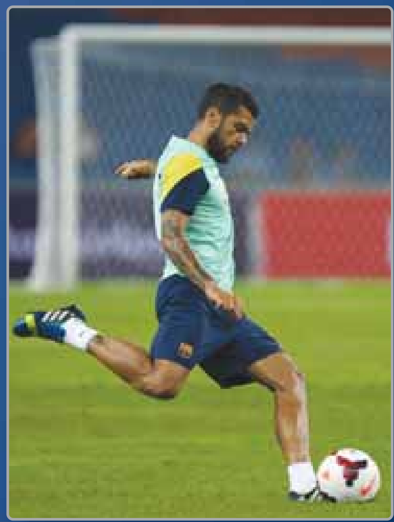
1 Basics of Geometry

Alamillo Bridge *(p. 53)*


Soccer *(p. 49)*


Shed *(p. 33)*


Skateboard *(p. 20)*


Sulfur Hexafluoride *(p. 7)*

Maintaining Mathematical Proficiency

Finding Absolute Value

Example 1 **Simplify $|-7 - 1|$.**

$$\begin{aligned} |-7 - 1| &= |-7 + (-1)| && \text{Add the opposite of 1.} \\ &= |-8| && \text{Add.} \\ &= 8 && \text{Find the absolute value.} \end{aligned}$$

▶ $|-7 - 1| = 8$

Simplify the expression.

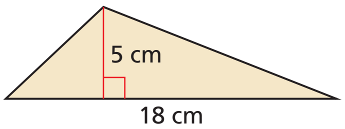
1. $|8 - 12|$ **2.** $|-6 - 5|$ **3.** $|4 + (-9)|$

4. $|13 + (-4)|$ **5.** $|6 - (-2)|$ **6.** $|5 - (-1)|$

7. $|-8 - (-7)|$ **8.** $|8 - 13|$ **9.** $|-14 - 3|$

Finding the Area of a Triangle

Example 2 **Find the area of the triangle.**

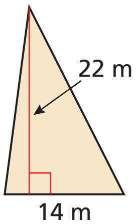


$$\begin{aligned} A &= \tfrac{1}{2}bh && \text{Write the formula for area of a triangle.} \\ &= \tfrac{1}{2}(18)(5) && \text{Substitute 18 for } b \text{ and 5 for } h. \\ &= \tfrac{1}{2}(90) && \text{Multiply 18 and 5.} \\ &= 45 && \text{Multiply } \tfrac{1}{2} \text{ and 90.} \end{aligned}$$

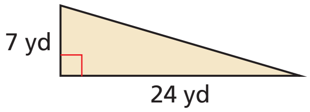
▶ The area of the triangle is 45 square centimeters.

Find the area of the triangle.

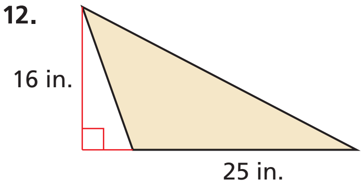
10.



11.



12.



13. **ABSTRACT REASONING** Describe the possible values for x and y when $|x - y| > 0$. What does it mean when $|x - y| = 0$? Can $|x - y| < 0$? Explain your reasoning.

Mathematical Practices

Mathematically proficient students carefully specify units of measure.

Specifying Units of Measure

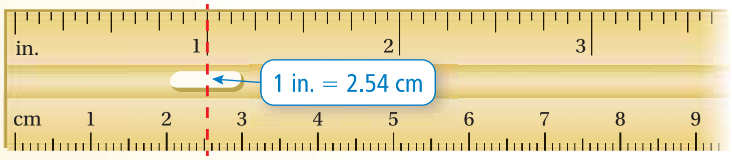
Core Concept

Customary Units of Length

1 foot = 12 inches
1 yard = 3 feet
1 mile = 5280 feet = 1760 yards

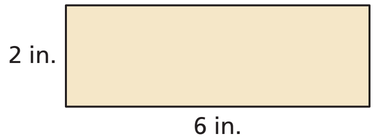
Metric Units of Length

1 centimeter = 10 millimeters
1 meter = 1000 millimeters
1 kilometer = 1000 meters



EXAMPLE 1 **Converting Units of Measure**

Find the area of the rectangle in square centimeters. Round your answer to the nearest hundredth.



SOLUTION

Use the formula for the area of a rectangle. Convert the units of length from customary units to metric units.

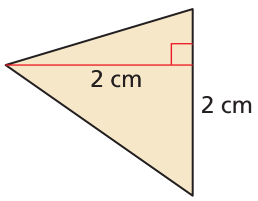
$$
\begin{aligned}
\text{Area} &= (\text{Length})(\text{Width}) && \text{Formula for area of a rectangle}\\
&= (6 \text{ in.})(2 \text{ in.}) && \text{Substitute given length and width.}\\
&= \left[(6\,\cancel{\text{in.}})\left(\frac{2.54\text{ cm}}{1\,\cancel{\text{in.}}}\right)\right]\left[(2\,\cancel{\text{in.}})\left(\frac{2.54\text{ cm}}{1\,\cancel{\text{in.}}}\right)\right] && \text{Multiply each dimension by the conversion factor.}\\
&= (15.24\text{ cm})(5.08\text{ cm}) && \text{Multiply.}\\
&\approx 77.42\text{ cm}^2 && \text{Multiply and round to the nearest hundredth.}
\end{aligned}
$$

▶ The area of the rectangle is about 77.42 square centimeters.

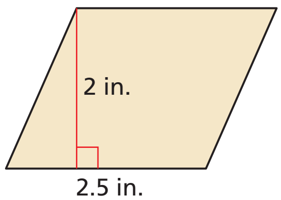
Monitoring Progress

Find the area of the polygon using the specified units. Round your answer to the nearest hundredth.

1. triangle (square inches)



2. parallelogram (square centimeters)



3. The distance between two cities is 120 miles. What is the distance in kilometers? Round your answer to the nearest whole number.

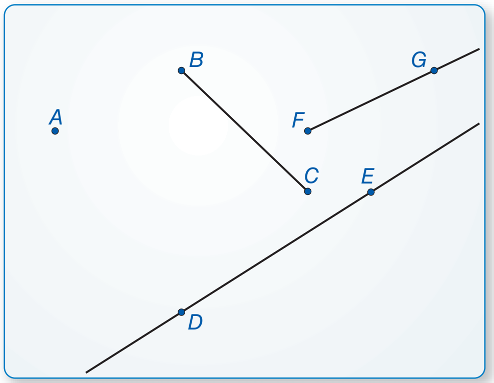
1.1 Points, Lines, and Planes

Essential Question How can you use dynamic geometry software to visualize geometric concepts?

EXPLORATION 1 Using Dynamic Geometry Software

Work with a partner. Use dynamic geometry software to draw several points. Also, draw some lines, line segments, and rays. What is the difference between a line, a line segment, and a ray?

Sample



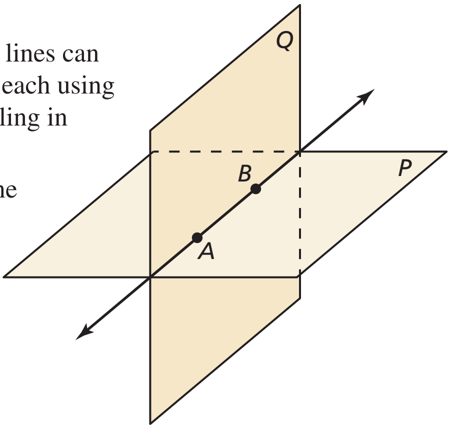
EXPLORATION 2 Intersections of Lines and Planes

Work with a partner.

a. Describe and sketch the ways in which two lines can intersect or not intersect. Give examples of each using the lines formed by the walls, floor, and ceiling in your classroom.

b. Describe and sketch the ways in which a line and a plane can intersect or not intersect. Give examples of each using the walls, floor, and ceiling in your classroom.

c. Describe and sketch the ways in which two planes can intersect or not intersect. Give examples of each using the walls, floor, and ceiling in your classroom.



UNDERSTANDING MATHEMATICAL TERMS

To be proficient in math, you need to understand definitions and previously established results. An appropriate tool, such as a software package, can sometimes help.

EXPLORATION 3 Exploring Dynamic Geometry Software

Work with a partner. Use dynamic geometry software to explore geometry. Use the software to find a term or concept that is unfamiliar to you. Then use the capabilities of the software to determine the meaning of the term or concept.

Communicate Your Answer

4. How can you use dynamic geometry software to visualize geometric concepts?

1.1 Lesson

Core Vocabulary

undefined terms, *p. 4*
point, *p. 4*
line, *p. 4*
plane, *p. 4*
collinear points, *p. 4*
coplanar points, *p. 4*
defined terms, *p. 5*
line segment, or segment, *p. 5*
endpoints, *p. 5*
ray, *p. 5*
opposite rays, *p. 5*
intersection, *p. 6*

What You Will Learn

- Name points, lines, and planes.
- Name segments and rays.
- Sketch intersections of lines and planes.
- Solve real-life problems involving lines and planes.

Using Undefined Terms

In geometry, the words *point*, *line*, and *plane* are **undefined terms**. These words do not have formal definitions, but there is agreement about what they mean.

Core Concept

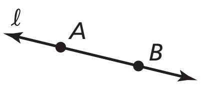
Undefined Terms: Point, Line, and Plane

Point A **point** has no dimension. A dot represents a point.

point A

Line A **line** has one dimension. It is represented by a line with two arrowheads, but it extends without end.

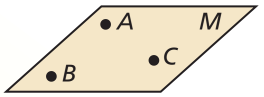
Through any two points, there is exactly one line. You can use any two points on a line to name it.



line ℓ, line AB ($\overleftrightarrow{AB}$), or line BA ($\overleftrightarrow{BA}$)

Plane A **plane** has two dimensions. It is represented by a shape that looks like a floor or a wall, but it extends without end.

Through any three points not on the same line, there is exactly one plane. You can use three points that are not all on the same line to name a plane.



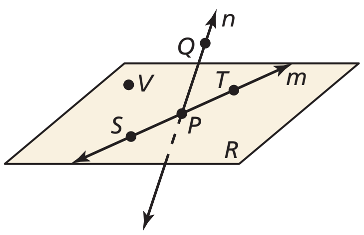
plane M, or plane ABC

Collinear points are points that lie on the same line. **Coplanar points** are points that lie in the same plane.

EXAMPLE 1 Naming Points, Lines, and Planes

a. Give two other names for $\overleftrightarrow{PQ}$ and plane R.

b. Name three points that are collinear. Name four points that are coplanar.



SOLUTION

a. Other names for $\overleftrightarrow{PQ}$ are $\overleftrightarrow{QP}$ and line n. Other names for plane R are plane SVT and plane PTV.

b. Points S, P, and T lie on the same line, so they are collinear. Points S, P, T, and V lie in the same plane, so they are coplanar.

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1. Use the diagram in Example 1. Give two other names for $\overleftrightarrow{ST}$. Name a point that is *not* coplanar with points Q, S, and T.

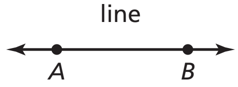
Using Defined Terms

In geometry, terms that can be described using known words such as *point* or *line* are called **defined terms**.

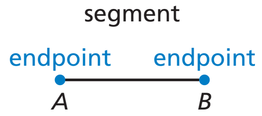
Core Concept

Defined Terms: Segment and Ray

The definitions below use line AB (written as $\overleftrightarrow{AB}$) and points A and B.

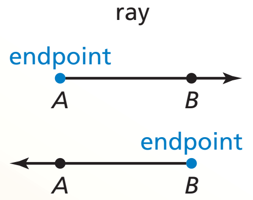


Segment The **line segment** AB, or **segment** AB, (written as $\overline{AB}$) consists of the **endpoints** A and B and all points on $\overleftrightarrow{AB}$ that are between A and B. Note that $\overline{AB}$ can also be named $\overline{BA}$.

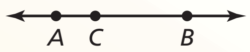


Ray The **ray** AB (written as $\overrightarrow{AB}$) consists of the endpoint A and all points on $\overleftrightarrow{AB}$ that lie on the same side of A as B.

Note that $\overrightarrow{AB}$ and $\overrightarrow{BA}$ are different rays.



Opposite Rays If point C lies on $\overleftrightarrow{AB}$ between A and B, then $\overrightarrow{CA}$ and $\overrightarrow{CB}$ are **opposite rays**.

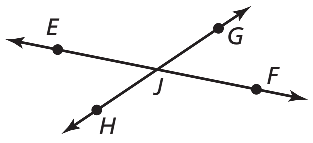


Segments and rays are collinear when they lie on the same line. So, opposite rays are collinear. Lines, segments, and rays are coplanar when they lie in the same plane.

EXAMPLE 2 Naming Segments, Rays, and Opposite Rays

a. Give another name for $\overline{GH}$.

b. Name all rays with endpoint J. Which of these rays are opposite rays?



SOLUTION

a. Another name for $\overline{GH}$ is $\overline{HG}$.

b. The rays with endpoint J are $\overrightarrow{JE}$, $\overrightarrow{JG}$, $\overrightarrow{JF}$, and $\overrightarrow{JH}$. The pairs of opposite rays with endpoint J are $\overrightarrow{JE}$ and $\overrightarrow{JF}$, and $\overrightarrow{JG}$ and $\overrightarrow{JH}$.

COMMON ERROR

In Example 2, $\overrightarrow{JG}$ and $\overrightarrow{JF}$ have a common endpoint, but they are not collinear. So, they are *not* opposite rays.

Monitoring Progress

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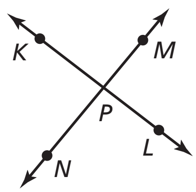
Use the diagram.

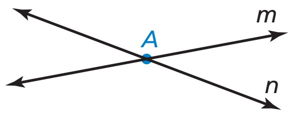


2. Give another name for $\overline{KL}$.

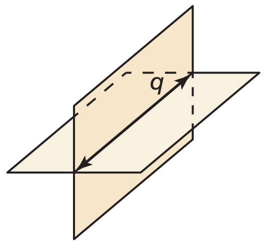
3. Are $\overrightarrow{KP}$ and $\overrightarrow{PK}$ the same ray? Are $\overrightarrow{NP}$ and $\overrightarrow{NM}$ the same ray? Explain.

Sketching Intersections

Two or more geometric figures *intersect* when they have one or more points in common. The **intersection** of the figures is the set of points the figures have in common. Some examples of intersections are shown below.



The intersection of two different lines is a point.



The intersection of two different planes is a line.

EXAMPLE 3 Sketching Intersections of Lines and Planes

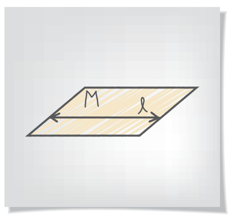
a. Sketch a plane and a line that is in the plane.

b. Sketch a plane and a line that does not intersect the plane.

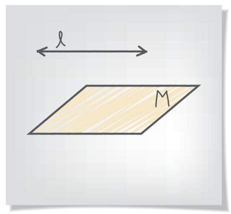
c. Sketch a plane and a line that intersects the plane at a point.

SOLUTION

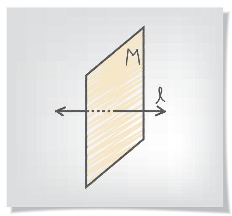
a.



b.



c.



EXAMPLE 4 Sketching Intersections of Planes

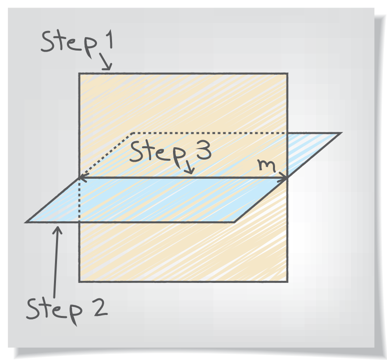
Sketch two planes that intersect in a line.

SOLUTION

Step 1 Draw a vertical plane. Shade the plane.

Step 2 Draw a second plane that is horizontal. Shade this plane a different color. Use dashed lines to show where one plane is hidden.

Step 3 Draw the line of intersection.



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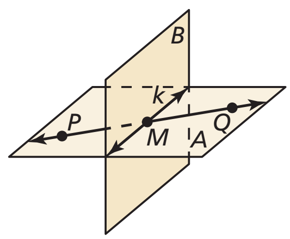
4. Sketch two different lines that intersect a plane at the same point.

