plane



30. **CRITICAL THINKING** If two lines intersect, then they intersect in exactly one point by the Line Intersection Postulate (Postulate 2.3). Do the two lines have to be in the same plane? Draw a picture to support your answer. Then explain your reasoning.

31. **MAKING AN ARGUMENT** Your friend claims that even though two planes intersect in a line, it is possible for three planes to intersect in a point. Is your friend correct? Explain your reasoning.

32. **MAKING AN ARGUMENT** Your friend claims that by the Plane Intersection Postulate (Post. 2.7), any two planes intersect in a line. Is your friend's interpretation of the Plane Intersection Postulate (Post. 2.7) correct? Explain your reasoning.

33. **ABSTRACT REASONING** Points $E, F,$ and G all lie in plane P and in plane Q . What must be true about points $E, F,$ and G so that planes P and Q are different planes? What must be true about points $E, F,$ and G to force planes P and Q to be the same plane? Make sketches to support your answers.

34. **THOUGHT PROVOKING** The postulates in this book represent Euclidean geometry. In spherical geometry, all points are points on the surface of a sphere. A line is a circle on the sphere whose diameter is equal to the diameter of the sphere. A plane is the surface of the sphere. Find a postulate on page 84 that is not true in spherical geometry. Explain your reasoning.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Solve the equation. Tell which algebraic property of equality you used. (*Skills Review Handbook*)

35. $t - 6 = -4$

36. $3x = 21$

37. $9 + x = 13$

38. $\frac{x}{7} = 5$

2.1–2.3 What Did You Learn?

Core Vocabulary

conditional statement, *p.* 66
if-then form, *p.* 66
hypothesis, *p.* 66
conclusion, *p.* 66
negation, *p.* 66
converse, *p.* 67

inverse, *p.* 67
contrapositive, *p.* 67
equivalent statements, *p.* 67
perpendicular lines, *p.* 68
biconditional statement, *p.* 69
truth value, *p.* 70

truth table, *p.* 70
conjecture, *p.* 76
inductive reasoning, *p.* 76
counterexample, *p.* 77
deductive reasoning, *p.* 78
line perpendicular to a plane, *p.* 86

Core Concepts

Section 2.1

Conditional Statement, *p.* 66
Negation, *p.* 66
Related Conditionals, *p.* 67

Biconditional Statement, *p.* 69
Making a Truth Table, *p.* 70

Section 2.2

Inductive Reasoning, *p.* 76
Counterexample, *p.* 77

Deductive Reasoning, *p.* 78
Laws of Logic, *p.* 78

Section 2.3

Postulates 2.1–2.7 Point, Line, and Plane Postulates, *p.* 84
Identifying Postulates, *p.* 85
Sketching and Interpreting Diagrams, *p.* 86

Mathematical Practices

1. Provide a counterexample for each *false* conditional statement in Exercises 17–24 on page 71. (You do not need to consider the converse, inverse, and contrapositive statements.)
2. Create a truth table for each of your answers to Exercise 59 on page 74.
3. For Exercise 32 on page 88, write a question you would ask your friend about his or her interpretation.

Study Skills

Using the Features of Your Textbook to Prepare for Quizzes and Tests

- Read and understand the core vocabulary and the contents of the Core Concept boxes.
- Review the Examples and the Monitoring Progress questions. Use the tutorials at *BigIdeasMath.com* for additional help.
- Review previously completed homework assignments.



2.1–2.3 Quiz

Rewrite the conditional statement in if-then form. Then write the converse, inverse, and contrapositive of the conditional statement. Decide whether each statement is true or false. (Section 2.1)

1. An angle measure of 167° is an obtuse angle.
2. You are in a physics class, so you always have homework.
3. I will take my driving test, so I will get my driver's license.

Find a counterexample to show that the conjecture is false. (Section 2.2)

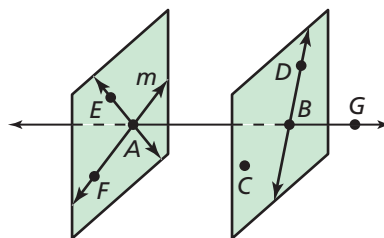
4. The sum of a positive number and a negative number is always positive.
5. If a figure has four sides, then it is a rectangle.

Use inductive reasoning to make a conjecture about the given quantity. Then use deductive reasoning to show that the conjecture is true. (Section 2.2)

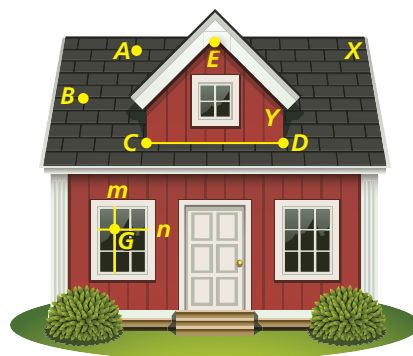
6. the sum of two negative integers
7. the difference of two even integers

Use the diagram to determine whether you can assume the statement. (Section 2.3)

8. Points D , B , and C are coplanar.
9. Plane EAF is parallel to plane DBC .
10. Line m intersects line $\overleftrightarrow{AB}$ at point A .
11. Line $\overleftrightarrow{DC}$ lies in plane DBC .
12. $m\angle DBG = 90^\circ$
13. You and your friend are bowling. Your friend claims that the statement "If I got a strike, then I used the green ball" can be written as a true biconditional statement. Is your friend correct? Explain your reasoning. (Section 2.1)
14. The table shows the 1-mile running times of the members of a high school track team. (Section 2.2)
 - a. What conjecture can you make about the running times of females and males?
 - b. What type of reasoning did you use? Explain.
15. List five of the seven Point, Line, and Plane Postulates on page 84 that the diagram of the house demonstrates. Explain how the postulate is demonstrated in the diagram. (Section 2.3)



Females	Males
06:43	05:41
07:22	06:07
07:04	05:13
06:39	05:21
06:56	06:01



2.4 Algebraic Reasoning

Essential Question How can algebraic properties help you solve an equation?

EXPLORATION 1 Justifying Steps in a Solution

Work with a partner. In previous courses, you studied different properties, such as the properties of equality and the Distributive, Commutative, and Associative Properties. Write the property that justifies each of the following solution steps.

Algebraic Step	Justification
$2(x + 3) - 5 = 5x + 4$	Write given equation.
$2x + 6 - 5 = 5x + 4$	
$2x + 1 = 5x + 4$	
$2x - 2x + 1 = 5x - 2x + 4$	
$1 = 3x + 4$	
$1 - 4 = 3x + 4 - 4$	
$-3 = 3x$	
$\frac{-3}{3} = \frac{3x}{3}$	
$-1 = x$	
$x = -1$	

LOOKING FOR STRUCTURE

To be proficient in math, you need to look closely to discern a pattern or structure.

EXPLORATION 2 Stating Algebraic Properties

Work with a partner. The symbols $\blacklozenge$ and $\bullet$ represent addition and multiplication (not necessarily in that order). Determine which symbol represents which operation. Justify your answer. Then state each algebraic property being illustrated.

Example of Property	Name of Property
$5 \blacklozenge 6 = 6 \blacklozenge 5$	
$5 \bullet 6 = 6 \bullet 5$	
$4 \blacklozenge (5 \blacklozenge 6) = (4 \blacklozenge 5) \blacklozenge 6$	
$4 \bullet (5 \bullet 6) = (4 \bullet 5) \bullet 6$	
$0 \blacklozenge 5 = 0$	
$0 \bullet 5 = 5$	
$1 \blacklozenge 5 = 5$	
$4 \blacklozenge (5 \bullet 6) = 4 \blacklozenge 5 \bullet 4 \blacklozenge 6$	

Communicate Your Answer

- How can algebraic properties help you solve an equation?
- Solve $3(x + 1) - 1 = -13$. Justify each step.

2.4 Lesson

Core Vocabulary

Previous

equation
solve an equation
formula

What You Will Learn

- ▶ Use Algebraic Properties of Equality to justify the steps in solving an equation.
- ▶ Use the Distributive Property to justify the steps in solving an equation.
- ▶ Use properties of equality involving segment lengths and angle measures.

Using Algebraic Properties of Equality

When you *solve an equation*, you use properties of real numbers. Segment lengths and angle measures are real numbers, so you can also use these properties to write logical arguments about geometric figures.

Core Concept

Algebraic Properties of Equality

Let a , b , and c be real numbers.

Addition Property of Equality	If $a = b$, then $a + c = b + c$.
Subtraction Property of Equality	If $a = b$, then $a - c = b - c$.
Multiplication Property of Equality	If $a = b$, then $a \cdot c = b \cdot c$, $c \neq 0$.
Division Property of Equality	If $a = b$, then $\frac{a}{c} = \frac{b}{c}$, $c \neq 0$.
Substitution Property of Equality	If $a = b$, then a can be substituted for b (or b for a) in any equation or expression.

REMEMBER

Inverse operations “undo” each other. Addition and subtraction are inverse operations. Multiplication and division are inverse operations.

EXAMPLE 1 Justifying Steps

Solve $3x + 2 = 23 - 4x$. Justify each step.

SOLUTION

Equation	Explanation	Reason
$3x + 2 = 23 - 4x$	Write the equation.	Given
$3x + 2 + 4x = 23 - 4x + 4x$	Add $4x$ to each side.	Addition Property of Equality
$7x + 2 = 23$	Combine like terms.	Simplify.
$7x + 2 - 2 = 23 - 2$	Subtract 2 from each side.	Subtraction Property of Equality
$7x = 21$	Combine constant terms.	Simplify.
$x = 3$	Divide each side by 7.	Division Property of Equality

- ▶ The solution is $x = 3$.

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Solve the equation. Justify each step.

- $6x - 11 = -35$
- $-2p - 9 = 10p - 17$
- $39 - 5z = -1 + 5z$

Using the Distributive Property

Core Concept

Distributive Property

Let a , b , and c be real numbers.

Sum $a(b + c) = ab + ac$

Difference $a(b - c) = ab - ac$

EXAMPLE 2 Using the Distributive Property

Solve $-5(7w + 8) = 30$. Justify each step.

SOLUTION

Equation	Explanation	Reason
$-5(7w + 8) = 30$	Write the equation.	Given
$-35w - 40 = 30$	Multiply.	Distributive Property
$-35w = 70$	Add 40 to each side.	Addition Property of Equality
$w = -2$	Divide each side by -35 .	Division Property of Equality

► The solution is $w = -2$.

EXAMPLE 3 Solving a Real-Life Problem

You get a raise at your part-time job. To write your raise as a percent, use the formula $p(r + 1) = n$, where p is your previous wage, r is the percent increase (as a decimal), and n is your new wage. Solve the formula for r . What is your raise written as a percent when your hourly wage increases from \$7.25 to \$7.54 per hour?

SOLUTION

Step 1 Solve for r in the formula $p(r + 1) = n$.