Use the diagram.

5. Name the intersection of $\overleftrightarrow{PQ}$ and line k.

6. Name the intersection of plane A and plane B.

7. Name the intersection of line k and plane A.

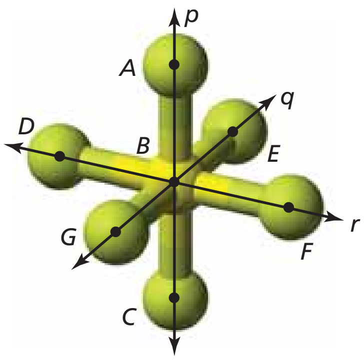


Solving Real-Life Problems

EXAMPLE 5 **Modeling with Mathematics**

The diagram shows a molecule of sulfur hexafluoride, the most potent greenhouse gas in the world. Name two different planes that contain line r.

Electric utilities use sulfur hexafluoride as an insulator. Leaks in electrical equipment contribute to the release of sulfur hexafluoride into the atmosphere.



SOLUTION

1. **Understand the Problem** In the diagram, you are given three lines, p, q, and r, that intersect at point B. You need to name two different planes that contain line r.
2. **Make a Plan** The planes should contain two points on line r and one point not on line r.
3. **Solve the Problem** Points D and F are on line r. Point E does not lie on line r. So, plane DEF contains line r. Another point that does not lie on line r is C. So, plane CDF contains line r.

   Note that you cannot form a plane through points D, B, and F. By definition, three points that do not lie on the same line form a plane. Points D, B, and F are collinear, so they do *not* form a plane.
4. **Look Back** The question asks for two *different* planes. You need to check whether plane DEF and plane CDF are two unique planes or the same plane named differently. Because point C does not lie on plane DEF, plane DEF and plane CDF are different planes.

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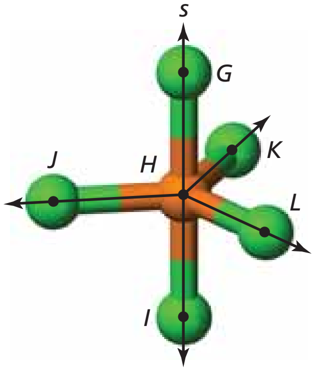
Use the diagram that shows a molecule of phosphorus pentachloride.



8. Name two different planes that contain line s.
9. Name three different planes that contain point K.
10. Name two different planes that contain $\overrightarrow{HJ}$.

1.1 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

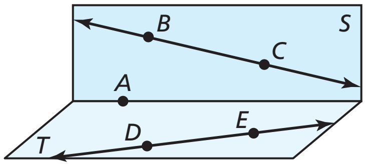
1. **WRITING** Compare collinear points and coplanar points.

2. **WHICH ONE DOESN'T BELONG?** Which term does *not* belong with the other three? Explain your reasoning.

$\overline{AB}$	plane CDE	$\overleftrightarrow{FG}$	$\overrightarrow{HI}$

Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, use the diagram.



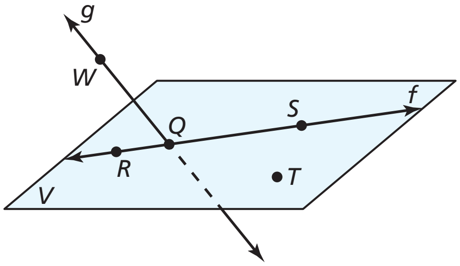
3. Name four points.

4. Name two lines.

5. Name the plane that contains points A, B, and C.

6. Name the plane that contains points A, D, and E.

In Exercises 7–10, use the diagram. *(See Example 1.)*



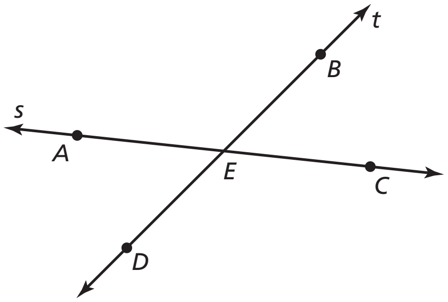
7. Give two other names for $\overleftrightarrow{WQ}$.

8. Give another name for plane V.

9. Name three points that are collinear. Then name a fourth point that is not collinear with these three points.

10. Name a point that is not coplanar with R, S, and T.

In Exercises 11–16, use the diagram. *(See Example 2.)*



11. What is another name for $\overline{BD}$?

12. What is another name for $\overline{AC}$?

13. What is another name for ray $\overrightarrow{AE}$?

14. Name all rays with endpoint E.

15. Name two pairs of opposite rays.

16. Name one pair of rays that are not opposite rays.

In Exercises 17–24, sketch the figure described. *(See Examples 3 and 4.)*

17. plane P and line ℓ intersecting at one point

18. plane K and line m intersecting at all points on line m

19. $\overleftrightarrow{AB}$ and $\overleftrightarrow{AC}$

20. $\overrightarrow{MN}$ and $\overrightarrow{NX}$

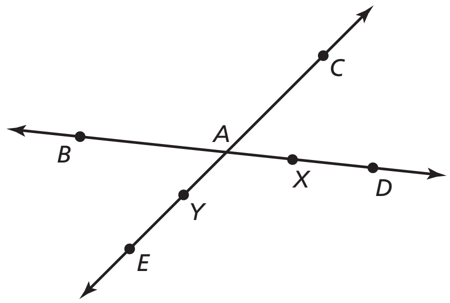
21. plane M and $\overrightarrow{NB}$ intersecting at B

22. plane M and $\overrightarrow{NB}$ intersecting at A

23. plane A and plane B not intersecting

24. plane C and plane D intersecting at $\overleftrightarrow{XY}$

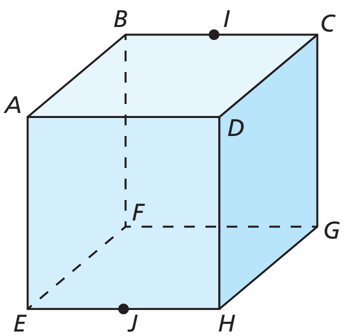
ERROR ANALYSIS **In Exercises 25 and 26, describe and correct the error in naming opposite rays in the diagram.**



25. ✗ $\overrightarrow{AD}$ and $\overrightarrow{AC}$ are opposite rays.

26. ✗ $\overline{YC}$ and $\overline{YE}$ are opposite rays.

In Exercises 27–34, use the diagram.



27. Name a point that is collinear with points E and H.

28. Name a point that is collinear with points B and I.

29. Name a point that is not collinear with points E and H.

30. Name a point that is not collinear with points B and I.

31. Name a point that is coplanar with points D, A, and B.

32. Name a point that is coplanar with points C, G, and F.

33. Name the intersection of plane AEH and plane FBE.

34. Name the intersection of plane BGF and plane HDG.

In Exercises 35–38, name the geometric term modeled by the object.

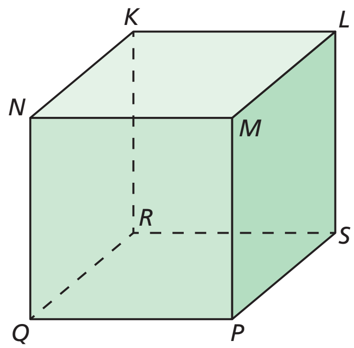
35.

36.

37.

38.

In Exercises 39–44, use the diagram to name all the points that are not coplanar with the given points.



39. N, K, and L

40. P, Q, and N

41. P, Q, and R

42. R, K, and N

43. P, S, and K

44. Q, K, and L

45. **CRITICAL THINKING** Given two points on a line and a third point not on the line, is it possible to draw a plane that includes the line and the third point? Explain your reasoning.

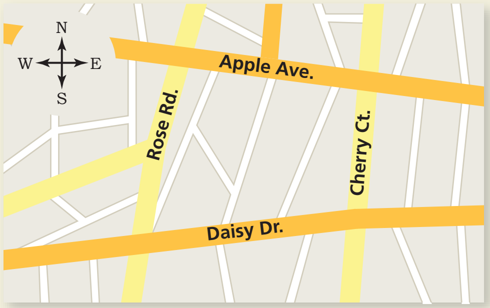
46. **CRITICAL THINKING** Is it possible for one point to be in two different planes? Explain your reasoning.

47. **REASONING** Explain why a four-legged chair may rock from side to side even if the floor is level. Would a three-legged chair on the same level floor rock from side to side? Why or why not?

48. **THOUGHT PROVOKING** You are designing the living room of an apartment. Counting the floor, walls, and ceiling, you want the design to contain at least eight different planes. Draw a diagram of your design. Label each plane in your design.

49. **LOOKING FOR STRUCTURE** Two coplanar intersecting lines will always intersect at one point. What is the greatest number of intersection points that exist if you draw four coplanar lines? Explain.

50. **HOW DO YOU SEE IT?** You and your friend walk in opposite directions, forming opposite rays. You were originally on the corner of Apple Avenue and Cherry Court.



a. Name two possibilities of the road and direction you and your friend may have traveled.

b. Your friend claims he went north on Cherry Court, and you went east on Apple Avenue. Make an argument as to why you know this could not have happened.

MATHEMATICAL CONNECTIONS **In Exercises 51–54, graph the inequality on a number line. Tell whether the graph is a *segment*, a *ray* or *rays*, a *point*, or a *line*.**

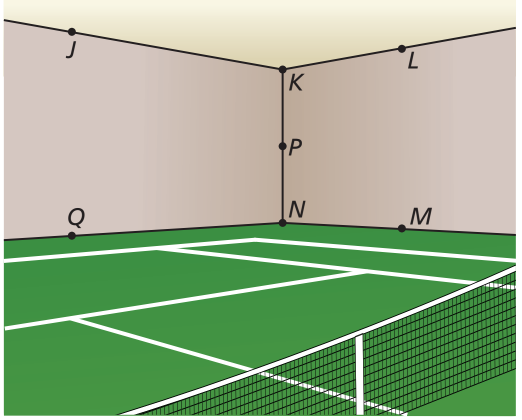
51. $x \le 3$

52. $-7 \le x \le 4$

53. $x \ge 5$ or $x \le -2$

54. $|x| \le 0$

55. **MODELING WITH MATHEMATICS** Use the diagram.



a. Name two points that are collinear with P.

b. Name two planes that contain J.

c. Name all the points that are in more than one plane.

CRITICAL THINKING **In Exercises 56–63, complete the statement with *always*, *sometimes*, or *never*. Explain your reasoning.**

56. A line ____________ has endpoints.

57. A line and a point ____________ intersect.

58. A plane and a point ____________ intersect.

59. Two planes ____________ intersect in a line.

60. Two points ____________ determine a line.

61. Any three points ____________ determine a plane.

62. Any three points not on the same line ____________ determine a plane.

63. Two lines that are not parallel ____________ intersect.

64. **ABSTRACT REASONING** Is it possible for three planes to never intersect? intersect in one line? intersect in one point? Sketch the possible situations.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Find the absolute value. *(Skills Review Handbook)*

65. $|6 + 2|$

66. $|3 - 9|$

67. $|-8 - 2|$

68. $|7 - 11|$

Solve the equation. *(Skills Review Handbook)*

69. $18 + x = 43$

70. $36 + x = 20$

71. $x - 15 = 7$

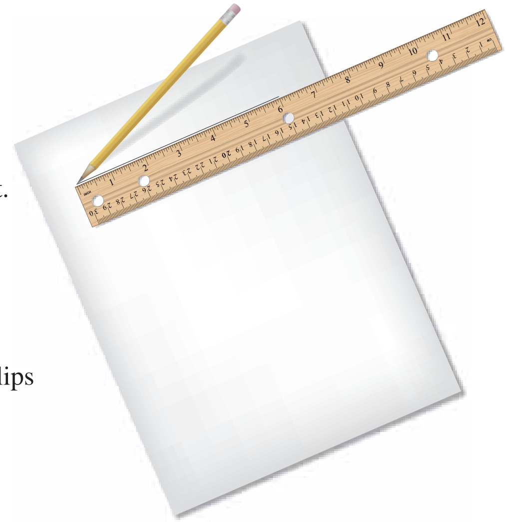
72. $x - 23 = 19$

1.2 Measuring and Constructing Segments

Essential Question How can you measure and construct a line segment?

EXPLORATION 1 Measuring Line Segments Using Nonstandard Units

Work with a partner.



a. Draw a line segment that has a length of 6 inches.

b. Use a standard-sized paper clip to measure the length of the line segment. Explain how you measured the line segment in "paper clips."

c. Write conversion factors from paper clips to inches and vice versa.

1 paper clip = ____ in.

1 in. = ____ paper clip

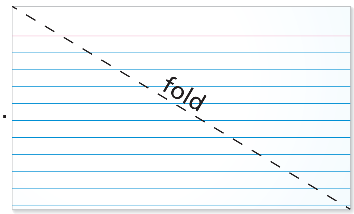
d. A *straightedge* is a tool that you can use to draw a straight line. An example of a straightedge is a ruler. Use only a pencil, straightedge, paper clip, and paper to draw another line segment that is 6 inches long. Explain your process.

MAKING SENSE OF PROBLEMS

To be proficient in math, you need to explain to yourself the meaning of a problem and look for entry points to its solution.

EXPLORATION 2 Measuring Line Segments Using Nonstandard Units

Work with a partner.



a. Fold a 3-inch by 5-inch index card on one of its diagonals.

b. Use the Pythagorean Theorem to algebraically determine the length of the diagonal in inches. Use a ruler to check your answer.

c. Measure the length and width of the index card in paper clips.

d. Use the Pythagorean Theorem to algebraically determine the length of the diagonal in paper clips. Then check your answer by measuring the length of the diagonal in paper clips. Does the Pythagorean Theorem work for any unit of measure? Justify your answer.

EXPLORATION 3 Measuring Heights Using Nonstandard Units

Work with a partner. Consider a unit of length that is equal to the length of the diagonal you found in Exploration 2. Call this length "1 diag." How tall are you in diags? Explain how you obtained your answer.

Communicate Your Answer

4. How can you measure and construct a line segment?

1.2 Lesson

Core Vocabulary

postulate, *p. 12*
axiom, *p. 12*
coordinate, *p. 12*
distance, *p. 12*
construction, *p. 13*
congruent segments, *p. 13*
between, *p. 14*

What You Will Learn

- Use the Ruler Postulate.
- Copy segments and compare segments for congruence.
- Use the Segment Addition Postulate.

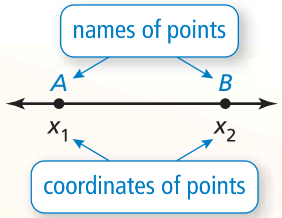
Using the Ruler Postulate

In geometry, a rule that is accepted without proof is called a **postulate** or an **axiom**. A rule that can be proved is called a *theorem*, as you will see later. Postulate 1.1 shows how to find the distance between two points on a line.

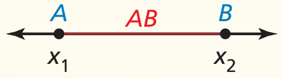
Postulate

Postulate 1.1 Ruler Postulate

The points on a line can be matched one to one with the real numbers. The real number that corresponds to a point is the **coordinate** of the point.



The **distance** between points A and B, written as AB, is the absolute value of the difference of the coordinates of A and B.



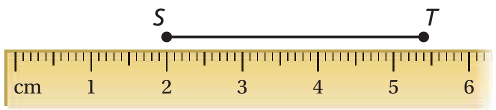
$AB = |x_2 - x_1|$

EXAMPLE 1 Using the Ruler Postulate

Measure the length of $\overline{ST}$ to the nearest tenth of a centimeter.

SOLUTION

Align one mark of a metric ruler with S. Then estimate the coordinate of T. For example, when you align S with 2, T appears to align with 5.4.



$ST = |5.4 - 2| = 3.4$ Ruler Postulate

- So, the length of $\overline{ST}$ is about 3.4 centimeters.

Monitoring Progress

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Use a ruler to measure the length of the segment to the nearest $\frac{1}{8}$ inch.

1. 

2. 

3. 

4. 

Constructing and Comparing Congruent Segments

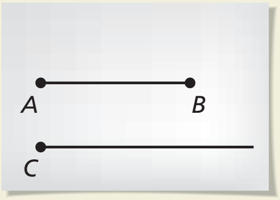
A **construction** is a geometric drawing that uses a limited set of tools, usually a *compass* and *straightedge*.

CONSTRUCTION **Copying a Segment**

Use a compass and straightedge to construct a line segment that has the same length as $\overline{AB}$.

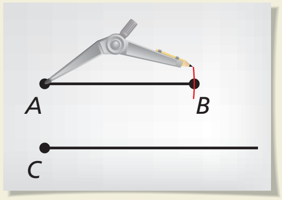
SOLUTION

Step 1



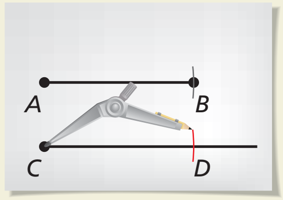
Draw a segment Use a straightedge to draw a segment longer than $\overline{AB}$. Label point C on the new segment.

Step 2



Measure length Set your compass at the length of $\overline{AB}$.

Step 3

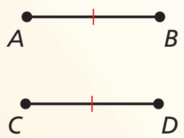


Copy length Place the compass at C. Mark point D on the new segment. So, $\overline{CD}$ has the same length as $\overline{AB}$.

Core Concept

Congruent Segments

Line segments that have the same length are called **congruent segments**. You can say "the length of $\overline{AB}$ is equal to the length of $\overline{CD}$," or you can say "$\overline{AB}$ *is congruent to* $\overline{CD}$." The symbol $\cong$ means "is congruent to."



Lengths are equal.
$AB = CD$
↑
"is equal to"

Segments are congruent.
$\overline{AB} \cong \overline{CD}$
↑
"is congruent to"

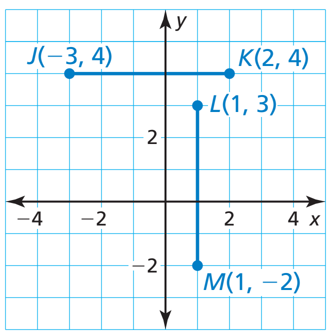
READING

In the diagram, the red tick marks indicate $\overline{AB} \cong \overline{CD}$. When there is more than one pair of congruent segments, use multiple tick marks.

EXAMPLE 2 **Comparing Segments for Congruence**

Plot $J(-3, 4)$, $K(2, 4)$, $L(1, 3)$, and $M(1, -2)$ in a coordinate plane. Then determine whether $\overline{JK}$ and $\overline{LM}$ are congruent.

SOLUTION



Plot the points, as shown. To find the length of a horizontal segment, find the absolute value of the difference of the x-coordinates of the endpoints.

$JK = |2 - (-3)| = 5$ Ruler Postulate

To find the length of a vertical segment, find the absolute value of the difference of the y-coordinates of the endpoints.

$LM = |-2 - 3| = 5$ Ruler Postulate

▶ $\overline{JK}$ and $\overline{LM}$ have the same length. So, $\overline{JK} \cong \overline{LM}$.

Monitoring Progress

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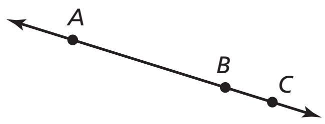
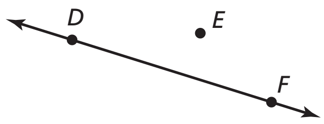
5. Plot $A(-2, 4)$, $B(3, 4)$, $C(0, 2)$, and $D(0, -2)$ in a coordinate plane. Then determine whether $\overline{AB}$ and $\overline{CD}$ are congruent.

Using the Segment Addition Postulate

When three points are collinear, you can say that one point is **between** the other two.



Point *B* is between points *A* and *C*.



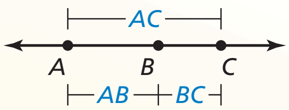
Point *E* is not between points *D* and *F*.

Postulate

Postulate 1.2 Segment Addition Postulate

If B is between A and C, then $AB + BC = AC$.

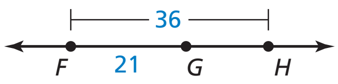
If $AB + BC = AC$, then B is between A and C.



EXAMPLE 3 Using the Segment Addition Postulate

a. Find DF.

b. Find GH.



SOLUTION

a. Use the Segment Addition Postulate to write an equation. Then solve the equation to find DF.

$DF = DE + EF$	Segment Addition Postulate
$DF = 23 + 35$	Substitute 23 for *DE* and 35 for *EF*.
$DF = 58$	Add.

b. Use the Segment Addition Postulate to write an equation. Then solve the equation to find GH.

$FH = FG + GH$	Segment Addition Postulate
$36 = 21 + GH$	Substitute 36 for *FH* and 21 for *FG*.
$15 = GH$	Subtract 21 from each side.

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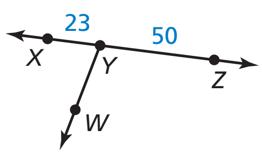
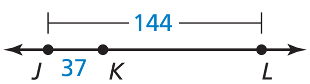
Use the diagram at the right.



6. Use the Segment Addition Postulate to find XZ.

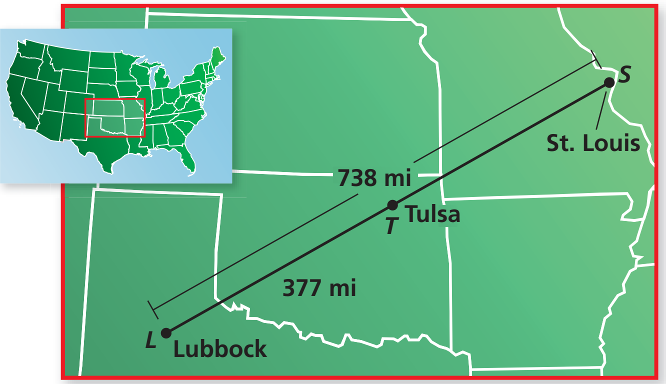
7. In the diagram, $WY = 30$. Can you use the Segment Addition Postulate to find the distance between points W and Z? Explain your reasoning.



8. Use the diagram at the left to find KL.

EXAMPLE 4 Using the Segment Addition Postulate

The cities shown on the map lie approximately in a straight line. Find the distance from Tulsa, Oklahoma, to St. Louis, Missouri.



SOLUTION

1. **Understand the Problem** You are given the distance from Lubbock to St. Louis and the distance from Lubbock to Tulsa. You need to find the distance from Tulsa to St. Louis.
2. **Make a Plan** Use the Segment Addition Postulate to find the distance from Tulsa to St. Louis.
3. **Solve the Problem** Use the Segment Addition Postulate to write an equation. Then solve the equation to find TS.

$LS = LT + TS$	Segment Addition Postulate
$738 = 377 + TS$	Substitute 738 for LS and 377 for LT.
$361 = TS$	Subtract 377 from each side.

▶ So, the distance from Tulsa to St. Louis is about 361 miles.

4. **Look Back** Does the answer make sense in the context of the problem? The distance from Lubbock to St. Louis is 738 miles. By the Segment Addition Postulate, the distance from Lubbock to Tulsa plus the distance from Tulsa to St. Louis should equal 738 miles.

$377 + 361 = 738$ ✓

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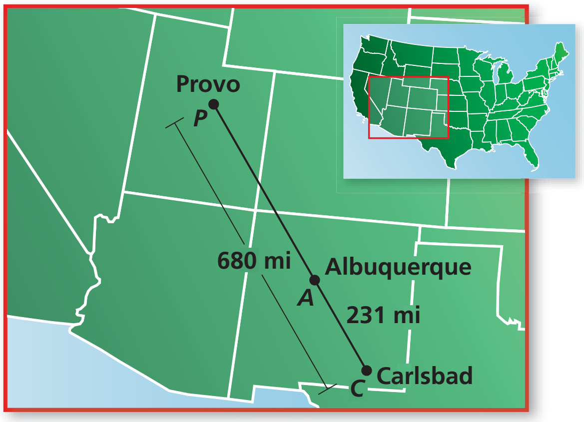
9. The cities shown on the map lie approximately in a straight line. Find the distance from Albuquerque, New Mexico, to Provo, Utah.



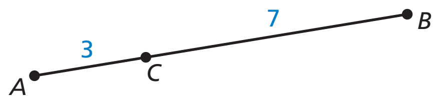
1.2 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **WRITING** Explain how $\overline{XY}$ and XY are different.

2. **DIFFERENT WORDS, SAME QUESTION** Which is different? Find "both" answers.



Find $AC + CB$.

Find $BC - AC$.

Find AB.

Find $CA + BC$.

Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, use a ruler to measure the length of the segment to the nearest tenth of a centimeter. *(See Example 1.)*

3.

4.

5.

6.

CONSTRUCTION **In Exercises 7 and 8, use a compass and straightedge to construct a copy of the segment.**

7. Copy the segment in Exercise 3.

8. Copy the segment in Exercise 4.

In Exercises 9–14, plot the points in a coordinate plane. Then determine whether $\overline{AB}$ and $\overline{CD}$ are congruent. *(See Example 2.)*

9. $A(-4, 5)$, $B(-4, 8)$, $C(2, -3)$, $D(2, 0)$

10. $A(6, -1)$, $B(1, -1)$, $C(2, -3)$, $D(4, -3)$

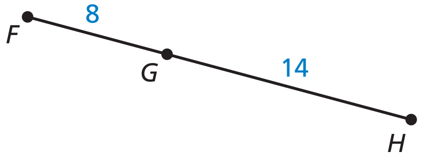
11. $A(8, 3)$, $B(-1, 3)$, $C(5, 10)$, $D(5, 3)$

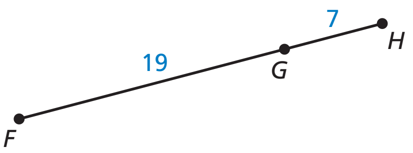
12. $A(6, -8)$, $B(6, 1)$, $C(7, -2)$, $D(-2, -2)$

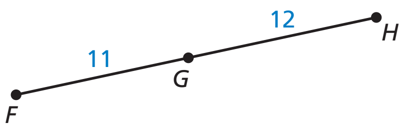
13. $A(-5, 6)$, $B(-5, -1)$, $C(-4, 3)$, $D(3, 3)$

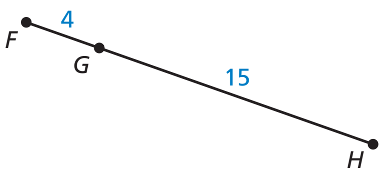
14. $A(10, -4)$, $B(3, -4)$, $C(-1, 2)$, $D(-1, 5)$

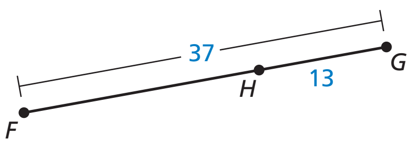
In Exercises 15–22, find FH. *(See Example 3.)*

15. 

16. 

17. 

18. 

19. 

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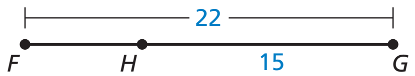

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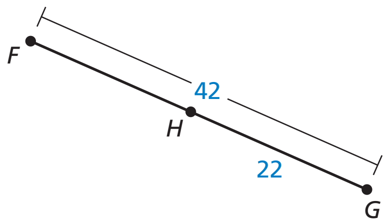

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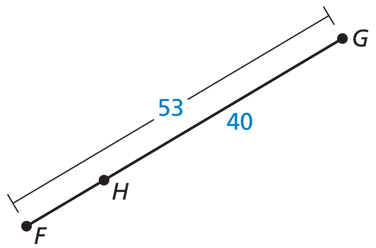
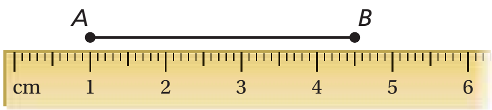


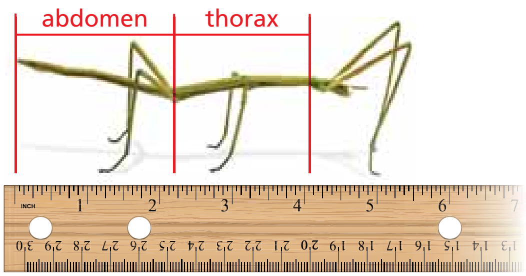
ERROR ANALYSIS **In Exercises 23 and 24, describe and correct the error in finding the length of $\overline{AB}$.**



23. ✗ $AB = 1 - 4.5 = -3.5$

24. ✗ $AB = |1 + 4.5| = 5.5$

25. **ATTENDING TO PRECISION** The diagram shows an insect called a walking stick. Use the ruler to estimate the length of the abdomen and the length of the thorax to the nearest $\frac{1}{4}$ inch. How much longer is the walking stick's abdomen than its thorax? How many times longer is its abdomen than its thorax?

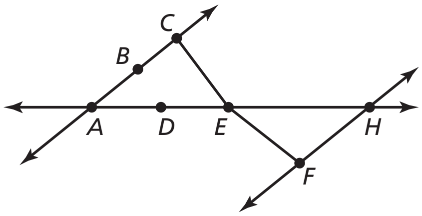


26. **MODELING WITH MATHEMATICS** In 2003, a remote-controlled model airplane became the first ever to fly nonstop across the Atlantic Ocean. The map shows the airplane's position at three different points during its flight. Point A represents Cape Spear, Newfoundland, point B represents the approximate position after 1 day, and point C represents Mannin Bay, Ireland. The airplane left from Cape Spear and landed in Mannin Bay. *(See Example 4.)*

a. Find the total distance the model airplane flew.

b. The model airplane's flight lasted nearly 38 hours. Estimate the airplane's average speed in miles per hour.

27. **USING STRUCTURE** Determine whether the statements are true or false. Explain your reasoning.



a. B is between A and C.

b. C is between B and E.

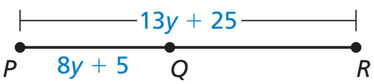
c. D is between A and H.

d. E is between C and F.

28. **MATHEMATICAL CONNECTIONS** Write an expression for the length of the segment.

a. $\overline{AC}$

b. $\overline{QR}$



29. **MATHEMATICAL CONNECTIONS** Point S is between points R and T on $\overline{RT}$. Use the information to write an equation in terms of x. Then solve the equation and find RS, ST, and RT.

a. $RS = 2x + 10$
$ST = x - 4$
$RT = 21$

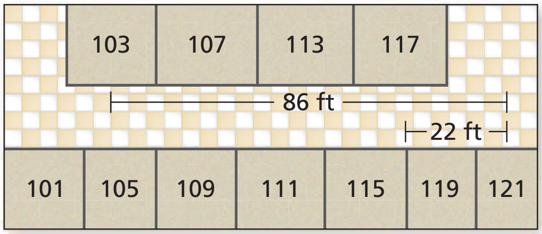
b. $RS = 3x - 16$
$ST = 4x - 8$
$RT = 60$

c. $RS = 2x - 8$
$ST = 11$
$RT = x + 10$

d. $RS = 4x - 9$
$ST = 19$
$RT = 8x - 14$

30. **THOUGHT PROVOKING** Is it possible to design a table where no two legs have the same length? Assume that the endpoints of the legs must all lie in the same plane. Include a diagram as part of your answer.

31. **MODELING WITH MATHEMATICS** You have to walk from Room 103 to Room 117.



a. How many feet do you travel from Room 103 to Room 117?

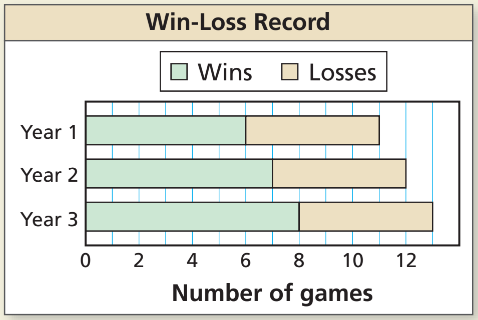
b. You can walk 4.4 feet per second. How many minutes will it take you to get to Room 117?

c. Why might it take you longer than the time in part (b)?

32. **MAKING AN ARGUMENT** Your friend and your cousin discuss measuring with a ruler. Your friend says that you must always line up objects at the zero on a ruler. Your cousin says it does not matter. Decide who is correct and explain your reasoning.