Equation	Explanation	Reason
$p(r + 1) = n$	Write the equation.	Given
$pr + p = n$	Multiply.	Distributive Property
$pr = n - p$	Subtract p from each side.	Subtraction Property of Equality
$r = \frac{n - p}{p}$	Divide each side by p .	Division Property of Equality

Step 2 Evaluate $r = \frac{n - p}{p}$ when $n = 7.54$ and $p = 7.25$.

$$r = \frac{n - p}{p} = \frac{7.54 - 7.25}{7.25} = \frac{0.29}{7.25} = 0.04$$

► Your raise is 4%.

REMEMBER

When evaluating expressions, use the order of operations.

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Solve the equation. Justify each step.

4. $3(3x + 14) = -3$

5. $4 = -10b + 6(2 - b)$

6. Solve the formula $A = \frac{1}{2}bh$ for b . Justify each step. Then find the base of a triangle whose area is 952 square feet and whose height is 56 feet.

Using Other Properties of Equality

The following properties of equality are true for all real numbers. Segment lengths and angle measures are real numbers, so these properties of equality are true for all segment lengths and angle measures.

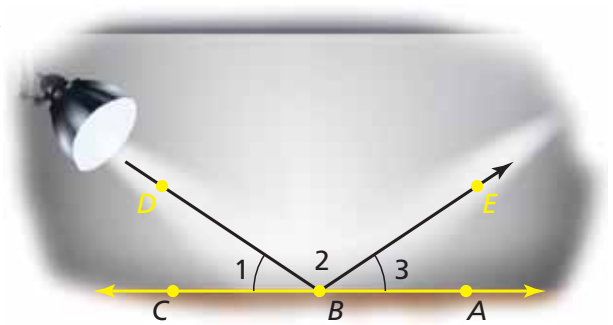
Core Concept

Reflexive, Symmetric, and Transitive Properties of Equality

	Real Numbers	Segment Lengths	Angle Measures
Reflexive Property	$a = a$	$AB = AB$	$m\angle A = m\angle A$
Symmetric Property	If $a = b$, then $b = a$.	If $AB = CD$, then $CD = AB$.	If $m\angle A = m\angle B$, then $m\angle B = m\angle A$.
Transitive Property	If $a = b$ and $b = c$, then $a = c$.	If $AB = CD$ and $CD = EF$, then $AB = EF$.	If $m\angle A = m\angle B$ and $m\angle B = m\angle C$, then $m\angle A = m\angle C$.

EXAMPLE 4 Using Properties of Equality with Angle Measures

You reflect the beam of a spotlight off a mirror lying flat on a stage, as shown. Determine whether $m\angle DBA = m\angle EBC$.



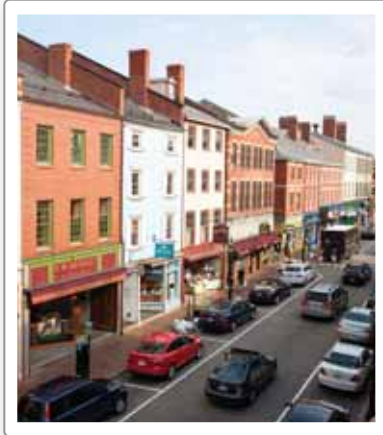
SOLUTION

Equation	Explanation	Reason
$m\angle 1 = m\angle 3$	Marked in diagram.	Given
$m\angle DBA = m\angle 3 + m\angle 2$	Add measures of adjacent angles.	Angle Addition Postulate (Post. 1.4)
$m\angle DBA = m\angle 1 + m\angle 2$	Substitute $m\angle 1$ for $m\angle 3$.	Substitution Property of Equality
$m\angle 1 + m\angle 2 = m\angle EBC$	Add measures of adjacent angles.	Angle Addition Postulate (Post. 1.4)
$m\angle DBA = m\angle EBC$	Both measures are equal to the sum $m\angle 1 + m\angle 2$.	Transitive Property of Equality

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Name the property of equality that the statement illustrates.

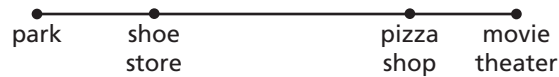
- If $m\angle 6 = m\angle 7$, then $m\angle 7 = m\angle 6$.
- $34^\circ = 34^\circ$
- $m\angle 1 = m\angle 2$ and $m\angle 2 = m\angle 5$. So, $m\angle 1 = m\angle 5$.

EXAMPLE 5**Modeling with Mathematics**

A park, a shoe store, a pizza shop, and a movie theater are located in order on a city street. The distance between the park and the shoe store is the same as the distance between the pizza shop and the movie theater. Show that the distance between the park and the pizza shop is the same as the distance between the shoe store and the movie theater.

SOLUTION

- 1. Understand the Problem** You know that the locations lie in order and that the distance between two of the locations (park and shoe store) is the same as the distance between the other two locations (pizza shop and movie theater). You need to show that two of the other distances are the same.
- 2. Make a Plan** Draw and label a diagram to represent the situation.



Modify your diagram by letting the points P , S , Z , and M represent the park, the shoe store, the pizza shop, and the movie theater, respectively. Show any mathematical relationships.



Use the Segment Addition Postulate (Postulate 1.2) to show that $PZ = SM$.

3. Solve the Problem

Equation	Explanation	Reason
$PS = ZM$	Marked in diagram.	Given
$PZ = PS + SZ$	Add lengths of adjacent segments.	Segment Addition Postulate (Post. 1.2)
$SM = SZ + ZM$	Add lengths of adjacent segments.	Segment Addition Postulate (Post. 1.2)
$PS + SZ = ZM + SZ$	Add SZ to each side of $PS = ZM$.	Addition Property of Equality
$PZ = SM$	Substitute PZ for $PS + SZ$ and SM for $ZM + SZ$.	Substitution Property of Equality

- 4. Look Back** Reread the problem. Make sure your diagram is drawn precisely using the given information. Check the steps in your solution.

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Name the property of equality that the statement illustrates.

- If $JK = KL$ and $KL = 16$, then $JK = 16$.
- $PQ = ST$, so $ST = PQ$.
- $ZY = ZY$
- In Example 5, a hot dog stand is located halfway between the shoe store and the pizza shop, at point H . Show that $PH = HM$.

2.4 Exercises

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Vocabulary and Core Concept Check

- VOCABULARY** The statement “The measure of an angle is equal to itself” is true because of what property?
- DIFFERENT WORDS, SAME QUESTION** Which is different? Find both answers.

What property justifies the following statement?

$$\text{If } c = d, \text{ then } d = c.$$

$$\text{If } JK = LM, \text{ then } LM = JK.$$

$$\text{If } e = f \text{ and } f = g, \text{ then } e = g.$$

$$\text{If } m\angle R = m\angle S, \text{ then } m\angle S = m\angle R.$$

Monitoring Progress and Modeling with Mathematics

In Exercises 3 and 4, write the property that justifies each step.

3.	$3x - 12 = 7x + 8$	Given
	$-4x - 12 = 8$	_____
	$-4x = 20$	_____
	$x = -5$	_____

4.	$5(x - 1) = 4x + 13$	Given
	$5x - 5 = 4x + 13$	_____
	$x - 5 = 13$	_____
	$x = 18$	_____

In Exercises 5–14, solve the equation. Justify each step. (See Examples 1 and 2.)

- | | |
|-----------------------------|-----------------------|
| 5. $5x - 10 = -40$ | 6. $6x + 17 = -7$ |
| 7. $2x - 8 = 6x - 20$ | 8. $4x + 9 = 16 - 3x$ |
| 9. $5(3x - 20) = -10$ | |
| 10. $3(2x + 11) = 9$ | |
| 11. $2(-x - 5) = 12$ | |
| 12. $44 - 2(3x + 4) = -18x$ | |
| 13. $4(5x - 9) = -2(x + 7)$ | |
| 14. $3(4x + 7) = 5(3x + 3)$ | |

In Exercises 15–20, solve the equation for y . Justify each step. (See Example 3.)

- | | |
|-------------------------|--|
| 15. $5x + y = 18$ | 16. $-4x + 2y = 8$ |
| 17. $2y + 0.5x = 16$ | 18. $\frac{1}{2}x - \frac{3}{4}y = -2$ |
| 19. $12 - 3y = 30x + 6$ | 20. $3x + 7 = -7 + 9y$ |

In Exercises 21–24, solve the equation for the given variable. Justify each step. (See Example 3.)

- | | |
|-------------------------|---------------------------------|
| 21. $C = 2\pi r; r$ | 22. $I = Prt; P$ |
| 23. $S = 180(n - 2); n$ | 24. $S = 2\pi r^2 + 2\pi rh; h$ |

In Exercises 25–32, name the property of equality that the statement illustrates.

- If $x = y$, then $3x = 3y$.
- If $AM = MB$, then $AM + 5 = MB + 5$.
- $x = x$
- If $x = y$, then $y = x$.
- $m\angle Z = m\angle Z$
- If $m\angle A = 29^\circ$ and $m\angle B = 29^\circ$, then $m\angle A = m\angle B$.
- If $AB = LM$, then $LM = AB$.
- If $BC = XY$ and $XY = 8$, then $BC = 8$.

In Exercises 33–40, use the property to copy and complete the statement.

33. Substitution Property of Equality:
If $AB = 20$, then $AB + CD = \underline{\hspace{2cm}}$.
34. Symmetric Property of Equality:
If $m\angle 1 = m\angle 2$, then $\underline{\hspace{2cm}}$.
35. Addition Property of Equality:
If $AB = CD$, then $AB + EF = \underline{\hspace{2cm}}$.
36. Multiplication Property of Equality:
If $AB = CD$, then $5 \cdot AB = \underline{\hspace{2cm}}$.
37. Subtraction Property of Equality:
If $LM = XY$, then $LM - GH = \underline{\hspace{2cm}}$.
38. Distributive Property:
If $5(x + 8) = 2$, then $\underline{\hspace{2cm}} + \underline{\hspace{2cm}} = 2$.
39. Transitive Property of Equality:
If $m\angle 1 = m\angle 2$ and $m\angle 2 = m\angle 3$, then $\underline{\hspace{2cm}}$.
40. Reflexive Property of Equality:
 $m\angle ABC = \underline{\hspace{2cm}}$.

ERROR ANALYSIS In Exercises 41 and 42, describe and correct the error in solving the equation.

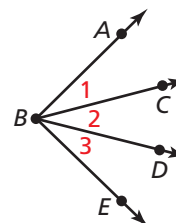
41.  $7x = x + 24$ Given
 $8x = 24$ Addition Property of Equality
 $x = 3$ Division Property of Equality

42.  $6x + 14 = 32$ Given
 $6x = 18$ Division Property of Equality
 $x = 3$ Simplify.