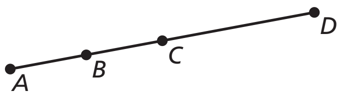
33. **REASONING** You travel from City X to City Y. You know that the round-trip distance is 647 miles. City Z, a city you pass on the way, is 27 miles from City X. Find the distance from City Z to City Y. Justify your answer.

34. **HOW DO YOU SEE IT?** The bar graph shows the win-loss record for a lacrosse team over a period of three years. Explain how you can apply the Ruler Postulate (Post. 1.1) and the Segment Addition Postulate (Post. 1.2) when interpreting a stacked bar graph like the one shown.



35. **ABSTRACT REASONING** The points (a, b) and (c, b) form a segment, and the points (d, e) and (d, f) form a segment. Create an equation assuming the segments are congruent. Are there any letters not used in the equation? Explain.

36. **MATHEMATICAL CONNECTIONS** In the diagram, $\overline{AB} \cong \overline{BC}$, $\overline{AC} \cong \overline{CD}$, and $AD = 12$. Find the lengths of all segments in the diagram. Suppose you choose one of the segments at random. What is the probability that the measure of the segment is greater than 3? Explain your reasoning.



37. **CRITICAL THINKING** Is it possible to use the Segment Addition Postulate (Post. 1.2) to show $FB > CB$ or that $AC > DB$? Explain your reasoning.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Simplify. *(Skills Review Handbook)*

38. $\dfrac{-4 + 6}{2}$ **39.** $\sqrt{20 + 5}$ **40.** $\sqrt{25 + 9}$ **41.** $\dfrac{7 + 6}{2}$

Solve the equation. *(Skills Review Handbook)*

42. $5x + 7 = 9x - 17$ **43.** $\dfrac{3 + y}{2} = 6$ **44.** $\dfrac{-5 + x}{2} = -9$ **45.** $-6x - 13 = -x - 23$

1.3 Using Midpoint and Distance Formulas

Essential Question

How can you find the midpoint and length of a line segment in a coordinate plane?

EXPLORATION 1 Finding the Midpoint of a Line Segment

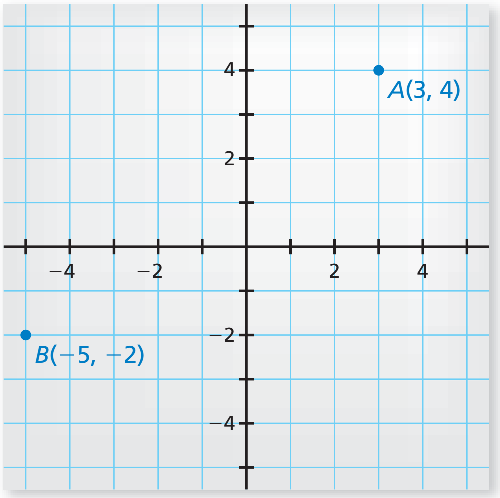
Work with a partner. Use centimeter graph paper.

a. Graph $\overline{AB}$, where the points A and B are as shown.

b. Explain how to *bisect* $\overline{AB}$, that is, to divide $\overline{AB}$ into two congruent line segments. Then bisect $\overline{AB}$ and use the result to find the *midpoint* M of $\overline{AB}$.

c. What are the coordinates of the midpoint M?

d. Compare the x-coordinates of A, B, and M. Compare the y-coordinates of A, B, and M. How are the coordinates of the midpoint M related to the coordinates of A and B?



MAKING SENSE OF PROBLEMS

To be proficient in math, you need to check your answers and continually ask yourself, "Does this make sense?"

EXPLORATION 2 Finding the Length of a Line Segment

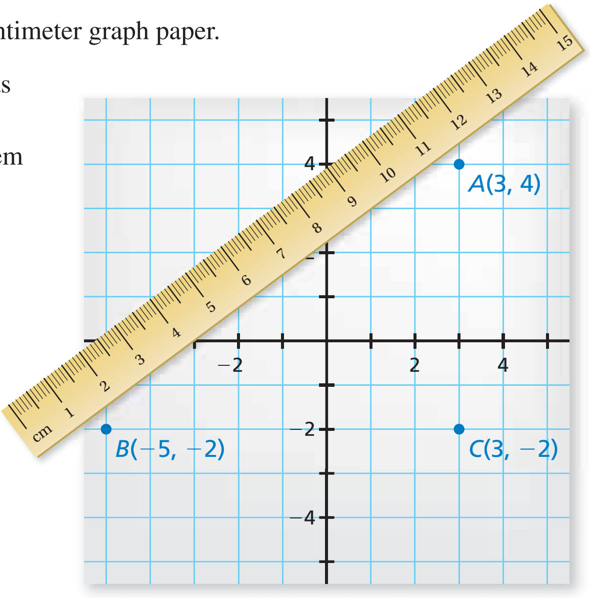
Work with a partner. Use centimeter graph paper.

a. Add point C to your graph as shown.

b. Use the Pythagorean Theorem to find the length of $\overline{AB}$.

c. Use a centimeter ruler to verify the length you found in part (b).

d. Use the Pythagorean Theorem and point M from Exploration 1 to find the lengths of $\overline{AM}$ and $\overline{MB}$. What can you conclude?



Communicate Your Answer

3. How can you find the midpoint and length of a line segment in a coordinate plane?

4. Find the coordinates of the midpoint M and the length of the line segment whose endpoints are given.

a. $D(-10, -4)$, $E(14, 6)$ **b.** $F(-4, 8)$, $G(9, 0)$

1.3 Lesson

Core Vocabulary

midpoint, *p. 20*
segment bisector, *p. 20*

What You Will Learn

- Find segment lengths using midpoints and segment bisectors.
- Use the Midpoint Formula.
- Use the Distance Formula.

Midpoints and Segment Bisectors

READING

The word *bisect* means "to cut into two equal parts."

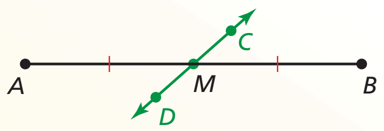
Core Concept

Midpoints and Segment Bisectors

The **midpoint** of a segment is the point that divides the segment into two congruent segments.

M is the midpoint of $\overline{AB}$.
So, $\overline{AM} \cong \overline{MB}$ and $AM = MB$.

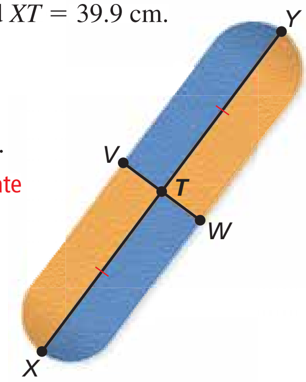
A **segment bisector** is a point, ray, line, line segment, or plane that intersects the segment at its midpoint. A midpoint or a segment bisector *bisects* a segment.



$\overleftrightarrow{CD}$ is a segment bisector of $\overline{AB}$.
So, $\overline{AM} \cong \overline{MB}$ and $AM = MB$.

EXAMPLE 1 Finding Segment Lengths

In the skateboard design, $\overline{VW}$ bisects $\overline{XY}$ at point T, and $XT = 39.9$ cm. Find XY.



SOLUTION

Point T is the midpoint of $\overline{XY}$. So, $XT = TY = 39.9$ cm.

$XY = XT + TY$	Segment Addition Postulate (Postulate 1.2)
$= 39.9 + 39.9$	Substitute.
$= 79.8$	Add.

- So, the length of $\overline{XY}$ is 79.8 centimeters.

Monitoring Progress

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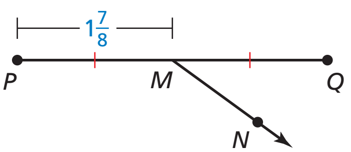
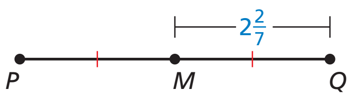
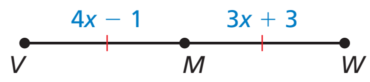
Identify the segment bisector of $\overline{PQ}$. Then find PQ.

1. 

2. 

EXAMPLE 2 Using Algebra with Segment Lengths

Point M is the midpoint of $\overline{VW}$. Find the length of $\overline{VM}$.



SOLUTION

Step 1 Write and solve an equation. Use the fact that $VM = MW$.

$VM = MW$	Write the equation.
$4x - 1 = 3x + 3$	Substitute.
$x - 1 = 3$	Subtract $3x$ from each side.
$x = 4$	Add 1 to each side.

Step 2 Evaluate the expression for VM when $x = 4$.

$VM = 4x - 1 = 4(4) - 1 = 15$

▶ So, the length of $\overline{VM}$ is 15.

Check

Because $VM = MW$, the length of $\overline{MW}$ should be 15.

$MW = 3x + 3 = 3(4) + 3 = 15$ ✓

Monitoring Progress

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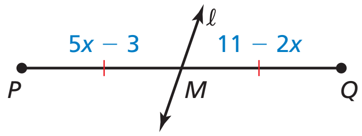
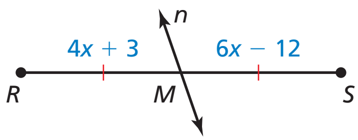
3. Identify the segment bisector of $\overline{PQ}$. Then find MQ.



4. Identify the segment bisector of $\overline{RS}$. Then find RS.

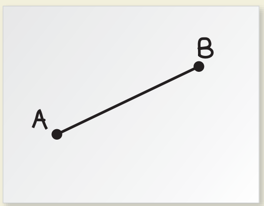


CONSTRUCTION Bisecting a Segment

Construct a segment bisector of $\overline{AB}$ by paper folding. Then find the midpoint M of $\overline{AB}$.

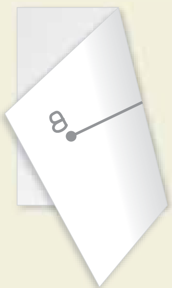
SOLUTION

Step 1



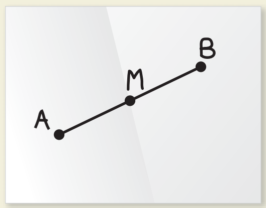
Draw the segment
Draw $\overline{AB}$ on a piece of paper.

Step 2



Fold the paper
Fold the paper so that B is on top of A.

Step 3



Label the midpoint
Label point M. Compare AM, MB, and AB.

$AM = MB = \frac{1}{2}AB$

Using the Midpoint Formula

You can use the coordinates of the endpoints of a segment to find the coordinates of the midpoint.

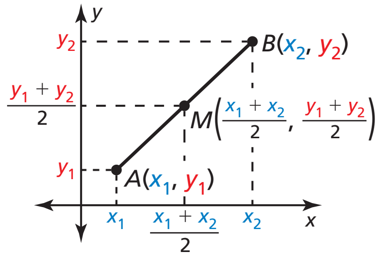
Core Concept

The Midpoint Formula

The coordinates of the midpoint of a segment are the averages of the x-coordinates and of the y-coordinates of the endpoints.

If $A(x_1, y_1)$ and $B(x_2, y_2)$ are points in a coordinate plane, then the midpoint M of $\overline{AB}$ has coordinates

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$



EXAMPLE 3 Using the Midpoint Formula

a. The endpoints of $\overline{RS}$ are $R(1, -3)$ and $S(4, 2)$. Find the coordinates of the midpoint M.

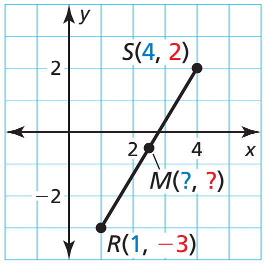
b. The midpoint of $\overline{JK}$ is $M(2, 1)$. One endpoint is $J(1, 4)$. Find the coordinates of endpoint K.

SOLUTION

a. Use the Midpoint Formula.

$$M\left(\frac{1 + 4}{2}, \frac{-3 + 2}{2}\right) = M\left(\frac{5}{2}, -\frac{1}{2}\right)$$

▶ The coordinates of the midpoint M are $\left(\frac{5}{2}, -\frac{1}{2}\right)$.



b. Let (x, y) be the coordinates of endpoint K. Use the Midpoint Formula.

Step 1 Find x.

$$\frac{1 + x}{2} = 2$$

$$1 + x = 4$$

$$x = 3$$

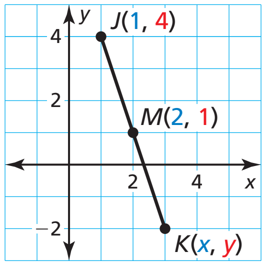
Step 2 Find y.

$$\frac{4 + y}{2} = 1$$

$$4 + y = 2$$

$$y = -2$$

▶ The coordinates of endpoint K are $(3, -2)$.



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5. The endpoints of $\overline{AB}$ are $A(1, 2)$ and $B(7, 8)$. Find the coordinates of the midpoint M.

6. The endpoints of $\overline{CD}$ are $C(-4, 3)$ and $D(-6, 5)$. Find the coordinates of the midpoint M.

7. The midpoint of $\overline{TU}$ is $M(2, 4)$. One endpoint is $T(1, 1)$. Find the coordinates of endpoint U.

8. The midpoint of $\overline{VW}$ is $M(-1, -2)$. One endpoint is $W(4, 4)$. Find the coordinates of endpoint V.

Using the Distance Formula

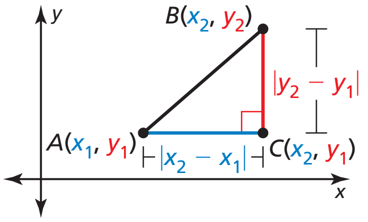
You can use the Distance Formula to find the distance between two points in a coordinate plane.

Core Concept

The Distance Formula

If $A(x_1, y_1)$ and $B(x_2, y_2)$ are points in a coordinate plane, then the distance between A and B is

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$



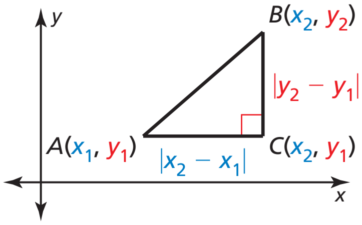
READING

The red mark at the corner of the triangle that makes a right angle indicates a right triangle.

The Distance Formula is related to the *Pythagorean Theorem*, which you will see again when you work with right triangles.

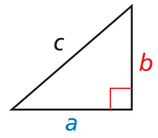
Distance Formula

$(AB)^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$



Pythagorean Theorem

$c^2 = a^2 + b^2$



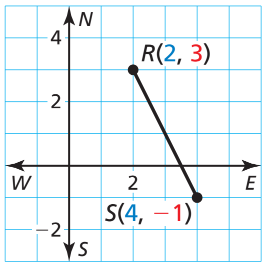
EXAMPLE 4 Using the Distance Formula

Your school is 4 miles east and 1 mile south of your apartment. A recycling center, where your class is going on a field trip, is 2 miles east and 3 miles north of your apartment. Estimate the distance between the recycling center and your school.

SOLUTION

You can model the situation using a coordinate plane with your apartment at the origin (0, 0). The coordinates of the recycling center and the school are $R(2, 3)$ and $S(4, -1)$, respectively. Use the Distance Formula. Let $(x_1, y_1) = (2, 3)$ and $(x_2, y_2) = (4, -1)$.

$RS = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	Distance Formula
$= \sqrt{(4 - 2)^2 + (-1 - 3)^2}$	Substitute.
$= \sqrt{2^2 + (-4)^2}$	Subtract.
$= \sqrt{4 + 16}$	Evaluate powers.
$= \sqrt{20}$	Add.
≈ 4.5	Use a calculator.



READING

The symbol ≈ means "is approximately equal to."

▶ So, the distance between the recycling center and your school is about 4.5 miles.

Monitoring Progress

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9. In Example 4, a park is 3 miles east and 4 miles south of your apartment. Find the distance between the park and your school.

1.3 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

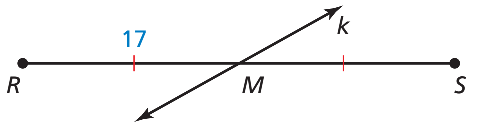
1. **VOCABULARY** If a point, ray, line, line segment, or plane intersects a segment at its midpoint, then what does it do to the segment?

2. **COMPLETE THE SENTENCE** To find the length of $\overline{AB}$, with endpoints $A(-7, 5)$ and $B(4, -6)$, you can use the ______________.