43. **REWRITING A FORMULA** The formula for the perimeter P of a rectangle is $P = 2\ell + 2w$, where ℓ is the length and w is the width. Solve the formula for ℓ . Justify each step. Then find the length of a rectangular lawn with a perimeter of 32 meters and a width of 5 meters.

44. **REWRITING A FORMULA** The formula for the area A of a trapezoid is $A = \frac{1}{2}h(b_1 + b_2)$, where h is the height and b_1 and b_2 are the lengths of the two bases. Solve the formula for b_1 . Justify each step. Then find the length of one of the bases of the trapezoid when the area of the trapezoid is 91 square meters, the height is 7 meters, and the length of the other base is 20 meters.

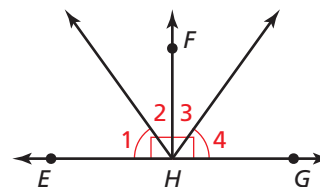
45. **ANALYZING RELATIONSHIPS** In the diagram, $m\angle ABD = m\angle CBE$. Show that $m\angle 1 = m\angle 3$. (See Example 4.)



46. **ANALYZING RELATIONSHIPS** In the diagram, $AC = BD$. Show that $AB = CD$. (See Example 5.)



47. **ANALYZING RELATIONSHIPS** Copy and complete the table to show that $m\angle 2 = m\angle 3$.

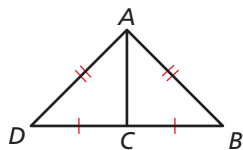


Equation	Reason
$m\angle 1 = m\angle 4$, $m\angle EHF = 90^\circ$, $m\angle GHF = 90^\circ$	Given
$m\angle EHF = m\angle GHF$	
$m\angle EHF = m\angle 1 + m\angle 2$ $m\angle GHF = m\angle 3 + m\angle 4$	
$m\angle 1 + m\angle 2 = m\angle 3 + m\angle 4$	
	Substitution Property of Equality
$m\angle 2 = m\angle 3$	

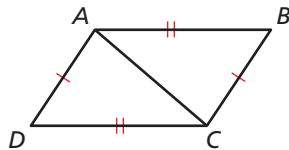
48. **WRITING** Compare the Reflexive Property of Equality with the Symmetric Property of Equality. How are the properties similar? How are they different?

REASONING In Exercises 49 and 50, show that the perimeter of $\triangle ABC$ is equal to the perimeter of $\triangle ADC$.

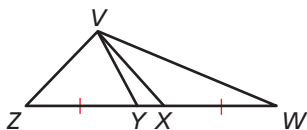
49.



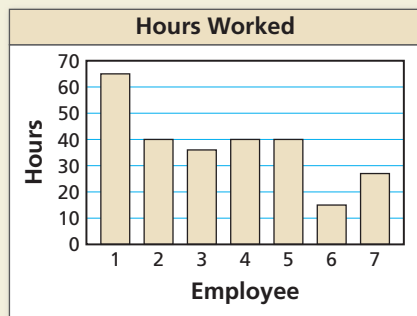
50.



51. **MATHEMATICAL CONNECTIONS** In the figure, $\overline{ZY} \cong \overline{XW}$, $ZX = 5x + 17$, $YW = 10 - 2x$, and $YX = 3$. Find ZY and XW .



52. **HOW DO YOU SEE IT?** The bar graph shows the number of hours each employee works at a grocery store. Give an example of the Reflexive, Symmetric, and Transitive Properties of Equality.



53. **ATTENDING TO PRECISION** Which of the following statements illustrate the Symmetric Property of Equality? Select all that apply.

- (A) If $AC = RS$, then $RS = AC$.
- (B) If $x = 9$, then $9 = x$.
- (C) If $AD = BC$, then $DA = CB$.
- (D) $AB = BA$
- (E) If $AB = LM$ and $LM = RT$, then $AB = RT$.
- (F) If $XY = EF$, then $FE = XY$.

54. **THOUGHT PROVOKING** Write examples from your everyday life to help you remember the Reflexive, Symmetric, and Transitive Properties of Equality. Justify your answers.

55. **MULTIPLE REPRESENTATIONS** The formula to convert a temperature in degrees Fahrenheit ($^{\circ}\text{F}$) to degrees Celsius ($^{\circ}\text{C}$) is $C = \frac{5}{9}(F - 32)$.

- Solve the formula for F . Justify each step.
- Make a table that shows the conversion to Fahrenheit for each temperature: 0°C , 20°C , 32°C , and 41°C .
- Use your table to graph the temperature in degrees Fahrenheit as a function of the temperature in degrees Celsius. Is this a linear function?

56. **REASONING** Select all the properties that would also apply to inequalities. Explain your reasoning.

- (A) Addition Property
- (B) Subtraction Property
- (C) Substitution Property
- (D) Reflexive Property
- (E) Symmetric Property
- (F) Transitive Property

Maintaining Mathematical Proficiency

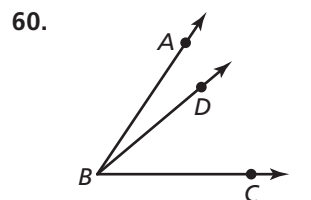
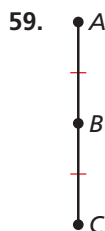
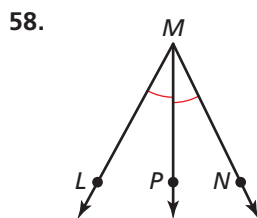
Reviewing what you learned in previous grades and lessons

Name the definition, property, or postulate that is represented by each diagram.

(Section 1.2, Section 1.3, and Section 1.5)



$$XY + YZ = XZ$$



$$m\angle ABD + m\angle DBC = m\angle ABC$$

2.5 Proving Statements about Segments and Angles

Essential Question How can you prove a mathematical statement?

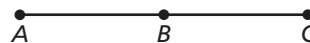
A **proof** is a logical argument that uses deductive reasoning to show that a statement is true.

EXPLORATION 1 Writing Reasons in a Proof

Work with a partner. Four steps of a proof are shown. Write the reasons for each statement.

Given $AC = AB + AB$

Prove $AB = BC$



REASONING ABSTRACTLY

To be proficient in math, you need to know and be able to use algebraic properties.

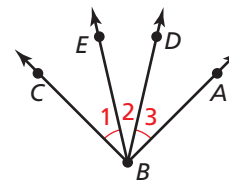
STATEMENTS	REASONS
1. $AC = AB + AB$	1. Given
2. $AB + BC = AC$	2. _____
3. $AB + AB = AB + BC$	3. _____
4. $AB = BC$	4. _____

EXPLORATION 2 Writing Steps in a Proof

Work with a partner. Six steps of a proof are shown. Complete the statements that correspond to each reason.

Given $m\angle 1 = m\angle 3$

Prove $m\angle EBA = m\angle CBD$



STATEMENTS	REASONS
1. _____	1. Given
2. $m\angle EBA = m\angle 2 + m\angle 3$	2. Angle Addition Postulate (Post.1.4)
3. $m\angle EBA = m\angle 2 + m\angle 1$	3. Substitution Property of Equality
4. $m\angle EBA =$ _____	4. Commutative Property of Addition
5. $m\angle 1 + m\angle 2 =$ _____	5. Angle Addition Postulate (Post.1.4)
6. _____	6. Transitive Property of Equality

Communicate Your Answer

3. How can you prove a mathematical statement?

4. Use the given information and the figure to write a proof for the statement.

Given B is the midpoint of $\overline{AC}$.
 C is the midpoint of $\overline{BD}$.

Prove $AB = CD$



2.5 Lesson

Core Vocabulary

proof, p. 100

two-column proof, p. 100

theorem, p. 101

What You Will Learn

- ▶ Write two-column proofs.
- ▶ Name and prove properties of congruence.

Writing Two-Column Proofs

A **proof** is a logical argument that uses deductive reasoning to show that a statement is true. There are several formats for proofs. A **two-column proof** has numbered statements and corresponding reasons that show an argument in a logical order.

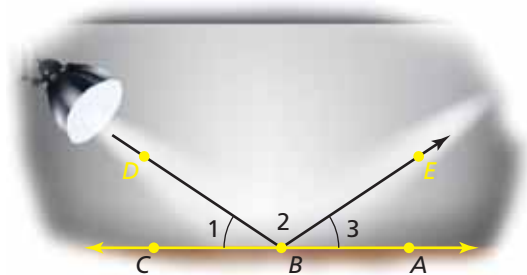
In a two-column proof, each statement in the left-hand column is either given information or the result of applying a known property or fact to statements already made. Each reason in the right-hand column is the explanation for the corresponding statement.

EXAMPLE 1 Writing a Two-Column Proof

Write a two-column proof for the situation in Example 4 from the Section 2.4 lesson.

Given $m\angle 1 = m\angle 3$

Prove $m\angle DBA = m\angle EBC$



STATEMENTS	REASONS
1. $m\angle 1 = m\angle 3$	1. Given
2. $m\angle DBA = m\angle 3 + m\angle 2$	2. Angle Addition Postulate (Post.1.4)
3. $m\angle DBA = m\angle 1 + m\angle 2$	3. Substitution Property of Equality
4. $m\angle 1 + m\angle 2 = m\angle EBC$	4. Angle Addition Postulate (Post.1.4)
5. $m\angle DBA = m\angle EBC$	5. Transitive Property of Equality

Monitoring Progress



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1. Six steps of a two-column proof are shown. Copy and complete the proof.

Given T is the midpoint of $\overline{SU}$.



Prove $x = 5$

STATEMENTS	REASONS
1. T is the midpoint of $\overline{SU}$.	1. _____
2. $\overline{ST} \cong \overline{TU}$	2. Definition of midpoint
3. $ST = TU$	3. Definition of congruent segments
4. $7x = 3x + 20$	4. _____
5. _____	5. Subtraction Property of Equality
6. $x = 5$	6. _____

Using Properties of Congruence

The reasons used in a proof can include definitions, properties, postulates, and *theorems*. A **theorem** is a statement that can be proven. Once you have proven a theorem, you can use the theorem as a reason in other proofs.

Theorems

Theorem 2.1 Properties of Segment Congruence

Segment congruence is reflexive, symmetric, and transitive.

Reflexive For any segment AB , $\overline{AB} \cong \overline{AB}$.

Symmetric If $\overline{AB} \cong \overline{CD}$, then $\overline{CD} \cong \overline{AB}$.

Transitive If $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$, then $\overline{AB} \cong \overline{EF}$.

Proofs Ex. 11, p. 103; Example 3, p. 101; Chapter Review 2.5 Example, p. 118

Theorem 2.2 Properties of Angle Congruence

Angle congruence is reflexive, symmetric, and transitive.

Reflexive For any angle A , $\angle A \cong \angle A$.

Symmetric If $\angle A \cong \angle B$, then $\angle B \cong \angle A$.

Transitive If $\angle A \cong \angle B$ and $\angle B \cong \angle C$, then $\angle A \cong \angle C$.