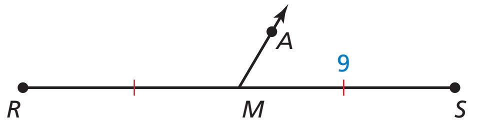
Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, identify the segment bisector of $\overline{RS}$. Then find RS. *(See Example 1.)*

3.



4.



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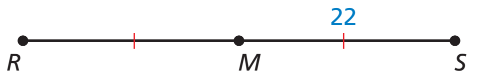

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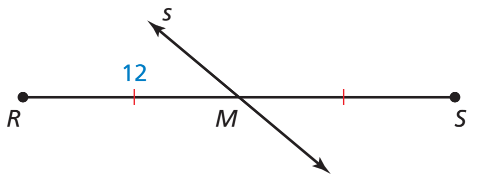
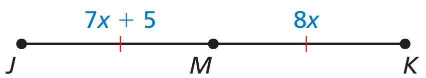


In Exercises 7 and 8, identify the segment bisector of $\overline{JK}$. Then find JM. *(See Example 2.)*

7.



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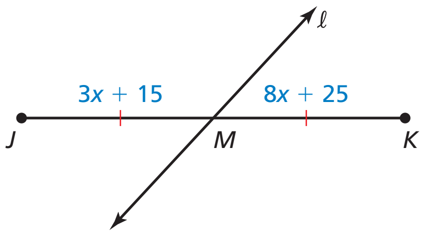
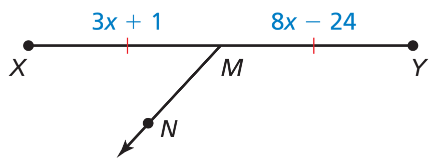
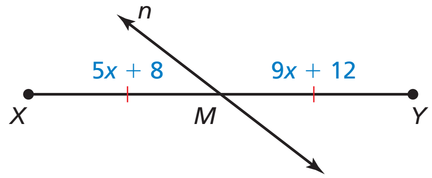


In Exercises 9 and 10, identify the segment bisector of $\overline{XY}$. Then find XY. *(See Example 2.)*

9.



10.



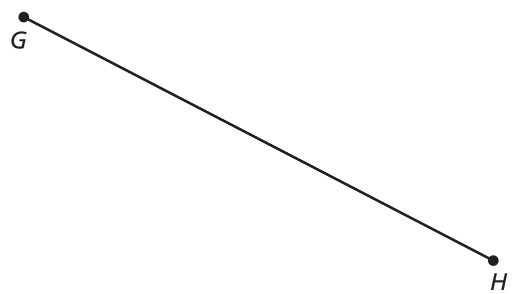
CONSTRUCTION **In Exercises 11–14, copy the segment and construct a segment bisector by paper folding. Then label the midpoint M.**

11.

12.

13.

14.



In Exercises 15–18, the endpoints of $\overline{CD}$ are given. Find the coordinates of the midpoint M. *(See Example 3.)*

15. $C(3, -5)$ and $D(7, 9)$

16. $C(-4, 7)$ and $D(0, -3)$

17. $C(-2, 0)$ and $D(4, 9)$

18. $C(-8, -6)$ and $D(-4, 10)$

In Exercises 19–22, the midpoint M and one endpoint of $\overline{GH}$ are given. Find the coordinates of the other endpoint. *(See Example 3.)*

19. $G(5, -6)$ and $M(4, 3)$

20. $H(-3, 7)$ and $M(-2, 5)$

21. $H(-2, 9)$ and $M(8, 0)$

22. $G(-4, 1)$ and $M\left(-\frac{13}{2}, -6\right)$

In Exercises 23–30, find the distance between the two points. *(See Example 4.)*

23. $A(13, 2)$ and $B(7, 10)$

24. $C(-6, 5)$ and $D(-3, 1)$

25. $E(3, 7)$ and $F(6, 5)$

26. $G(-5, 4)$ and $H(2, 6)$

27. $J(-8, 0)$ and $K(1, 4)$

28. $L(7, -1)$ and $M(-2, 4)$

29. $R(0, 1)$ and $S(6, 3.5)$

30. $T(13, 1.6)$ and $V(5.4, 3.7)$

ERROR ANALYSIS In Exercises 31 and 32, describe and correct the error in finding the distance between $A(6, 2)$ and $B(1, -4)$.

31. ✗

$$\begin{aligned} AB &= (6-1)^2 + [2-(-4)]^2 \\ &= 5^2 + 6^2 \\ &= 25 + 36 \\ &= 61 \end{aligned}$$

32. ✗

$$\begin{aligned} AB &= \sqrt{(6-2)^2 + [1-(-4)]^2} \\ &= \sqrt{4^2 + 5^2} \\ &= \sqrt{16 + 25} \\ &= \sqrt{41} \\ &\approx 6.4 \end{aligned}$$

COMPARING SEGMENTS In Exercises 33 and 34, the endpoints of two segments are given. Find each segment length. Tell whether the segments are congruent. If they are not congruent, state which segment length is greater.

33. $\overline{AB}$: $A(0, 2)$, $B(-3, 8)$ and $\overline{CD}$: $C(-2, 2)$, $D(0, -4)$

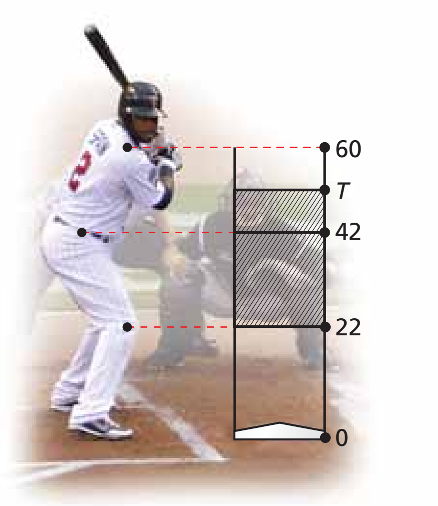
34. $\overline{EF}$: $E(1, 4)$, $F(5, 1)$ and $\overline{GH}$: $G(-3, 1)$, $H(1, 6)$

35. **WRITING** Your friend is having trouble understanding the Midpoint Formula.

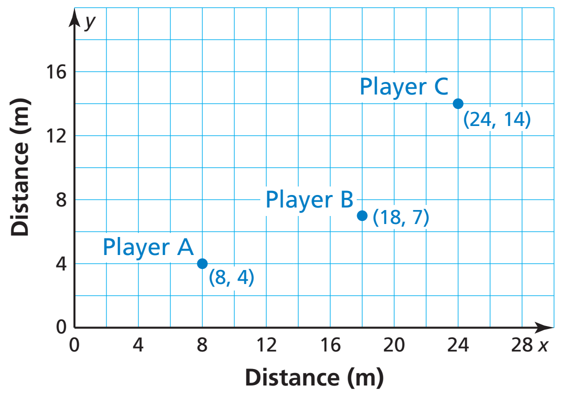
a. Explain how to find the midpoint when given the two endpoints in your own words.

b. Explain how to find the other endpoint when given one endpoint and the midpoint in your own words.

36. **PROBLEM SOLVING** In baseball, the strike zone is the region a baseball needs to pass through for the umpire to declare it a strike when the batter does not swing. The top of the strike zone is a horizontal plane passing through the midpoint of the top of the batter's shoulders and the top of the uniform pants when the player is in a batting stance. Find the height of T. (*Note:* All heights are in inches.)



37. **MODELING WITH MATHEMATICS** The figure shows the position of three players during part of a water polo match. Player A throws the ball to Player B, who then throws the ball to Player C.

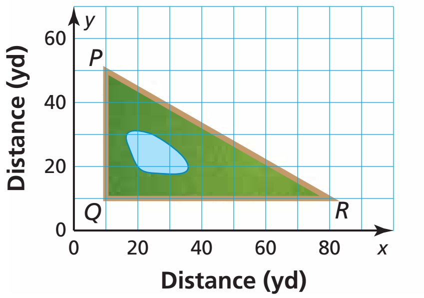


a. How far did Player A throw the ball? Player B?

b. How far would Player A have to throw the ball to throw it directly to Player C?

38. MODELING WITH MATHEMATICS Your school is 20 blocks east and 12 blocks south of your house. The mall is 10 blocks north and 7 blocks west of your house. You plan on going to the mall right after school. Find the distance between your school and the mall assuming there is a road directly connecting the school and the mall. One block is 0.1 mile.

39. PROBLEM SOLVING A path goes around a triangular park, as shown.



a. Find the distance around the park to the nearest yard.

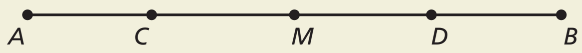
b. A new path and a bridge are constructed from point Q to the midpoint M of $\overline{PR}$. Find QM to the nearest yard.

c. A man jogs from P to Q to M to R to Q and back to P at an average speed of 150 yards per minute. About how many minutes does it take? Explain your reasoning.

40. MAKING AN ARGUMENT Your friend claims there is an easier way to find the length of a segment than the Distance Formula when the x-coordinates of the endpoints are equal. He claims all you have to do is subtract the y-coordinates. Do you agree with his statement? Explain your reasoning.

41. MATHEMATICAL CONNECTIONS Two points are located at (a, c) and (b, c). Find the midpoint and the distance between the two points.

42. HOW DO YOU SEE IT? $\overline{AB}$ contains midpoint M and points C and D, as shown. Compare the lengths. If you cannot draw a conclusion, write *impossible to tell*. Explain your reasoning.



a. AM and MB

b. AC and MB

c. MC and MD

d. MB and DB

43. ABSTRACT REASONING Use the diagram in Exercise 42. The points on $\overline{AB}$ represent locations you pass on your commute to work. You travel from your home at location A to location M before realizing that you left your lunch at home. You could turn around to get your lunch and then continue to work at location B. Or you could go to work and go to location D for lunch today. You want to choose the option that involves the least distance you must travel. Which option should you choose? Explain your reasoning.

44. THOUGHT PROVOKING Describe three ways to divide a rectangle into two congruent regions. Do the regions have to be triangles? Use a diagram to support your answer.

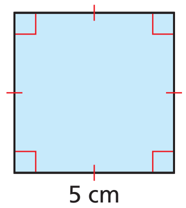
45. ANALYZING RELATIONSHIPS The length of $\overline{XY}$ is 24 centimeters. The midpoint of $\overline{XY}$ is M, and C is on $\overline{XM}$ so that XC is $\frac{2}{3}$ of XM. Point D is on $\overline{MY}$ so that MD is $\frac{3}{4}$ of MY. What is the length of $\overline{CD}$?

Maintaining Mathematical Proficiency

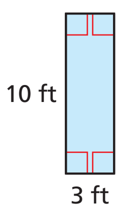
Reviewing what you learned in previous grades and lessons

Find the perimeter and area of the figure. *(Skills Review Handbook)*

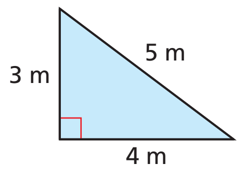
46.



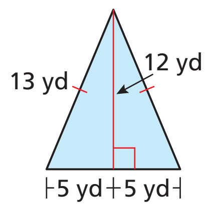
47.



48.



49.



Solve the inequality. Graph the solution. *(Skills Review Handbook)*

50. $a + 18 < 7$

51. $y - 5 \geq 8$

52. $-3x > 24$

53. $\frac{z}{4} \leq 12$

1.1–1.3 What Did You Learn?

Core Vocabulary

undefined terms, *p. 4*
point, *p. 4*
line, *p. 4*
plane, *p. 4*
collinear points, *p. 4*
coplanar points, *p. 4*
defined terms, *p. 5*
line segment, or segment, *p. 5*
endpoints, *p. 5*
ray, *p. 5*
opposite rays, *p. 5*
intersection, *p. 6*
postulate, *p. 12*
axiom, *p. 12*
coordinate, *p. 12*
distance, *p. 12*
construction, *p. 13*
congruent segments, *p. 13*
between, *p. 14*
midpoint, *p. 20*
segment bisector, *p. 20*

Core Concepts

Section 1.1

Undefined Terms: Point, Line, and Plane, *p. 4*
Defined Terms: Segment and Ray, *p. 5*
Intersections of Lines and Planes, *p. 6*

Section 1.2

Postulate 1.1 Ruler Postulate, *p. 12*
Congruent Segments, *p. 13*
Postulate 1.2 Segment Addition Postulate, *p. 14*

Section 1.3

Midpoints and Segment Bisectors, *p. 20*
The Midpoint Formula, *p. 22*
The Distance Formula, *p. 23*

Mathematical Practices

1. Sketch an example of the situation described in Exercise 49 on page 10 in a coordinate plane. Label your figure.

2. Explain how you arrived at your answer for Exercise 35 on page 18.

3. What assumptions did you make when solving Exercise 43 on page 26?

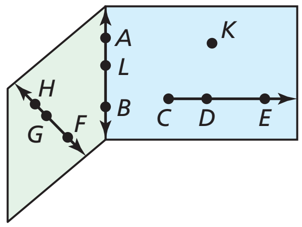
Study Skills

Keeping Your Mind Focused

- Keep a notebook just for vocabulary, formulas, and core concepts.
- Review this notebook before completing homework and before tests.

1.1–1.3 Quiz

Use the diagram. *(Section 1.1)*



1. Name four points.

2. Name three collinear points.

3. Name two lines.

4. Name three coplanar points.

5. Name the plane that is shaded green.

6. Give two names for the plane that is shaded blue.

7. Name three line segments.

8. Name three rays.

Sketch the figure described. *(Section 1.1)*

9. $\overrightarrow{QR}$ and $\overleftrightarrow{QS}$

10. plane P intersecting $\overleftrightarrow{YZ}$ at Z

Plot the points in a coordinate plane. Then determine whether $\overline{AB}$ and $\overline{CD}$ are congruent. *(Section 1.2)*

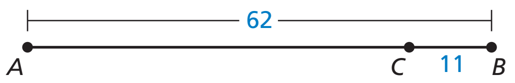
11. $A(-3, 3)$, $B(1, 3)$, $C(3, 2)$, $D(3, -2)$

12. $A(-8, 7)$, $B(1, 7)$, $C(-3, -6)$, $D(5, -6)$

Find AC. *(Section 1.2)*

13.

14.



Find the coordinates of the midpoint M and the distance between the two points. *(Section 1.3)*

15. $J(4, 3)$ and $K(2, -3)$

16. $L(-4, 5)$ and $N(5, -3)$

17. $P(-6, -1)$ and $Q(1, 2)$

18. Identify the segment bisector of $\overline{RS}$. Then find RS. *(Section 1.3)*

19. The midpoint of $\overline{JK}$ is $M(0, 1)$. One endpoint is $J(-6, 3)$. Find the coordinates of endpoint K. *(Section 1.3)*

20. Your mom asks you to run some errands on your way home from school. She wants you to stop at the post office and the grocery store, which are both on the same straight road between your school and your house. The distance from your school to the post office is 376 yards, the distance from the post office to your house is 929 yards, and the distance from the grocery store to your house is 513 yards. *(Section 1.2)*

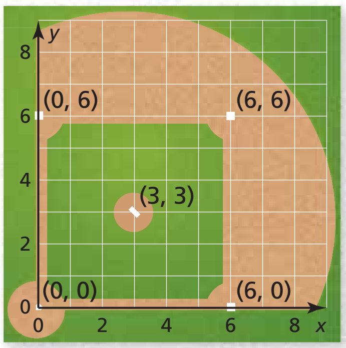
a. Where should you stop first?

b. What is the distance from the post office to the grocery store?

c. What is the distance from your school to your house?

d. You walk at a speed of 75 yards per minute. How long does it take you to walk straight home from school? Explain your answer.

21. The figure shows a coordinate plane on a baseball field. The distance from home plate to first base is 90 feet. The pitching mound is the midpoint between home plate and second base. Find the distance from home plate to second base. Find the distance between home plate and the pitching mound. Explain how you found your answers. *(Section 1.3)*



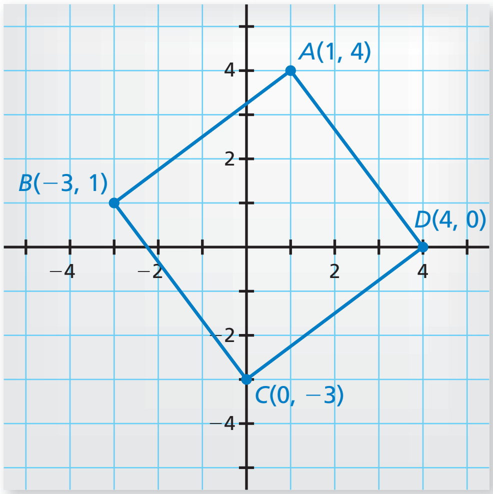
1.4 Perimeter and Area in the Coordinate Plane

Essential Question How can you find the perimeter and area of a polygon in a coordinate plane?

EXPLORATION 1 Finding the Perimeter and Area of a Quadrilateral

Work with a partner.

a. On a piece of centimeter graph paper, draw quadrilateral $ABCD$ in a coordinate plane. Label the points $A(1, 4)$, $B(-3, 1)$, $C(0, -3)$, and $D(4, 0)$.



b. Find the perimeter of quadrilateral $ABCD$.

c. Are adjacent sides of quadrilateral $ABCD$ perpendicular to each other? How can you tell?

d. What is the definition of a square? Is quadrilateral $ABCD$ a square? Justify your answer. Find the area of quadrilateral $ABCD$.

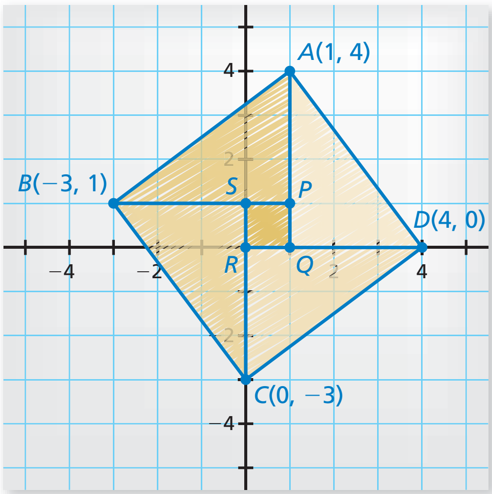
LOOKING FOR STRUCTURE

To be proficient in math, you need to visualize single objects as being composed of more than one object.

EXPLORATION 2 Finding the Area of a Polygon

Work with a partner.

a. Partition quadrilateral $ABCD$ into four right triangles and one square, as shown. Find the coordinates of the vertices for the five smaller polygons.



b. Find the areas of the five smaller polygons.

Area of Triangle BPA:

Area of Triangle AQD:

Area of Triangle DRC:

Area of Triangle CSB:

Area of Square $PQRS$:

c. Is the sum of the areas of the five smaller polygons equal to the area of quadrilateral $ABCD$? Justify your answer.

Communicate Your Answer

3. How can you find the perimeter and area of a polygon in a coordinate plane?

4. Repeat Exploration 1 for quadrilateral $EFGH$, where the coordinates of the vertices are $E(-3, 6)$, $F(-7, 3)$, $G(-1, -5)$, and $H(3, -2)$.

1.4 Lesson

Core Vocabulary

Previous
polygon
side
vertex
n-gon
convex
concave

What You Will Learn

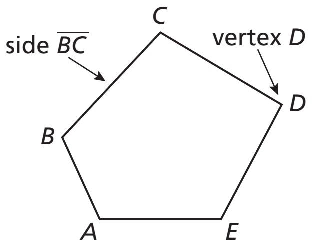
- Classify polygons.
- Find perimeters and areas of polygons in the coordinate plane.

Classifying Polygons

Core Concept

Polygons

In geometry, a figure that lies in a plane is called a plane figure. Recall that a *polygon* is a closed plane figure formed by three or more line segments called *sides*. Each side intersects exactly two sides, one at each *vertex*, so that no two sides with a common vertex are collinear. You can name a polygon by listing the vertices in consecutive order.

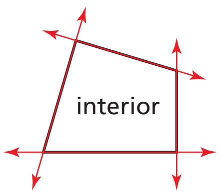


polygon *ABCDE*

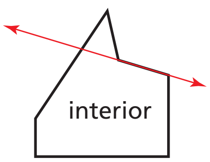
The number of sides determines the name of a polygon, as shown in the table.

Number of sides	Type of polygon
3	Triangle
4	Quadrilateral
5	Pentagon
6	Hexagon
7	Heptagon
8	Octagon
9	Nonagon
10	Decagon
12	Dodecagon
n	*n*-gon

You can also name a polygon using the term *n*-gon, where *n* is the number of sides. For instance, a 14-gon is a polygon with 14 sides.



convex polygon



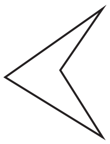
concave polygon

A polygon is *convex* when no line that contains a side of the polygon contains a point in the interior of the polygon. A polygon that is not convex is *concave*.

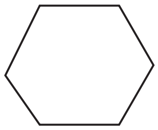
EXAMPLE 1 Classifying Polygons

Classify each polygon by the number of sides. Tell whether it is *convex* or *concave*.

a.



b.



SOLUTION

a. The polygon has four sides. So, it is a quadrilateral. The polygon is concave.

b. The polygon has six sides. So, it is a hexagon. The polygon is convex.

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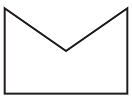
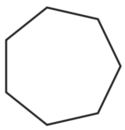
Classify the polygon by the number of sides. Tell whether it is *convex* or *concave*.

1.



2.



Finding Perimeter and Area in the Coordinate Plane

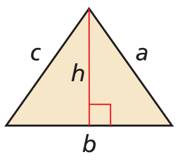
You can use the formulas given below and the Distance Formula to find the perimeters and areas of polygons in the coordinate plane.

REMEMBER

Perimeter has linear units, such as feet or meters. Area has square units, such as square feet or square meters.

Perimeter and Area

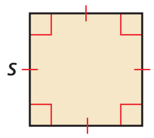
Triangle



$P = a + b + c$

$A = \frac{1}{2}bh$

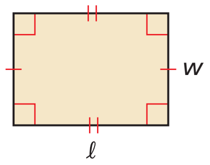
Square



$P = 4s$

$A = s^2$

Rectangle



$P = 2\ell + 2w$

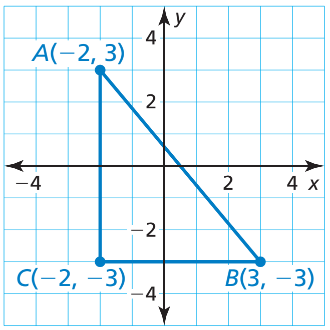
$A = \ell w$

EXAMPLE 2 Finding Perimeter in the Coordinate Plane

Find the perimeter of $\triangle ABC$ with vertices $A(-2, 3)$, $B(3, -3)$, and $C(-2, -3)$.

READING

You can read the notation $\triangle ABC$ as "triangle A B C."



SOLUTION

Step 1 Draw the triangle in a coordinate plane. Then find the length of each side.

Side $\overline{AB}$

$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ — Distance Formula

$= \sqrt{[3 - (-2)]^2 + (-3 - 3)^2}$ — Substitute.

$= \sqrt{5^2 + (-6)^2}$ — Subtract.

$= \sqrt{61}$ — Simplify.

≈ 7.81 — Use a calculator.

Side $\overline{BC}$

$BC = |-2 - 3| = 5$ — Ruler Postulate (Postulate 1.1)

Side $\overline{CA}$

$CA = |3 - (-3)| = 6$ — Ruler Postulate (Postulate 1.1)

Step 2 Find the sum of the side lengths.

$AB + BC + CA \approx 7.81 + 5 + 6 = 18.81$

▶ So, the perimeter of $\triangle ABC$ is about 18.81 units.

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Find the perimeter of the polygon with the given vertices.

3. $D(-3, 2)$, $E(4, 2)$, $F(4, -3)$

4. $G(-3, 2)$, $H(2, 2)$, $J(-1, -3)$

5. $K(-1, 1)$, $L(4, 1)$, $M(2, -2)$, $N(-3, -2)$

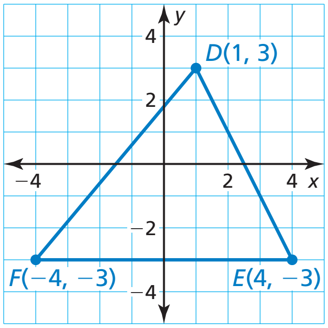
6. $Q(-4, -1)$, $R(1, 4)$, $S(4, 1)$, $T(-1, -4)$

EXAMPLE 3 Finding Area in the Coordinate Plane

Find the area of $\triangle DEF$ with vertices $D(1, 3)$, $E(4, -3)$, and $F(-4, -3)$.

SOLUTION

Step 1 Draw the triangle in a coordinate plane by plotting the vertices and connecting them.



Step 2 Find the lengths of the base and height.

Base

The base is $\overline{FE}$. Use the Ruler Postulate (Postulate 1.1) to find the length of $\overline{FE}$.

$$FE = |4 - (-4)| \qquad \text{Ruler Postulate (Postulate 1.1)}$$
$$= |8| \qquad \text{Subtract.}$$
$$= 8 \qquad \text{Simplify.}$$

So, the length of the base is 8 units.

Height

The height is the distance from point D to line segment $\overline{FE}$. By counting grid lines, you can determine that the height is 6 units.

Step 3 Substitute the values for the base and height into the formula for the area of a triangle.

$$A = \frac{1}{2}bh \qquad \text{Write the formula for area of a triangle.}$$
$$= \frac{1}{2}(8)(6) \qquad \text{Substitute.}$$
$$= 24 \qquad \text{Multiply.}$$

▶ So, the area of $\triangle DEF$ is 24 square units.

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Find the area of the polygon with the given vertices.

7. $G(2, 2)$, $H(3, -1)$, $J(-2, -1)$

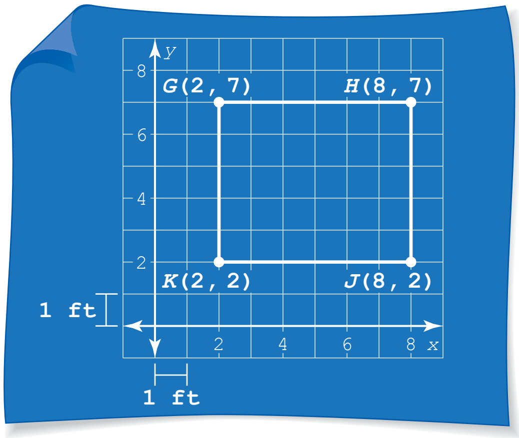
8. $N(-1, 1)$, $P(2, 1)$, $Q(2, -2)$, $R(-1, -2)$

9. $F(-2, 3)$, $G(1, 3)$, $H(1, -1)$, $J(-2, -1)$

10. $K(-3, 3)$, $L(3, 3)$, $M(3, -1)$, $N(-3, -1)$

EXAMPLE 4 Modeling with Mathematics

You are building a shed in your backyard. The diagram shows the four vertices of the shed. Each unit in the coordinate plane represents 1 foot. Find the area of the floor of the shed.



SOLUTION

1. **Understand the Problem** You are given the coordinates of a shed. You need to find the area of the floor of the shed.

2. **Make a Plan** The shed is rectangular, so use the coordinates to find the length and width of the shed. Then use a formula to find the area.

3. **Solve the Problem**

Step 1 Find the length and width.

Length $GH = |8 - 2| = 6$ Ruler Postulate (Postulate 1.1)

Width $GK = |7 - 2| = 5$ Ruler Postulate (Postulate 1.1)

The shed has a length of 6 feet and a width of 5 feet.

Step 2 Substitute the values for the length and width into the formula for the area of a rectangle.

$A = \ell w$ Write the formula for area of a rectangle.

$= (6)(5)$ Substitute.

$= 30$ Multiply.

▶ So, the area of the floor of the shed is 30 square feet.

4. **Look Back** Make sure your answer makes sense in the context of the problem. Because you are finding an area, your answer should be in square units. An answer of 30 square feet makes sense in the context of the problem. ✓

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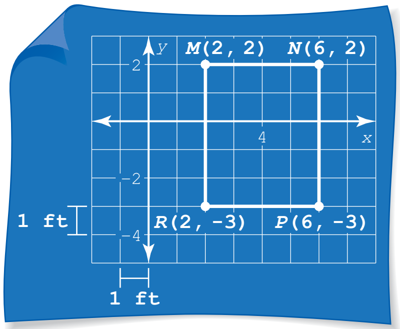


11. You are building a patio in your school's courtyard. In the diagram at the left, the coordinates represent the four vertices of the patio. Each unit in the coordinate plane represents 1 foot. Find the area of the patio.

Dynamic Solutions available at *BigIdeasMath.com*

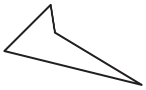
Vocabulary and Core Concept Check

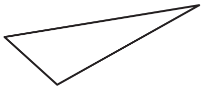
1. **COMPLETE THE SENTENCE** The perimeter of a square with side length s is $P =$ ____.

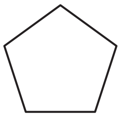
2. **WRITING** What formulas can you use to find the area of a triangle in a coordinate plane?

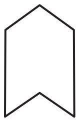
Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, classify the polygon by the number of sides. Tell whether it is *convex* or *concave*. *(See Example 1.)*

3. 

4. 

5. 

6. 

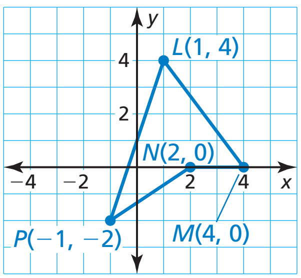
In Exercises 7–12, find the perimeter of the polygon with the given vertices. *(See Example 2.)*

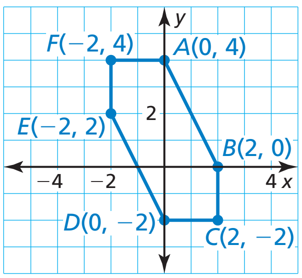
7. $G(2, 4)$, $H(2, -3)$, $J(-2, -3)$, $K(-2, 4)$

8. $Q(-3, 2)$, $R(1, 2)$, $S(1, -2)$, $T(-3, -2)$

9. $U(-2, 4)$, $V(3, 4)$, $W(3, -4)$

10. $X(-1, 3)$, $Y(3, 0)$, $Z(-1, -2)$

11. 

12. 

In Exercises 13–16, find the area of the polygon with the given vertices. *(See Example 3.)*

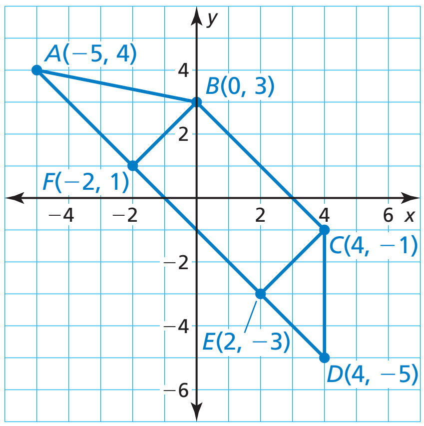
13. $E(3, 1)$, $F(3, -2)$, $G(-2, -2)$

14. $J(-3, 4)$, $K(4, 4)$, $L(3, -3)$

15. $W(0, 0)$, $X(0, 3)$, $Y(-3, 3)$, $Z(-3, 0)$

16. $N(-2, 1)$, $P(3, 1)$, $Q(3, -1)$, $R(-2, -1)$

In Exercises 17–24, use the diagram.



17. Find the perimeter of $\triangle CDE$.

18. Find the perimeter of rectangle $BCEF$.

19. Find the perimeter of $\triangle ABF$.

20. Find the perimeter of quadrilateral $ABCD$.

21. Find the area of $\triangle CDE$.

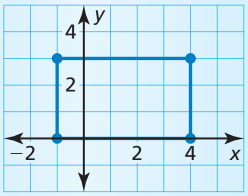
22. Find the area of rectangle $BCEF$.

23. Find the area of $\triangle ABF$.

24. Find the area of quadrilateral $ABCD$.

ERROR ANALYSIS In Exercises 25 and 26, describe and correct the error in finding the perimeter or area of the polygon.

25. ✗



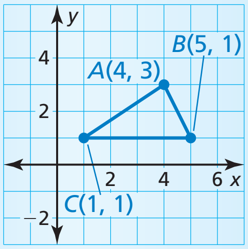
$P = 2\ell + 2w$

$= 2(4) + 2(3)$

$= 14$

The perimeter is 14 units.