Proofs Ex. 25, p. 118; 2.5 Concept Summary, p. 102; Ex. 12, p. 103

EXAMPLE 2 Naming Properties of Congruence

Name the property that the statement illustrates.

- If $\angle T \cong \angle V$ and $\angle V \cong \angle R$, then $\angle T \cong \angle R$.
- If $\overline{JL} \cong \overline{YZ}$, then $\overline{YZ} \cong \overline{JL}$.

SOLUTION

- Transitive Property of Angle Congruence
- Symmetric Property of Segment Congruence

STUDY TIP

When writing a proof, organize your reasoning by copying or drawing a diagram for the situation described. Then identify the **Given** and **Prove** statements.

EXAMPLE 3 Proving a Symmetric Property of Congruence

Write a two-column proof for the Symmetric Property of Segment Congruence.

Given $\overline{LM} \cong \overline{NP}$

Prove $\overline{NP} \cong \overline{LM}$



STATEMENTS	REASONS
1. $\overline{LM} \cong \overline{NP}$	1. Given
2. $LM = NP$	2. Definition of congruent segments
3. $NP = LM$	3. Symmetric Property of Equality
4. $\overline{NP} \cong \overline{LM}$	4. Definition of congruent segments

EXAMPLE 4

Writing a Two-Column Proof

Prove this property of midpoints: If you know that M is the midpoint of $\overline{AB}$, prove that AB is two times AM and AM is one-half AB .

Given M is the midpoint of $\overline{AB}$.



Prove $AB = 2AM$, $AM = \frac{1}{2}AB$

STATEMENTS	REASONS
1. M is the midpoint of $\overline{AB}$.	1. Given
2. $\overline{AM} \cong \overline{MB}$	2. Definition of midpoint
3. $AM = MB$	3. Definition of congruent segments
4. $AM + MB = AB$	4. Segment Addition Postulate (Post. 1.2)
5. $AM + AM = AB$	5. Substitution Property of Equality
6. $2AM = AB$	6. Distributive Property
7. $AM = \frac{1}{2}AB$	7. Division Property of Equality

Monitoring Progress



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Name the property that the statement illustrates.

2. $\overline{GH} \cong \overline{GH}$

3. If $\angle K \cong \angle P$, then $\angle P \cong \angle K$.

4. Look back at Example 4. What would be different if you were proving that $AB = 2 \cdot MB$ and that $MB = \frac{1}{2}AB$ instead?

Concept Summary

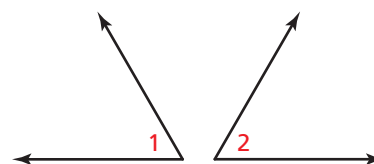
Writing a Two-Column Proof

In a proof, you make one statement at a time until you reach the conclusion. Because you make statements based on facts, you are using deductive reasoning. Usually the first statement-and-reason pair you write is given information.

Proof of the Symmetric Property of Angle Congruence

Given $\angle 1 \cong \angle 2$

Prove $\angle 2 \cong \angle 1$



Copy or draw diagrams and label given information to help develop proofs. Do not mark or label the information in the Prove statement on the diagram.

STATEMENTS	REASONS
1. $\angle 1 \cong \angle 2$	1. Given
2. $m\angle 1 = m\angle 2$	2. Definition of congruent angles
3. $m\angle 2 = m\angle 1$	3. Symmetric Property of Equality
4. $\angle 2 \cong \angle 1$	4. Definition of congruent angles

statements based on facts that you know or on conclusions from deductive reasoning

The number of statements will vary.

Remember to give a reason for the last statement.

definitions, postulates, or proven theorems that allow you to state the corresponding statement

2.5 Exercises

Dynamic Solutions available at BigIdeasMath.com

Vocabulary and Core Concept Check

- WRITING** How is a theorem different from a postulate?
- COMPLETE THE SENTENCE** In a two-column proof, each _____ is on the left and each _____ is on the right.

Monitoring Progress and Modeling with Mathematics

In Exercises 3 and 4, copy and complete the proof. (See Example 1.)

3. **Given** $PQ = RS$

Prove $PR = QS$



STATEMENTS

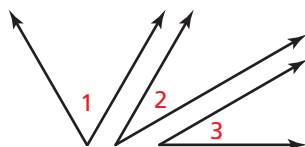
- $PQ = RS$
- $PQ + QR = RS + QR$
- _____
- $RS + QR = QS$
- $PR = QS$

REASONS

- _____
- _____
- Segment Addition Postulate (Post. 1.2)
- Segment Addition Postulate (Post. 1.2)
- _____

4. **Given** $\angle 1$ is a complement of $\angle 2$.
 $\angle 2 \cong \angle 3$

Prove $\angle 1$ is a complement of $\angle 3$.



STATEMENTS

- $\angle 1$ is a complement of $\angle 2$.
- $\angle 2 \cong \angle 3$
- $m\angle 1 + m\angle 2 = 90^\circ$
- $m\angle 2 = m\angle 3$
- _____
- $\angle 1$ is a complement of $\angle 3$.

REASONS

- Given
- _____
- _____
- Definition of congruent angles
- Substitution Property of Equality
- _____

In Exercises 5–10, name the property that the statement illustrates. (See Example 2.)

- If $\overline{PQ} \cong \overline{ST}$ and $\overline{ST} \cong \overline{UV}$, then $\overline{PQ} \cong \overline{UV}$.
- $\angle F \cong \angle F$
- If $\angle G \cong \angle H$, then $\angle H \cong \angle G$.
- $\overline{DE} \cong \overline{DE}$
- If $\overline{XY} \cong \overline{UV}$, then $\overline{UV} \cong \overline{XY}$.
- If $\angle L \cong \angle M$ and $\angle M \cong \angle N$, then $\angle L \cong \angle N$.

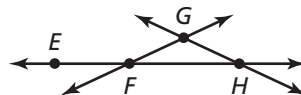
PROOF In Exercises 11 and 12, write a two-column proof for the property. (See Example 3.)

- Reflexive Property of Segment Congruence (Thm. 2.1)
- Transitive Property of Angle Congruence (Thm. 2.2)

PROOF In Exercises 13 and 14, write a two-column proof. (See Example 4.)

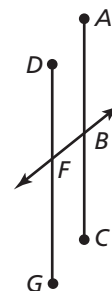
13. **Given** $\angle GFH \cong \angle GHF$

Prove $\angle EFG$ and $\angle GHF$ are supplementary.



14. **Given** $\overline{AB} \cong \overline{FG}$,
 $\overline{BF}$ bisects $\overline{AC}$ and $\overline{DG}$.

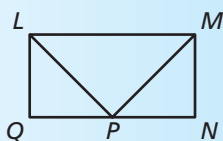
Prove $\overline{BC} \cong \overline{DF}$



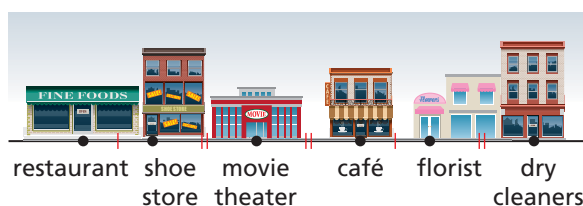
15. **ERROR ANALYSIS** In the diagram, $\overline{MN} \cong \overline{LQ}$ and $\overline{LQ} \cong \overline{PN}$. Describe and correct the error in the reasoning.



Because $\overline{MN} \cong \overline{LQ}$ and $\overline{LQ} \cong \overline{PN}$, then $\overline{MN} \cong \overline{PN}$ by the Reflexive Property of Segment Congruence (Thm. 2.1).



16. **MODELING WITH MATHEMATICS** The distance from the restaurant to the shoe store is the same as the distance from the café to the florist. The distance from the shoe store to the movie theater is the same as the distance from the movie theater to the café, and from the florist to the dry cleaners.



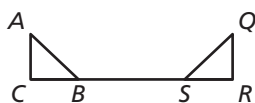
Use the steps below to prove that the distance from the restaurant to the movie theater is the same as the distance from the café to the dry cleaners.

- State what is given and what is to be proven for the situation.
- Write a two-column proof.

17. **REASONING** In the sculpture shown, $\angle 1 \cong \angle 2$ and $\angle 2 \cong \angle 3$. Classify the triangle and justify your answer.

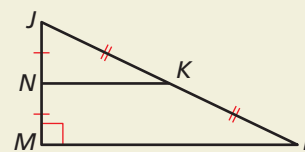


18. **MAKING AN ARGUMENT** In the figure, $\overline{SR} \cong \overline{CB}$ and $\overline{AC} \cong \overline{QR}$. Your friend claims that, because of this, $\overline{CB} \cong \overline{AC}$ by the Transitive Property of Segment Congruence (Thm. 2.1). Is your friend correct? Explain your reasoning.



19. **WRITING** Explain why you do not use inductive reasoning when writing a proof.

20. **HOW DO YOU SEE IT?** Use the figure to write Given and Prove statements for each conclusion.



- The acute angles of a right triangle are complementary.
 - A segment connecting the midpoints of two sides of a triangle is half as long as the third side.
21. **REASONING** Fold two corners of a piece of paper so their edges match, as shown.

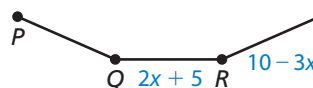
- What do you notice about the angle formed at the top of the page by the folds?
- Write a two-column proof to show that the angle measure is always the same no matter how you make the folds.



22. **THOUGHT PROVOKING** The distance from Springfield to Lakewood City is equal to the distance from Springfield to Bettsville. Janisburg is 50 miles farther from Springfield than Bettsville. Moon Valley is 50 miles farther from Springfield than Lakewood City is. Use line segments to draw a diagram that represents this situation.

23. **MATHEMATICAL CONNECTIONS** Solve for x using the given information. Justify each step.

Given $\overline{QR} \cong \overline{PQ}$, $\overline{RS} \cong \overline{PQ}$



Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Use the figure. (Section 1.6)

24. $\angle 1$ is a complement of $\angle 4$, and $m\angle 1 = 33^\circ$. Find $m\angle 4$.

25. $\angle 3$ is a supplement of $\angle 2$, and $m\angle 2 = 147^\circ$. Find $m\angle 3$.

26. Name a pair of vertical angles.



2.6 Proving Geometric Relationships

Essential Question How can you use a flowchart to prove a mathematical statement?

EXPLORATION 1 Matching Reasons in a Flowchart Proof

Work with a partner. Match each reason with the correct step in the flowchart.

Given $AC = AB + AB$



Prove $AB = BC$

MODELING WITH MATHEMATICS

To be proficient in math, you need to map relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and formulas.

$$AC = AB + AB$$

$$AB + BC = AC$$

$$AB + AB = AB + BC$$

$$AB = BC$$

A. Segment Addition Postulate (Post. 1.2) **B.** Given

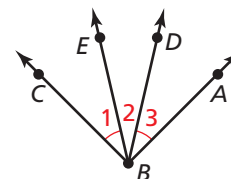
C. Transitive Property of Equality **D.** Subtraction Property of Equality

EXPLORATION 2 Matching Reasons in a Flowchart Proof

Work with a partner. Match each reason with the correct step in the flowchart.