26. ✗



$b = |5 - 1| = 4$

$h = \sqrt{(5-4)^2 + (1-3)^2}$

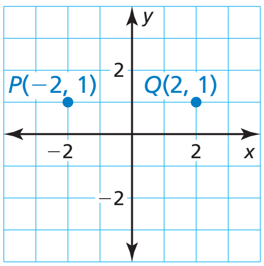
$= \sqrt{5}$

≈ 2.2

$A = \frac{1}{2}bh \approx \frac{1}{2}(4)(2.2) = 4.4$

The area is about 4.4 square units.

In Exercises 27 and 28, use the diagram.



27. Determine which point is the remaining vertex of a triangle with an area of 4 square units.

Ⓐ $R(2, 0)$

Ⓑ $S(-2, -1)$

Ⓒ $T(-1, 0)$

Ⓓ $U(2, -2)$

28. Determine which points are the remaining vertices of a rectangle with a perimeter of 14 units.

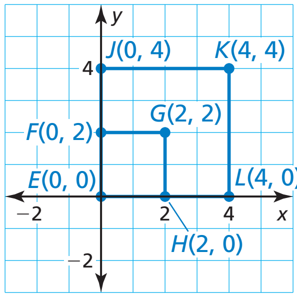
Ⓐ $A(2, -2)$ and $B(2, -1)$

Ⓑ $C(-2, -2)$ and $D(-2, 2)$

Ⓒ $E(-2, -2)$ and $F(2, -2)$

Ⓓ $G(2, 0)$ and $H(-2, 0)$

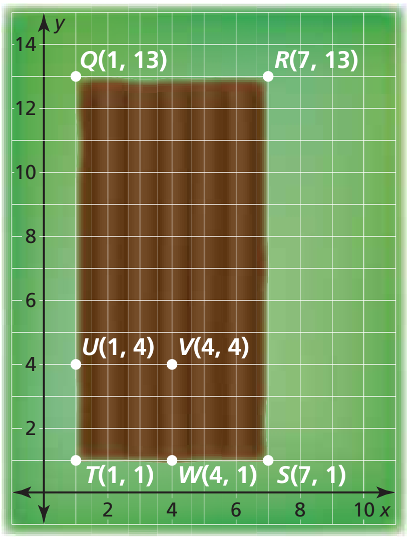
29. **USING STRUCTURE** Use the diagram.



a. Find the areas of square *EFGH* and square *EJKL*. What happens to the area when the perimeter of square *EFGH* is doubled?

b. Is this true for every square? Explain.

30. **MODELING WITH MATHEMATICS** You are growing zucchini plants in your garden. In the figure, the entire garden is rectangle *QRST*. Each unit in the coordinate plane represents 1 foot. *(See Example 4.)*



a. Find the area of the garden.

b. Zucchini plants require 9 square feet around each plant. How many zucchini plants can you plant?

c. You decide to use square *TUVW* to grow lettuce. You can plant four heads of lettuce per square foot. How many of each vegetable can you plant? Explain.

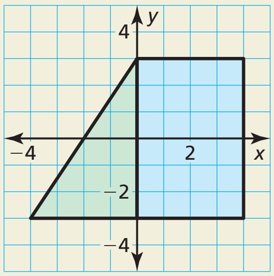
31. **MODELING WITH MATHEMATICS** You are going for a hike in the woods. You hike to a waterfall that is 4 miles east of where you left your car. You then hike to a lookout point that is 2 miles north of your car. From the lookout point, you return to your car.

   a. Map out your route in a coordinate plane with your car at the origin. Let each unit in the coordinate plane represent 1 mile. Assume you travel along straight paths.

   b. How far do you travel during the entire hike?

   c. When you leave the waterfall, you decide to hike to an old wishing well before going to the lookout point. The wishing well is 3 miles north and 2 miles west of the lookout point. How far do you travel during the entire hike?

32. **HOW DO YOU SEE IT?** Without performing any calculations, determine whether the triangle or the rectangle has a greater area. Which one has a greater perimeter? Explain your reasoning.



33. **MATHEMATICAL CONNECTIONS** The lines $y_1 = 2x - 6$, $y_2 = -3x + 4$, and $y_3 = -\frac{1}{2}x + 4$ are the sides of a right triangle.

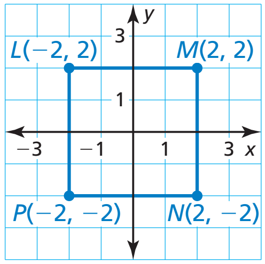
   a. Use slopes to determine which sides are perpendicular.

   b. Find the vertices of the triangle.

   c. Find the perimeter and area of the triangle.

34. **THOUGHT PROVOKING** Your bedroom has an area of 350 square feet. You are remodeling to include an attached bathroom that has an area of 150 square feet. Draw a diagram of the remodeled bedroom and bathroom in a coordinate plane.

35. **PROBLEM SOLVING** Use the diagram.

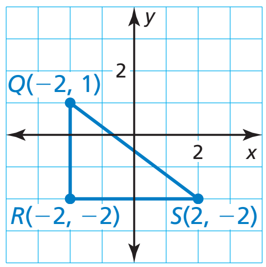


   a. Find the perimeter and area of the square.

   b. Connect the midpoints of the sides of the given square to make a quadrilateral. Is this quadrilateral a square? Explain your reasoning.

   c. Find the perimeter and area of the quadrilateral you made in part (b). Compare this area to the area you found in part (a).

36. **MAKING AN ARGUMENT** Your friend claims that a rectangle with the same perimeter as $\triangle QRS$ will have the same area as the triangle. Is your friend correct? Explain your reasoning.



37. **REASONING** Triangle ABC has a perimeter of 12 units. The vertices of the triangle are $A(x, 2)$, $B(2, -2)$, and $C(-1, 2)$. Find the value of x.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Solve the equation. *(Skills Review Handbook)*

38. $3x - 7 = 2$

39. $5x + 9 = 4$

40. $x + 4 = x - 12$

41. $4x - 9 = 3x + 5$

42. $11 - 2x = 5x - 3$

43. $\frac{x + 1}{2} = 4x - 3$

44. Use a compass and straightedge to construct a copy of the line segment. *(Section 1.2)*

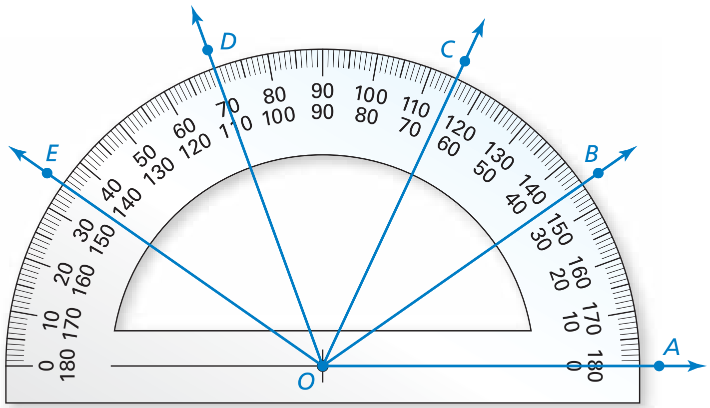
1.5 Measuring and Constructing Angles

Essential Question

How can you measure and classify an angle?

EXPLORATION 1 Measuring and Classifying Angles

Work with a partner. Find the degree measure of each of the following angles. Classify each angle as acute, right, or obtuse.



a. $\angle AOB$ **b.** $\angle AOC$ **c.** $\angle BOC$ **d.** $\angle BOE$

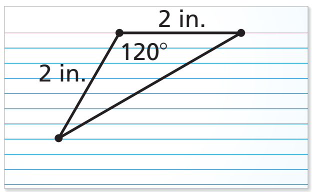
e. $\angle COE$ **f.** $\angle COD$ **g.** $\angle BOD$ **h.** $\angle AOE$

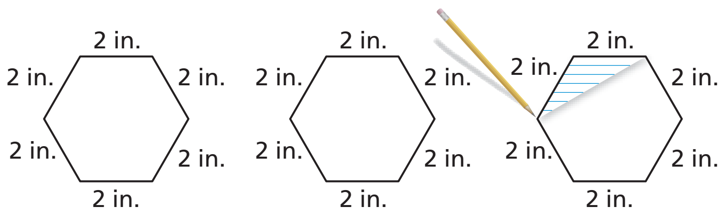
EXPLORATION 2 Drawing a Regular Polygon

Work with a partner.

a. Use a ruler and protractor to draw the triangular pattern shown at the right.

b. Cut out the pattern and use it to draw three regular hexagons, as shown below.



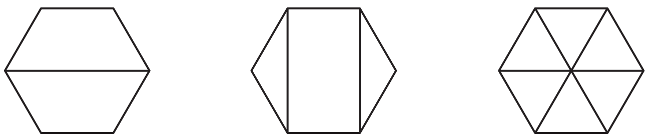


ATTENDING TO PRECISION

To be proficient in math, you need to calculate and measure accurately and efficiently.

c. The sum of the angle measures of a polygon with n sides is equal to $180(n - 2)°$. Do the angle measures of your hexagons agree with this rule? Explain.

d. Partition your hexagons into smaller polygons, as shown below. For each hexagon, find the sum of the angle measures of the smaller polygons. Does each sum equal the sum of the angle measures of a hexagon? Explain.



Communicate Your Answer

3. How can you measure and classify an angle?

1.5 Lesson

Core Vocabulary

angle, *p. 38*
vertex, *p. 38*
sides of an angle, *p. 38*
interior of an angle, *p. 38*
exterior of an angle, *p. 38*
measure of an angle, *p. 39*
acute angle, *p. 39*
right angle, *p. 39*
obtuse angle, *p. 39*
straight angle, *p. 39*
congruent angles, *p. 40*
angle bisector, *p. 42*

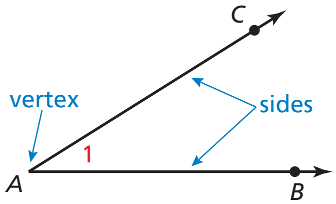
Previous
protractor
degrees

What You Will Learn

- Name angles.
- Measure and classify angles.
- Identify congruent angles.
- Use the Angle Addition Postulate to find angle measures.
- Bisect angles.

Naming Angles

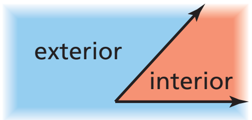
An **angle** is a set of points consisting of two different rays that have the same endpoint, called the **vertex**. The rays are the **sides** of the angle.



You can name an angle in several different ways.

- Use its vertex, such as $\angle A$.
- Use a point on each ray and the vertex, such as $\angle BAC$ or $\angle CAB$.
- Use a number, such as $\angle 1$.

The region that contains all the points between the sides of the angle is the **interior of the angle**. The region that contains all the points outside the angle is the **exterior of the angle**.

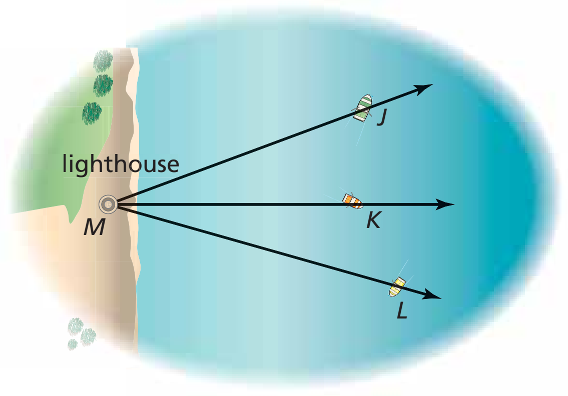


COMMON ERROR

When a point is the vertex of more than one angle, you cannot use the vertex alone to name the angle.

EXAMPLE 1 **Naming Angles**

A lighthouse keeper measures the angles formed by the lighthouse at point M and three boats. Name three angles shown in the diagram.



SOLUTION

$\angle JMK$ or $\angle KMJ$

$\angle KML$ or $\angle LMK$

$\angle JML$ or $\angle LMJ$

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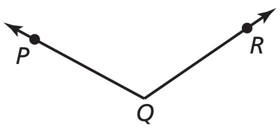
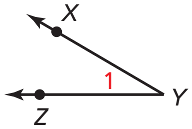
Write three names for the angle.

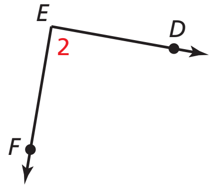
1.



2.



3.



Measuring and Classifying Angles

A protractor helps you approximate the *measure* of an angle. The measure is usually given in *degrees*.

COMMON ERROR

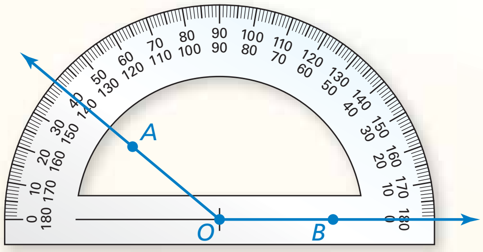
Most protractors have an inner and an outer scale. When measuring, make sure you are using the correct scale.

Postulate

Postulate 1.3 Protractor Postulate

Consider $\overleftrightarrow{OB}$ and a point A on one side of $\overleftrightarrow{OB}$. The rays of the form $\overrightarrow{OA}$ can be matched one to one with the real numbers from 0 to 180.

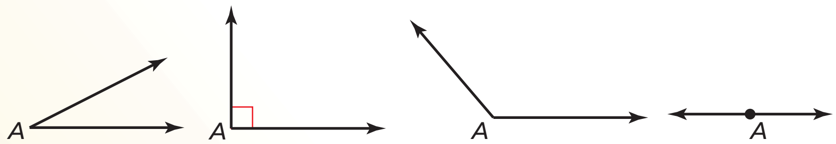
The **measure** of $\angle AOB$, which can be written as $m\angle AOB$, is equal to the absolute value of the difference between the real numbers matched with $\overrightarrow{OA}$ and $\overrightarrow{OB}$ on a protractor.



You can classify angles according to their measures.

Core Concept

Types of Angles

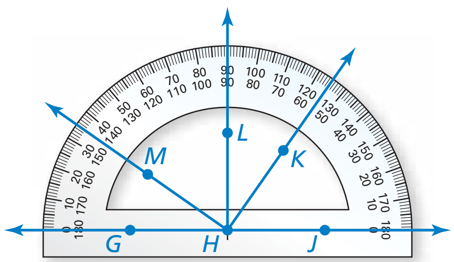


acute angle	right angle	obtuse angle	straight angle
Measures greater than 0° and less than 90°	Measures 90°	Measures greater than 90° and less than 180°	Measures 180°

EXAMPLE 2 Measuring and Classifying Angles

Find the measure of each angle. Then classify each angle.

a. $\angle GHK$ **b.** $\angle JHL$ **c.** $\angle LHK$



SOLUTION

a. $\overrightarrow{HG}$ lines up with 0° on the outer scale of the protractor. $\overrightarrow{HK}$ passes through 125° on the outer scale. So, $m\angle GHK = 125°$. It is an *obtuse* angle.

b. $\overrightarrow{HJ}$ lines up with 0° on the inner scale of the protractor. $\overrightarrow{HL}$ passes through 90°. So, $m\angle JHL = 90°$. It is a *right* angle.

c. $\overrightarrow{HL}$ passes through 90°. $\overrightarrow{HK}$ passes through 55° on the inner scale. So, $m\angle LHK = |90 - 55| = 35°$. It is an *acute* angle.

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Use the diagram in Example 2 to find the angle measure. Then classify the angle.

4. $\angle JHM$ **5.** $\angle MHK$ **6.** $\angle MHL$

Identifying Congruent Angles

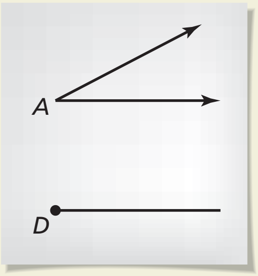
You can use a compass and straightedge to construct an angle that has the same measure as a given angle.

CONSTRUCTION **Copying an Angle**

Use a compass and straightedge to construct an angle that has the same measure as $\angle A$. In this construction, the *center* of an arc is the point where the compass point rests. The *radius* of an arc is the distance from the center of the arc to a point on the the arc drawn by the compass.