Given $m\angle 1 = m\angle 3$

Prove $m\angle EBA = m\angle CBD$



$$m\angle 1 = m\angle 3$$

$$m\angle EBA = m\angle 2 + m\angle 3$$

$$m\angle EBA = m\angle 2 + m\angle 1$$

$$m\angle EBA = m\angle 1 + m\angle 2$$

$$m\angle 1 + m\angle 2 = m\angle CBD$$

$$m\angle EBA = m\angle CBD$$

A. Angle Addition Postulate (Post. 1.4)

B. Transitive Property of Equality

C. Substitution Property of Equality

D. Angle Addition Postulate (Post. 1.4)

E. Given

F. Commutative Property of Addition

Communicate Your Answer

- How can you use a flowchart to prove a mathematical statement?
- Compare the flowchart proofs above with the two-column proofs in the Section 2.5 Explorations. Explain the advantages and disadvantages of each.

2.6 Lesson

Core Vocabulary

flowchart proof, or flow proof,
p. 106
paragraph proof, p. 108

What You Will Learn

- ▶ Write flowchart proofs to prove geometric relationships.
- ▶ Write paragraph proofs to prove geometric relationships.

Writing Flowchart Proofs

Another proof format is a **flowchart proof**, or **flow proof**, which uses boxes and arrows to show the flow of a logical argument. Each reason is below the statement it justifies. A flowchart proof of the *Right Angles Congruence Theorem* is shown in Example 1. This theorem is useful when writing proofs involving right angles.

Theorem

Theorem 2.3 Right Angles Congruence Theorem

All right angles are congruent.

Proof Example 1, p. 106

STUDY TIP

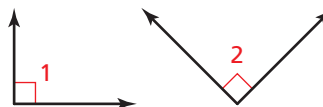
When you prove a theorem, write the hypothesis of the theorem as the **Given** statement. The conclusion is what you must **Prove**.

EXAMPLE 1 Proving the Right Angles Congruence Theorem

Use the given flowchart proof to write a two-column proof of the Right Angles Congruence Theorem.

Given $\angle 1$ and $\angle 2$ are right angles.

Prove $\angle 1 \cong \angle 2$



Flowchart Proof

$\angle 1$ and $\angle 2$ are right angles.

Given

$m\angle 1 = 90^\circ, m\angle 2 = 90^\circ$

Definition of right angle

$m\angle 1 = m\angle 2$

Transitive Property of Equality

$\angle 1 \cong \angle 2$

Definition of congruent angles

Two-Column Proof

STATEMENTS	REASONS
1. $\angle 1$ and $\angle 2$ are right angles.	1. Given
2. $m\angle 1 = 90^\circ, m\angle 2 = 90^\circ$	2. Definition of right angle
3. $m\angle 1 = m\angle 2$	3. Transitive Property of Equality
4. $\angle 1 \cong \angle 2$	4. Definition of congruent angles

Monitoring Progress

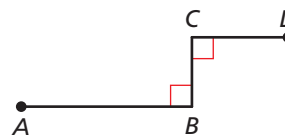


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- Copy and complete the flowchart proof. Then write a two-column proof.

Given $\overline{AB} \perp \overline{BC}, \overline{DC} \perp \overline{BC}$

Prove $\angle B \cong \angle C$



$\overline{AB} \perp \overline{BC}, \overline{DC} \perp \overline{BC}$

Given

$\angle B \cong \angle C$

Definition of $\perp$ lines

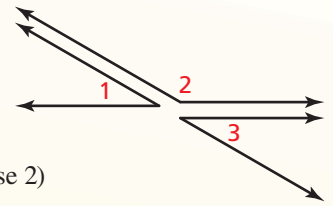
Theorems

Theorem 2.4 Congruent Supplements Theorem

If two angles are supplementary to the same angle (or to congruent angles), then they are congruent.

If $\angle 1$ and $\angle 2$ are supplementary and $\angle 3$ and $\angle 2$ are supplementary, then $\angle 1 \cong \angle 3$.

Proof Example 2, p. 107 (case 1); Ex. 20, p. 113 (case 2)

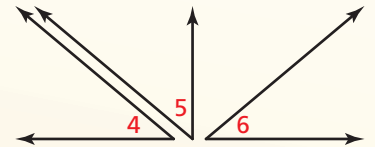


Theorem 2.5 Congruent Complements Theorem

If two angles are complementary to the same angle (or to congruent angles), then they are congruent.

If $\angle 4$ and $\angle 5$ are complementary and $\angle 6$ and $\angle 5$ are complementary, then $\angle 4 \cong \angle 6$.

Proof Ex. 19, p. 112 (case 1); Ex. 22, p. 113 (case 2)



To prove the Congruent Supplements Theorem, you must prove two cases: one with angles supplementary to the same angle and one with angles supplementary to congruent angles. The proof of the Congruent Complements Theorem also requires two cases.

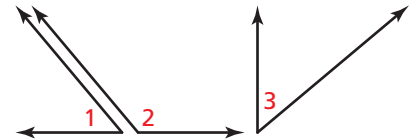
EXAMPLE 2

Proving a Case of Congruent Supplements Theorem

Use the given two-column proof to write a flowchart proof that proves that two angles supplementary to the same angle are congruent.

Given $\angle 1$ and $\angle 2$ are supplementary.
 $\angle 3$ and $\angle 2$ are supplementary.

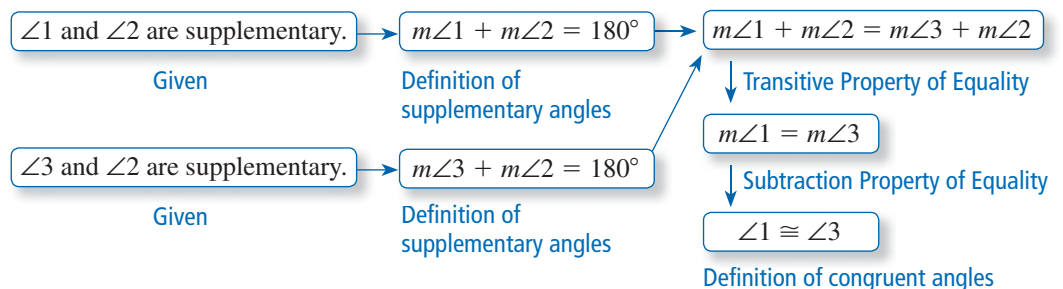
Prove $\angle 1 \cong \angle 3$



Two-Column Proof

STATEMENTS	REASONS
1. $\angle 1$ and $\angle 2$ are supplementary. $\angle 3$ and $\angle 2$ are supplementary.	1. Given
2. $m\angle 1 + m\angle 2 = 180^\circ$, $m\angle 3 + m\angle 2 = 180^\circ$	2. Definition of supplementary angles
3. $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 2$	3. Transitive Property of Equality
4. $m\angle 1 = m\angle 3$	4. Subtraction Property of Equality
5. $\angle 1 \cong \angle 3$	5. Definition of congruent angles

Flowchart Proof



Writing Paragraph Proofs

Another proof format is a **paragraph proof**, which presents the statements and reasons of a proof as sentences in a paragraph. It uses words to explain the logical flow of the argument.

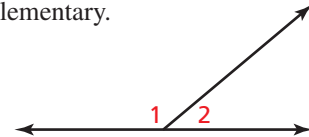
Two intersecting lines form pairs of vertical angles and linear pairs. The *Linear Pair Postulate* formally states the relationship between linear pairs. You can use this postulate to prove the *Vertical Angles Congruence Theorem*.

Postulate and Theorem

Postulate 2.8 Linear Pair Postulate

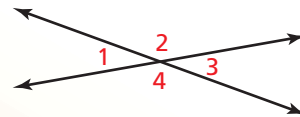
If two angles form a linear pair, then they are supplementary.

$\angle 1$ and $\angle 2$ form a linear pair, so $\angle 1$ and $\angle 2$ are supplementary and $m\angle 1 + m\angle 2 = 180^\circ$.



Theorem 2.6 Vertical Angles Congruence Theorem

Vertical angles are congruent.



Proof Example 3, p. 108

$$\angle 1 \cong \angle 3, \angle 2 \cong \angle 4$$

EXAMPLE 3

Proving the Vertical Angles Congruence Theorem

Use the given paragraph proof to write a two-column proof of the Vertical Angles Congruence Theorem.

Given $\angle 5$ and $\angle 7$ are vertical angles.

Prove $\angle 5 \cong \angle 7$



STUDY TIP

In paragraph proofs, *transitional words* such as *so*, *then*, and *therefore* help make the logic clear.

Paragraph Proof

$\angle 5$ and $\angle 7$ are vertical angles formed by intersecting lines. As shown in the diagram, $\angle 5$ and $\angle 6$ are a linear pair, and $\angle 6$ and $\angle 7$ are a linear pair. Then, by the Linear Pair Postulate, $\angle 5$ and $\angle 6$ are supplementary and $\angle 6$ and $\angle 7$ are supplementary. So, by the Congruent Supplements Theorem, $\angle 5 \cong \angle 7$.

JUSTIFYING STEPS

You can use information labeled in a diagram in your proof.

Two-Column Proof

STATEMENTS	REASONS
1. $\angle 5$ and $\angle 7$ are vertical angles.	1. Given
2. $\angle 5$ and $\angle 6$ are a linear pair. $\angle 6$ and $\angle 7$ are a linear pair.	2. Definition of linear pair, as shown in the diagram
3. $\angle 5$ and $\angle 6$ are supplementary. $\angle 6$ and $\angle 7$ are supplementary.	3. Linear Pair Postulate
4. $\angle 5 \cong \angle 7$	4. Congruent Supplements Theorem

2. Copy and complete the two-column proof. Then write a flowchart proof.

Given $AB = DE, BC = CD$

Prove $\overline{AC} \cong \overline{CE}$



STATEMENTS	REASONS
1. $AB = DE, BC = CD$	1. Given
2. $AB + BC = BC + DE$	2. Addition Property of Equality
3. _____	3. Substitution Property of Equality
4. $AB + BC = AC, CD + DE = CE$	4. _____
5. _____	5. Substitution Property of Equality
6. $\overline{AC} \cong \overline{CE}$	6. _____

3. Rewrite the two-column proof in Example 3 without using the Congruent Supplements Theorem. How many steps do you save by using the theorem?

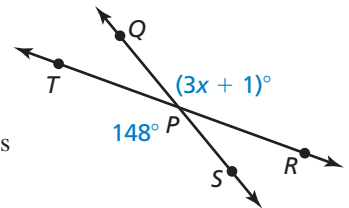
EXAMPLE 4

Using Angle Relationships

Find the value of x .

SOLUTION

$\angle TPS$ and $\angle QPR$ are vertical angles. By the Vertical Angles Congruence Theorem, the angles are congruent. Use this fact to write and solve an equation.



$$m\angle TPS = m\angle QPR$$

Definition of congruent angles

$$148^\circ = (3x + 1)^\circ$$

Substitute angle measures.

$$147 = 3x$$

Subtract 1 from each side.

$$49 = x$$

Divide each side by 3.

► So, the value of x is 49.

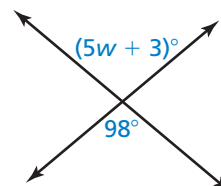
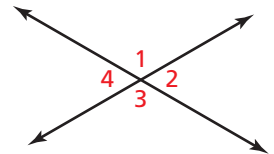
Use the diagram and the given angle measure to find the other three angle measures.

4. $m\angle 1 = 117^\circ$

5. $m\angle 2 = 59^\circ$

6. $m\angle 4 = 88^\circ$

7. Find the value of w .

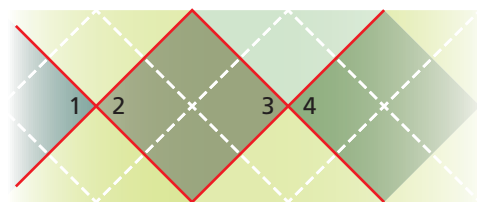


EXAMPLE 5**Using the Vertical Angles Congruence Theorem**

Write a paragraph proof.

Given $\angle 1 \cong \angle 4$

Prove $\angle 2 \cong \angle 3$

**Paragraph Proof**

$\angle 1$ and $\angle 4$ are congruent. By the Vertical Angles Congruence Theorem, $\angle 1 \cong \angle 2$ and $\angle 3 \cong \angle 4$. By the Transitive Property of Angle Congruence (Theorem 2.2), $\angle 2 \cong \angle 4$. Using the Transitive Property of Angle Congruence (Theorem 2.2) once more, $\angle 2 \cong \angle 3$.

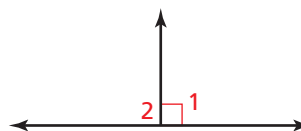
Monitoring Progress

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8. Write a paragraph proof.

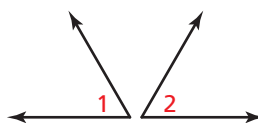
Given $\angle 1$ is a right angle.

Prove $\angle 2$ is a right angle.

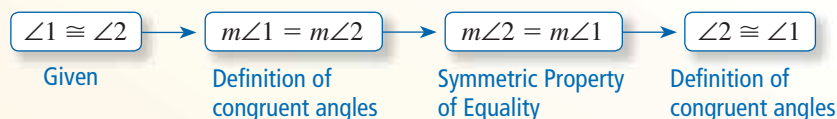
**Concept Summary****Types of Proofs****Symmetric Property of Angle Congruence (Theorem 2.2)**

Given $\angle 1 \cong \angle 2$

Prove $\angle 2 \cong \angle 1$

**Two-Column Proof**

STATEMENTS	REASONS
1. $\angle 1 \cong \angle 2$	1. Given
2. $m\angle 1 = m\angle 2$	2. Definition of congruent angles
3. $m\angle 2 = m\angle 1$	3. Symmetric Property of Equality
4. $\angle 2 \cong \angle 1$	4. Definition of congruent angles

Flowchart Proof**Paragraph Proof**

$\angle 1$ is congruent to $\angle 2$. By the definition of congruent angles, the measure of $\angle 1$ is equal to the measure of $\angle 2$. The measure of $\angle 2$ is equal to the measure of $\angle 1$ by the Symmetric Property of Equality. Then by the definition of congruent angles, $\angle 2$ is congruent to $\angle 1$.

2.6 Exercises

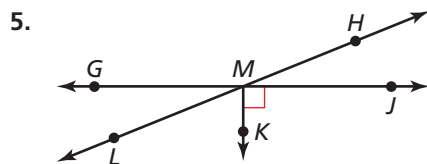
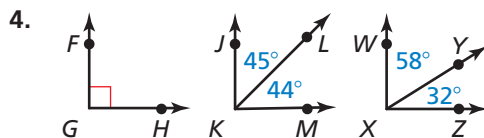
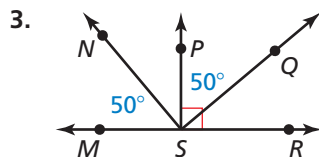
Dynamic Solutions available at BigIdeasMath.com

Vocabulary and Core Concept Check

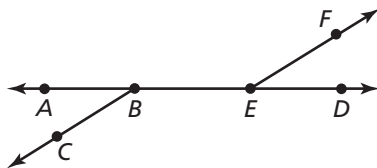
- WRITING** Explain why all right angles are congruent.
- VOCABULARY** What are the two types of angles that are formed by intersecting lines?

Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, identify the pair(s) of congruent angles in the figures. Explain how you know they are congruent. (See Examples 1, 2, and 3.)

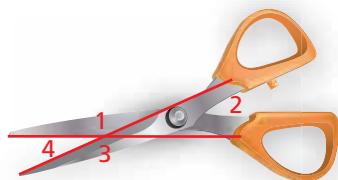


6. $\angle ABC$ is supplementary to $\angle CBD$.
 $\angle CBD$ is supplementary to $\angle DEF$.

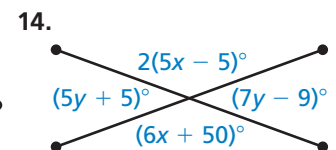
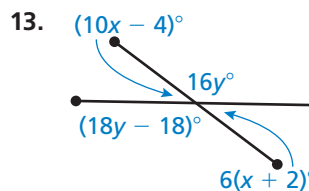
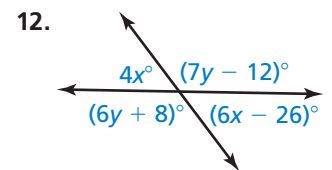
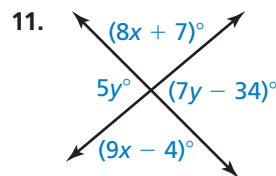


In Exercises 7–10, use the diagram and the given angle measure to find the other three measures. (See Example 3.)

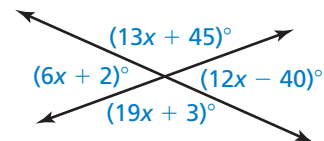
- $m\angle 1 = 143^\circ$
- $m\angle 3 = 159^\circ$
- $m\angle 2 = 34^\circ$
- $m\angle 4 = 29^\circ$



In Exercises 11–14, find the values of x and y . (See Example 4.)



ERROR ANALYSIS In Exercises 15 and 16, describe and correct the error in using the diagram to find the value of x .

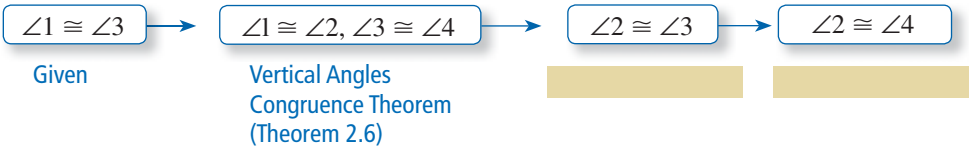
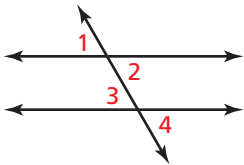


15. $(13x + 45)^\circ + (19x + 3)^\circ = 180^\circ$
 $32x + 48 = 180$
 $32x = 132$
 $x = 4.125$

16. $(13x + 45)^\circ + (12x - 40)^\circ = 90^\circ$
 $25x + 5 = 90$
 $25x = 85$
 $x = 3.4$

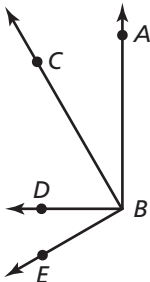
17. **PROOF** Copy and complete the flowchart proof. Then write a two-column proof.
(See Example 1.)

Given $\angle 1 \cong \angle 3$
Prove $\angle 2 \cong \angle 4$



18. **PROOF** Copy and complete the two-column proof. Then write a flowchart proof.
(See Example 2.)

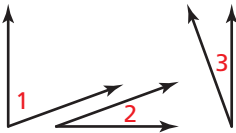
Given $\angle ABD$ is a right angle.
 $\angle CBE$ is a right angle.
Prove $\angle ABC \cong \angle DBE$



STATEMENTS	REASONS
1. $\angle ABD$ is a right angle. $\angle CBE$ is a right angle.	1. _____
2. $\angle ABC$ and $\angle CBD$ are complementary.	2. Definition of complementary angles
3. $\angle DBE$ and $\angle CBD$ are complementary.	3. _____
4. $\angle ABC \cong \angle DBE$	4. _____

19. **PROVING A THEOREM** Copy and complete the paragraph proof for the Congruent Complements Theorem (Theorem 2.5). Then write a two-column proof. (See Example 3.)

Given $\angle 1$ and $\angle 2$ are complementary.
 $\angle 1$ and $\angle 3$ are complementary.
Prove $\angle 2 \cong \angle 3$



$\angle 1$ and $\angle 2$ are complementary, and $\angle 1$ and $\angle 3$ are complementary. By the definition of _____ angles, $m\angle 1 + m\angle 2 = 90^\circ$ and _____ $= 90^\circ$. By the _____, $m\angle 1 + m\angle 2 = m\angle 1 + m\angle 3$. By the Subtraction Property of Equality, _____. So, $\angle 2 \cong \angle 3$ by the definition of _____.

20. **PROVING A THEOREM** Copy and complete the two-column proof for the Congruent Supplement Theorem (Theorem 2.4). Then write a paragraph proof. (See Example 5.)

Given $\angle 1$ and $\angle 2$ are supplementary.
 $\angle 3$ and $\angle 4$ are supplementary.
 $\angle 1 \cong \angle 4$

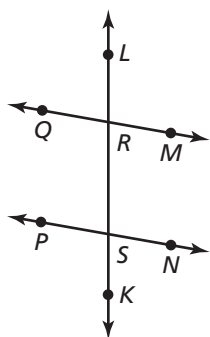


Prove $\angle 2 \cong \angle 3$

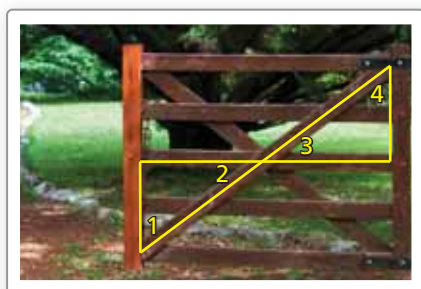
STATEMENTS	REASONS
1. $\angle 1$ and $\angle 2$ are supplementary. $\angle 3$ and $\angle 4$ are supplementary. $\angle 1 \cong \angle 4$	1. Given
2. $m\angle 1 + m\angle 2 = 180^\circ$, $m\angle 3 + m\angle 4 = 180^\circ$	2. _____
3. _____ = $m\angle 3 + m\angle 4$	3. Transitive Property of Equality
4. $m\angle 1 = m\angle 4$	4. Definition of congruent angles
5. $m\angle 1 + m\angle 2 =$ _____	5. Substitution Property of Equality
6. $m\angle 2 = m\angle 3$	6. _____
7. _____	7. _____

PROOF In Exercises 21–24, write a proof using any format.

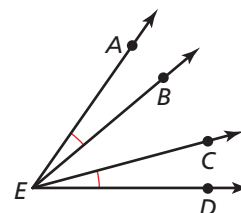
21. **Given** $\angle QRS$ and $\angle PSR$ are supplementary.
Prove $\angle QRL \cong \angle PSR$



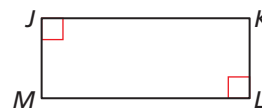
22. **Given** $\angle 1$ and $\angle 3$ are complementary.
 $\angle 2$ and $\angle 4$ are complementary.
Prove $\angle 1 \cong \angle 4$



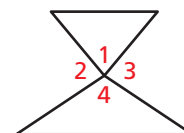
23. **Given** $\angle AEB \cong \angle DEC$
Prove $\angle AEC \cong \angle DEB$



24. **Given** $\overline{JK} \perp \overline{JM}$, $\overline{KL} \perp \overline{ML}$,
 $\angle J \cong \angle M$, $\angle K \cong \angle L$
Prove $\overline{JM} \perp \overline{ML}$ and $\overline{JK} \perp \overline{KL}$



25. **MAKING AN ARGUMENT** You overhear your friend discussing the diagram shown with a classmate. Your classmate claims $\angle 1 \cong \angle 4$ because they are vertical angles. Your friend claims they are not congruent because he can tell by looking at the diagram. Who is correct? Support your answer with definitions or theorems.

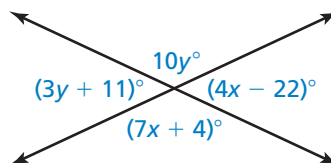


26. **THOUGHT PROVOKING** Draw three lines all intersecting at the same point. Explain how you can give two of the angle measures so that you can find the remaining four angle measures.

27. **CRITICAL THINKING** Is the converse of the Linear Pair Postulate (Postulate 2.8) true? If so, write a biconditional statement. Explain your reasoning.

28. **WRITING** How can you save time writing proofs?

29. **MATHEMATICAL CONNECTIONS** Find the measure of each angle in the diagram.



30. **HOW DO YOU SEE IT?** Use the student's two-column proof.

Given $\angle 1 \cong \angle 2$

$\angle 1$ and $\angle 2$ are supplementary.

Prove _____

STATEMENTS	REASONS
1. $\angle 1 \cong \angle 2$ $\angle 1$ and $\angle 2$ are supplementary.	1. Given
2. $m\angle 1 = m\angle 2$	2. Definition of congruent angles
3. $m\angle 1 + m\angle 2 = 180^\circ$	3. Definition of supplementary angles
4. $m\angle 1 + m\angle 1 = 180^\circ$	4. Substitution Property of Equality
5. $2m\angle 1 = 180^\circ$	5. Simplify.
6. $m\angle 1 = 90^\circ$	6. Division Property of Equality
7. $m\angle 2 = 90^\circ$	7. Transitive Property of Equality
8. _____	8. _____

a. What is the student trying to prove?

b. Your friend claims that the last line of the proof should be $\angle 1 \cong \angle 2$, because the measures of the angles are both 90° . Is your friend correct? Explain.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Use the cube. (Section 1.1)

31. Name three collinear points.

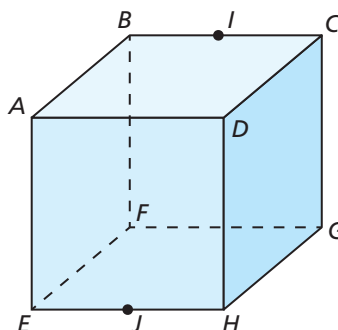
32. Name the intersection of plane ABF and plane EHG .

33. Name two planes containing $\overline{BC}$.

34. Name three planes containing point D .

35. Name three points that are not collinear.

36. Name two planes containing point J .



2.4–2.6 What Did You Learn?

Core Vocabulary

proof, *p. 100*
two-column proof, *p. 100*
theorem, *p. 101*

flowchart proof, or flow proof, *p. 106*
paragraph proof, *p. 108*

Core Concepts

Section 2.4

Algebraic Properties of Equality, *p. 92*
Distributive Property, *p. 93*

Reflexive, Symmetric, and Transitive Properties
of Equality, *p. 94*

Section 2.5

Writing Two-Column Proofs, *p. 100*
Theorem 2.1 Properties of Segment Congruence Theorem, *p. 101*
Theorem 2.2 Properties of Angle Congruence Theorem, *p. 101*

Section 2.6

Writing Flowchart Proofs, *p. 106*
Theorem 2.3 Right Angles Congruence Theorem,
p. 106
Theorem 2.4 Congruent Supplements Theorem,
p. 107

Theorem 2.5 Congruent Complements Theorem, *p. 107*
Writing Paragraph Proofs, *p. 108*
Postulate 2.8 Linear Pair Postulate, *p. 108*
Theorem 2.6 Vertical Angles Congruence Theorem,
p. 108

Mathematical Practices

1. Explain the purpose of justifying each step in Exercises 5–14 on page 96.
2. Create a diagram to model each statement in Exercises 5–10 on page 103.
3. Explain why you would not be able to prove the statement in Exercise 21 on page 113 if you were not provided with the given information or able to use any postulates or theorems.

Performance Task

Induction and the Next Dimension

Before you took Geometry, you could find the midpoint of a segment on a number line (a one-dimensional system). In Chapter 1, you learned how to find the midpoint of a segment in a coordinate plane (a two-dimensional system). How would you find the midpoint of a segment in a three-dimensional system?

To explore the answers to this question and more, go to
BigIdeasMath.com.



2.1 Conditional Statements (pp. 65–74)

Write the if-then form, the converse, the inverse, the contrapositive, and the biconditional of the conditional statement “A leap year is a year with 366 days.”

If-then form: If it is a leap year, then it is a year with 366 days.

Converse: If it is a year with 366 days, then it is a leap year.

Inverse: If it is not a leap year, then it is not a year with 366 days.

Contrapositive: If it is not a year with 366 days, then it is not a leap year.

Biconditional: It is a leap year if and only if it is a year with 366 days.

Write the if-then form, the converse, the inverse, the contrapositive, and the biconditional of the conditional statement.

- Two lines intersect in a point.
- $4x + 9 = 21$ because $x = 3$.
- Supplementary angles sum to 180° .
- Right angles are 90° .

2.2 Inductive and Deductive Reasoning (pp. 75–82)

What conclusion can you make about the sum of any two even integers?

Step 1 Look for a pattern in several examples. Use inductive reasoning to make a conjecture.

$$2 + 4 = 6$$

$$6 + 10 = 16$$

$$12 + 16 = 28$$

$$-2 + 4 = 2$$

$$6 + (-10) = -4$$

$$-12 + (-16) = -28$$

Conjecture Even integer + Even integer = Even integer

Step 2 Let n and m each be any integer. Use deductive reasoning to show that the conjecture is true.

$2n$ and $2m$ are even integers because any integer multiplied by 2 is even.

$2n + 2m$ represents the sum of two even integers.

$2n + 2m = 2(n + m)$ by the Distributive Property.

$2(n + m)$ is the product of 2 and an integer $(n + m)$.

So, $2(n + m)$ is an even integer.

► The sum of any two even integers is an even integer.

- What conclusion can you make about the difference of any two odd integers?
- What conclusion can you make about the product of an even and an odd integer?
- Use the Law of Detachment to make a valid conclusion.
If an angle is a right angle, then the angle measures 90° . $\angle B$ is a right angle.
- Use the Law of Syllogism to write a new conditional statement that follows from the pair of true statements: If $x = 3$, then $2x = 6$. If $4x = 12$, then $x = 3$.

2.3 Postulates and Diagrams (pp. 83–88)

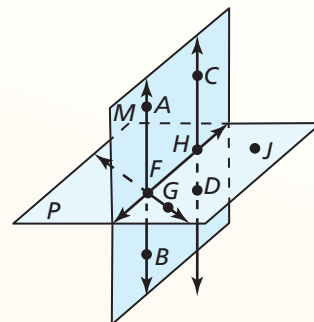
Use the diagram to make three statements that can be concluded and three statements that *cannot* be concluded. Justify your answers.

You can conclude:

1. Points A , B , and C are coplanar because they lie in plane M .
2. $\overleftrightarrow{FG}$ lies in plane P by the Plane-Line Postulate (Post. 2.6).
3. $\overleftrightarrow{CD}$ and $\overleftrightarrow{FH}$ intersect at point H by the Line Intersection Postulate (Post. 2.3).

You *cannot* conclude:

1. $\overleftrightarrow{CD} \perp$ to plane P because no right angle is marked.
2. Points A , F , and G are coplanar because point A lies in plane M and point G lies in plane P .
3. Points G , D , and J are collinear because no drawn line connects the points.

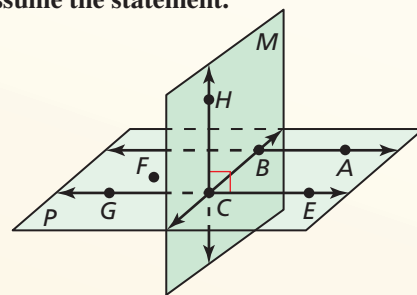


Use the diagram at the right to determine whether you can assume the statement.

9. Points A , B , C , and E are coplanar.
10. $\overleftrightarrow{HC} \perp \overleftrightarrow{GE}$
11. Points F , B , and G are collinear.
12. $\overleftrightarrow{AB} \parallel \overleftrightarrow{GE}$

Sketch a diagram of the description.

13. $\angle ABC$, an acute angle, is bisected by $\overleftrightarrow{BE}$.
14. $\angle CDE$, a straight angle, is bisected by $\overleftrightarrow{DK}$.
15. Plane P and plane R intersect perpendicularly in $\overleftrightarrow{XY}$.
 $\overleftrightarrow{ZW}$ lies in plane P .



2.4 Algebraic Reasoning (pp. 91–98)

Solve $2(2x + 9) = -10$. Justify each step.

Equation	Explanation	Reason
$2(2x + 9) = -10$	Write the equation.	Given
$4x + 18 = -10$	Multiply.	Distributive Property
$4x = -28$	Subtract 18 from each side.	Subtraction Property of Equality
$x = -7$	Divide each side by 4.	Division Property of Equality

► The solution is $x = -7$.

Solve the equation. Justify each step.

16. $-9x - 21 = -20x - 87$
17. $15x + 22 = 7x + 62$
18. $3(2x + 9) = 30$
19. $5x + 2(2x - 23) = -154$

Name the property of equality that the statement illustrates.

20. If $LM = RS$ and $RS = 25$, then $LM = 25$.
21. $AM = AM$

2.5 Proving Statements about Segments and Angles (pp. 99–104)

Write a two-column proof for the Transitive Property of Segment Congruence (Theorem 2.1).

Given $\overline{AB} \cong \overline{CD}, \overline{CD} \cong \overline{EF}$

Prove $\overline{AB} \cong \overline{EF}$

STATEMENTS	REASONS
1. $\overline{AB} \cong \overline{CD}, \overline{CD} \cong \overline{EF}$	1. Given
2. $AB = CD, CD = EF$	2. Definition of congruent segments
3. $AB = EF$	3. Transitive Property of Equality
4. $\overline{AB} \cong \overline{EF}$	4. Definition of congruent segments

Name the property that the statement illustrates.

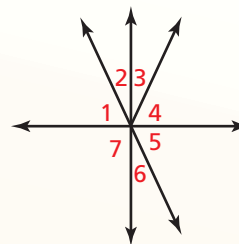
22. If $\angle DEF \cong \angle JKL$, then $\angle JKL \cong \angle DEF$.
23. $\angle C \cong \angle C$
24. If $MN = PQ$ and $PQ = RS$, then $MN = RS$.
25. Write a two-column proof for the Reflexive Property of Angle Congruence (Thm. 2.2).

2.6 Proving Geometric Relationships (pp. 105–114)

Rewrite the two-column proof into a paragraph proof.

Given $\angle 2 \cong \angle 3$

Prove $\angle 3 \cong \angle 6$



Two-Column Proof

STATEMENTS	REASONS
1. $\angle 2 \cong \angle 3$	1. Given
2. $\angle 2 \cong \angle 6$	2. Vertical Angles Congruence Theorem (Thm. 2.6)
3. $\angle 3 \cong \angle 6$	3. Transitive Property of Angle Congruence (Thm. 2.2)

Paragraph Proof

$\angle 2$ and $\angle 3$ are congruent. By the Vertical Angles Congruence Theorem (Theorem 2.6), $\angle 2 \cong \angle 6$. So, by the Transitive Property of Angle Congruence (Theorem 2.2), $\angle 3 \cong \angle 6$.

26. Write a proof using any format.

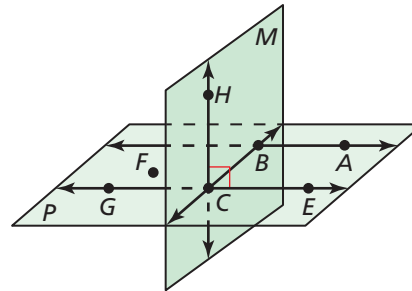
Given $\angle 3$ and $\angle 2$ are complementary.
 $m\angle 1 + m\angle 2 = 90^\circ$

Prove $\angle 3 \cong \angle 1$

2 Chapter Test

Use the diagram to determine whether you can assume the statement.
Explain your reasoning.

- $\overleftrightarrow{AB} \perp \text{plane } M$
- Points F , G , and A are coplanar.
- Points E , C , and G are collinear.
- Planes M and P intersect at $\overleftrightarrow{BC}$.
- $\overleftrightarrow{FA}$ lies in plane P .
- $\overleftrightarrow{FG}$ intersects $\overleftrightarrow{AB}$ at point B .



Solve the equation. Justify each step.

- $9x + 31 = -23 + 3x$
- $26 + 2(3x + 11) = -18$
- $3(7x - 9) - 19x = -15$

Write the if-then form, the converse, the inverse, the contrapositive, and the biconditional of the conditional statement.

- Two planes intersect at a line.
- A relation that pairs each input with exactly one output is a function.

Use inductive reasoning to make a conjecture about the given quantity. Then use deductive reasoning to show that the conjecture is true.

- the sum of three odd integers
- the product of three even integers
- Give an example of two statements for which the Law of Detachment does not apply.
- The formula for the area A of a triangle is $A = \frac{1}{2}bh$, where b is the base and h is the height. Solve the formula for h and justify each step. Then find the height of a standard yield sign when the area is 558 square inches and each side is 36 inches.



- You visit the zoo and notice the following.
 - The elephants, giraffes, lions, tigers, and zebras are located along a straight walkway.
 - The giraffes are halfway between the elephants and the lions.
 - The tigers are halfway between the lions and the zebras.
 - The lions are halfway between the giraffes and the tigers.

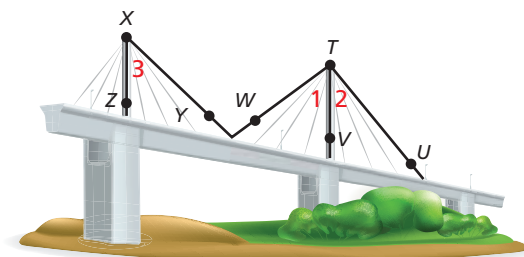
Draw and label a diagram that represents this information. Then prove that the distance between the elephants and the giraffes is equal to the distance between the tigers and the zebras. Use any proof format.

- Write a proof using any format.

Given $\angle 2 \cong \angle 3$

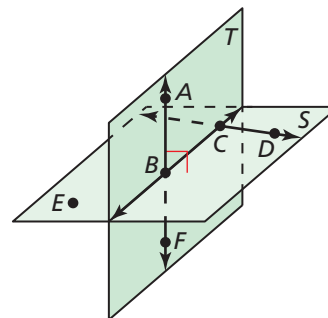
$\overleftrightarrow{TV}$ bisects $\angle UTW$.

Prove $\angle 1 \cong \angle 3$



2 Cumulative Assessment

- Use the diagram to write an example of each postulate.
 - Two Point Postulate (Postulate 2.1)** Through any two points, there exists exactly one line.
 - Line Intersection Postulate (Postulate 2.3)** If two lines intersect, then their intersection is exactly one point.
 - Three Point Postulate (Postulate 2.4)** Through any three noncollinear points, there exists exactly one plane.
 - Plane-Line Postulate (Postulate 2.6)** If two points lie in a plane, then the line containing them lies in the plane.
 - Plane Intersection Postulate (Postulate 2.7)** If two planes intersect, then their intersection is a line.

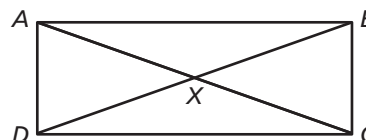


- Enter the reasons in the correct positions to complete the two-column proof.

Given $\overline{AX} \cong \overline{DX}$, $\overline{XB} \cong \overline{XC}$

Prove $\overline{AC} \cong \overline{BD}$

STATEMENTS	REASONS
1. $\overline{AX} \cong \overline{DX}$	1. Given
2. $AX = DX$	2. _____
3. $\overline{XB} \cong \overline{XC}$	3. Given
4. $XB = XC$	4. _____
5. $AX + XC = AC$	5. _____
6. $DX + XB = DB$	6. _____
7. $AC = DX + XB$	7. _____
8. $AC = BD$	8. _____
9. $\overline{AC} \cong \overline{BD}$	9. _____

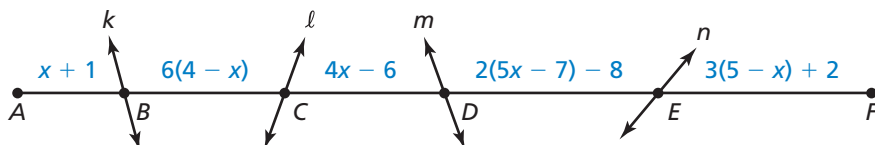


Segment Addition Postulate
(Postulate 1.2)

Definition of congruent segments

Substitution Property of Equality

- Classify each related conditional statement, based on the conditional statement "If I study, then I will pass the final exam."
 - I will pass the final exam if and only if I study.
 - If I do not study, then I will not pass the final exam.
 - If I pass the final exam, then I studied.
 - If I do not pass the final exam, then I did not study.
- List all segment bisectors given $x = 3$.



5. You are given $m\angle FHE = m\angle BHG = m\angle AHF = 90^\circ$. Choose the symbol that makes each statement true. State which theorem or postulate, if any, supports your answer.

a. $\angle 3$ $\angle 6$

b. $m\angle 4$ $m\angle 7$

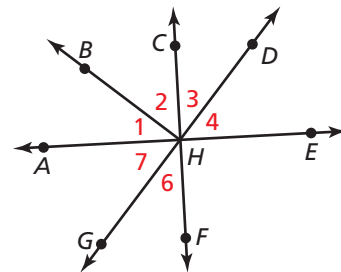
c. $m\angle FHE$ $m\angle AHG$

d. $m\angle AHG + m\angle GHE$ 180°

=

$\cong$

$\neq$



6. Find the distance between each pair of points. Then order each line segment from longest to shortest.

a. $A(-6, 1), B(-1, 6)$

b. $C(-5, 8), D(5, 8)$

c. $E(2, 7), F(4, -2)$

d. $G(7, 3), H(7, -1)$

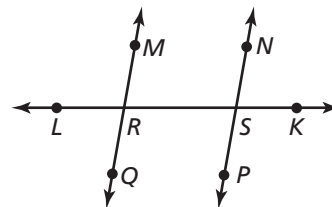
e. $J(-4, -2), K(1, -5)$

f. $L(3, -8), M(7, -5)$

7. The proof shows that $\angle MRL$ is congruent to $\angle NSR$. Select all other angles that are also congruent to $\angle NSR$.

Given $\angle MRS$ and $\angle NSR$ are supplementary.

Prove $\angle MRL \cong \angle NSR$



STATEMENTS

REASONS

1. $\angle MRS$ and $\angle NSR$ are supplementary.

1. Given

2. $\angle MRL$ and $\angle MRS$ are a linear pair.

2. Definition of linear pair, as shown in the diagram

3. $\angle MRL$ and $\angle MRS$ are supplementary.

3. Linear Pair Postulate (Postulate 2.8)

4. $\angle MRL \cong \angle NSR$

4. Congruent Supplements Theorem (Theorem 2.4)

$\angle PSK$

$\angle KSN$

$\angle PSR$

$\angle QRS$

$\angle QRL$

8. Your teacher assigns your class a homework problem that asks you to prove the Vertical Angles Congruence Theorem (Theorem 2.6) using the picture and information given at the right. Your friend claims that this can be proved without using the Linear Pair Postulate (Postulate 2.8). Is your friend correct? Explain your reasoning.

Given $\angle 1$ and $\angle 3$ are vertical angles.

Prove $\angle 1 \cong \angle 3$