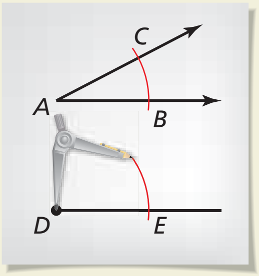
SOLUTION

Step 1



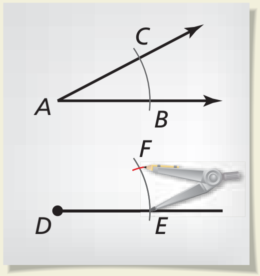
Draw a segment Draw an angle such as $\angle A$, as shown. Then draw a segment. Label a point D on the segment.

Step 2



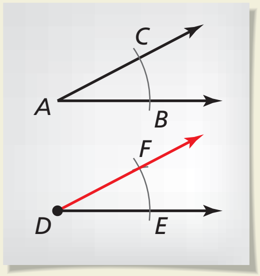
Draw arcs Draw an arc with center A. Using the same radius, draw an arc with center D.

Step 3



Draw an arc Label B, C, and E. Draw an arc with radius BC and center E. Label the intersection F.

Step 4



Draw a ray Draw $\overrightarrow{DF}$. $\angle EDF \cong \angle BAC$.

Two angles are **congruent angles** when they have the same measure. In the construction above, $\angle A$ and $\angle D$ are congruent angles. So,

$m\angle A = m\angle D$ The measure of angle *A* is *equal to* the measure of angle *D*.

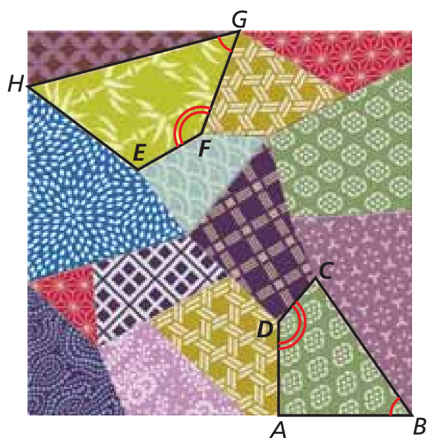
and

$\angle A \cong \angle D$. Angle *A* is *congruent to* angle *D*.

EXAMPLE 3 **Identifying Congruent Angles**

a. Identify the congruent angles labeled in the quilt design.

b. $m\angle ADC = 140°$. What is $m\angle EFG$?



SOLUTION

a. There are two pairs of congruent angles:

$\angle ABC \cong \angle FGH$ and $\angle ADC \cong \angle EFG$.

b. Because $\angle ADC \cong \angle EFG$, $m\angle ADC = m\angle EFG$.

So, $m\angle EFG = 140°$.

READING

In diagrams, matching arcs indicate congruent angles. When there is more than one pair of congruent angles, use multiple arcs.

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7. Without measuring, is $\angle DAB \cong \angle FEH$ in Example 3? Explain your reasoning. Use a protractor to verify your answer.

Using the Angle Addition Postulate

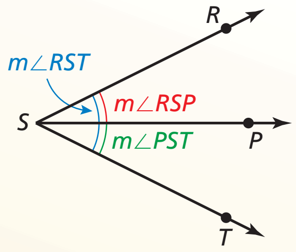
Postulate

Postulate 1.4 Angle Addition Postulate

Words If P is in the interior of $\angle RST$, then the measure of $\angle RST$ is equal to the sum of the measures of $\angle RSP$ and $\angle PST$.

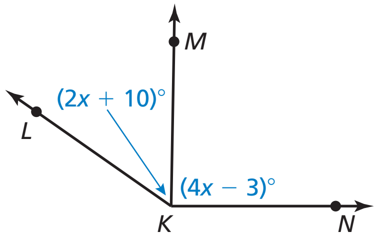
Symbols If P is in the interior of $\angle RST$, then

$$m\angle RST = m\angle RSP + m\angle PST.$$



EXAMPLE 4 Finding Angle Measures

Given that $m\angle LKN = 145°$, find $m\angle LKM$ and $m\angle MKN$.



SOLUTION

Step 1 Write and solve an equation to find the value of x.

$m\angle LKN = m\angle LKM + m\angle MKN$	Angle Addition Postulate
$145° = (2x + 10)° + (4x - 3)°$	Substitute angle measures.
$145 = 6x + 7$	Combine like terms.
$138 = 6x$	Subtract 7 from each side.
$23 = x$	Divide each side by 6.

Step 2 Evaluate the given expressions when $x = 23$.

$$m\angle LKM = (2x + 10)° = (2 \cdot 23 + 10)° = 56°$$

$$m\angle MKN = (4x - 3)° = (4 \cdot 23 - 3)° = 89°$$

So, $m\angle LKM = 56°$, and $m\angle MKN = 89°$.

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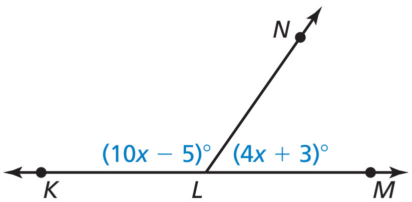
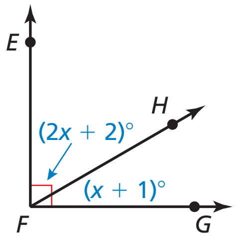
Find the indicated angle measures.

8. Given that $\angle KLM$ is a straight angle, find $m\angle KLN$ and $m\angle NLM$.

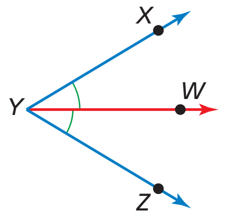


9. Given that $\angle EFG$ is a right angle, find $m\angle EFH$ and $m\angle HFG$.



Bisecting Angles

An **angle bisector** is a ray that divides an angle into two angles that are congruent. In the figure, $\overrightarrow{YW}$ bisects $\angle XYZ$, so $\angle XYW \cong \angle ZYW$.



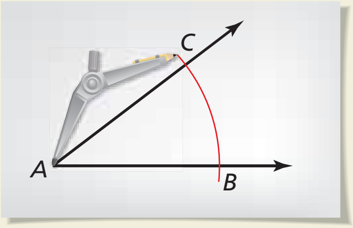
You can use a compass and straightedge to bisect an angle.

CONSTRUCTION Bisecting an Angle

Construct an angle bisector of $\angle A$ with a compass and straightedge.

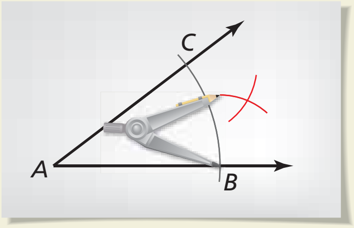
SOLUTION

Step 1



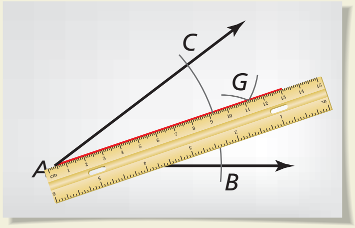
Draw an arc Draw an angle such as $\angle A$, as shown. Place the compass at A. Draw an arc that intersects both sides of the angle. Label the intersections B and C.

Step 2



Draw arcs Place the compass at C. Draw an arc. Then place the compass point at B. Using the same radius, draw another arc.

Step 3



Draw a ray Label the intersection G. Use a straightedge to draw a ray through A and G.

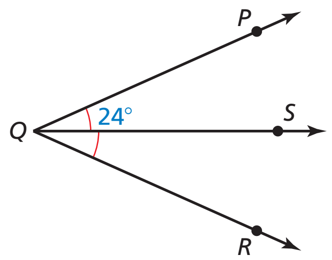
$\overrightarrow{AG}$ bisects $\angle A$.

EXAMPLE 5 Using a Bisector to Find Angle Measures

$\overrightarrow{QS}$ bisects $\angle PQR$, and $m\angle PQS = 24°$. Find $m\angle PQR$.

SOLUTION

Step 1 Draw a diagram.



Step 2 Because $\overrightarrow{QS}$ bisects $\angle PQR$, $m\angle PQS = m\angle RQS$. So, $m\angle RQS = 24°$. Use the Angle Addition Postulate to find $m\angle PQR$.

$$m\angle PQR = m\angle PQS + m\angle RQS \quad \text{Angle Addition Postulate}$$
$$= 24° + 24° \quad \text{Substitute angle measures.}$$
$$= 48° \quad \text{Add.}$$

▶ So, $m\angle PQR = 48°$.

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10. Angle MNP is a straight angle, and $\overrightarrow{NQ}$ bisects $\angle MNP$. Draw $\angle MNP$ and $\overrightarrow{NQ}$. Use arcs to mark the congruent angles in your diagram. Find the angle measures of these congruent angles.

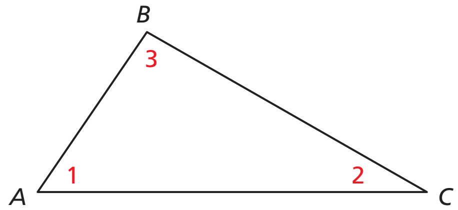
1.5 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **COMPLETE THE SENTENCE** Two angles are ___________ angles when they have the same measure.

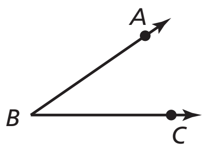
2. **WHICH ONE DOESN'T BELONG?** Which angle name does *not* belong with the other three? Explain your reasoning.

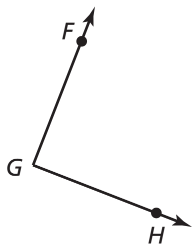


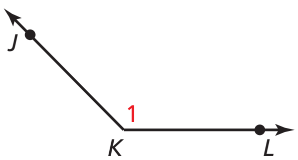
$\angle BCA$	$\angle BAC$	$\angle 1$	$\angle CAB$

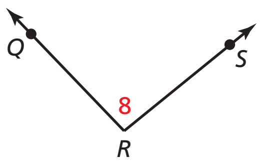
Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, write three names for the angle. *(See Example 1.)*

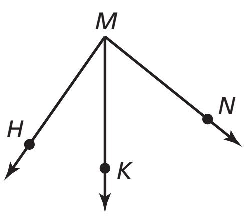
3. 

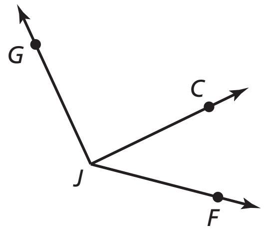
4. 

5. 

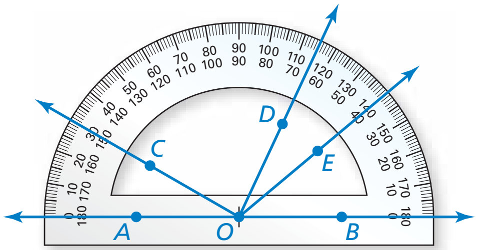
6. 

In Exercises 7 and 8, name three different angles in the diagram. *(See Example 1.)*

7. 

8. 

In Exercises 9–12, find the angle measure. Then classify the angle. *(See Example 2.)*



9. $m\angle AOC$

10. $m\angle BOD$

11. $m\angle COD$

12. $m\angle EOD$

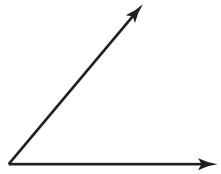
ERROR ANALYSIS **In Exercises 13 and 14, describe and correct the error in finding the angle measure. Use the diagram from Exercises 9–12.**

13. ✗ $m\angle BOC = 30°$

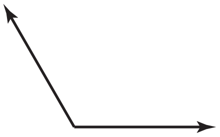
14. ✗ $m\angle DOE = 65°$

CONSTRUCTION **In Exercises 15 and 16, use a compass and straightedge to copy the angle.**

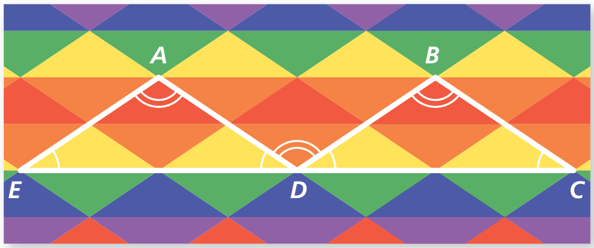
15.



16.



In Exercises 17–20, $m\angle AED = 34°$ and $m\angle EAD = 112°$. *(See Example 3.)*



17. Identify the angles congruent to $\angle AED$.

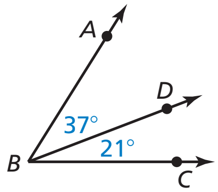
18. Identify the angles congruent to $\angle EAD$.

19. Find $m\angle BDC$.

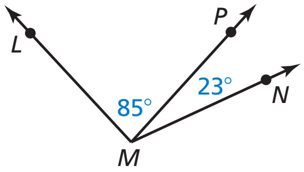
20. Find $m\angle ADB$.

In Exercises 21–24, find the indicated angle measure.

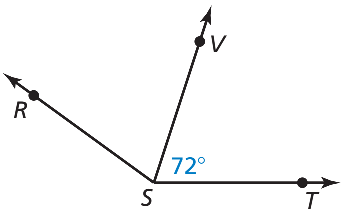
21. Find $m\angle ABC$.



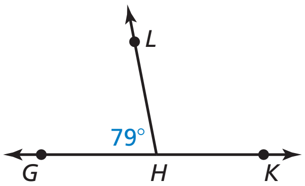
22. Find $m\angle LMN$.



23. $m\angle RST = 114°$. Find $m\angle RSV$.

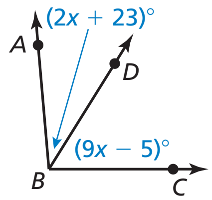


24. $\angle GHK$ is a straight angle. Find $m\angle LHK$.

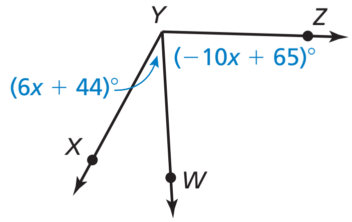


In Exercises 25–30, find the indicated angle measures. *(See Example 4.)*

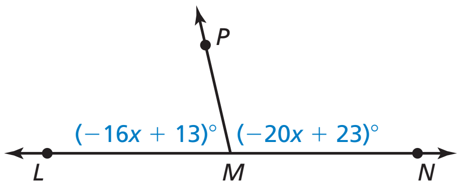
25. $m\angle ABC = 95°$. Find $m\angle ABD$ and $m\angle DBC$.



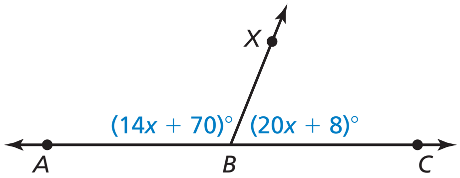
26. $m\angle XYZ = 117°$. Find $m\angle XYW$ and $m\angle WYZ$.



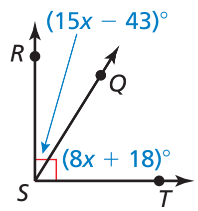
27. $\angle LMN$ is a straight angle. Find $m\angle LMP$ and $m\angle NMP$.



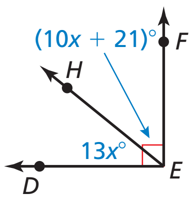
28. $\angle ABC$ is a straight angle. Find $m\angle ABX$ and $m\angle CBX$.



29. Find $m\angle RSQ$ and $m\angle TSQ$.

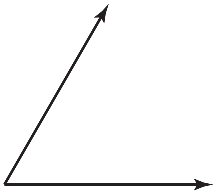


30. Find $m\angle DEH$ and $m\angle FEH$.

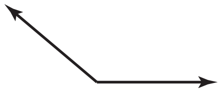


CONSTRUCTION **In Exercises 31 and 32, copy the angle. Then construct the angle bisector with a compass and straightedge.**

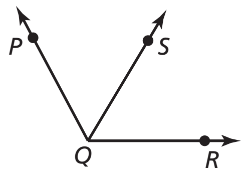
31.



32.



In Exercises 33–36, $\overrightarrow{QS}$ bisects $\angle PQR$. Use the diagram and the given angle measure to find the indicated angle measures. *(See Example 5.)*



33. $m\angle PQS = 63°$. Find $m\angle RQS$ and $m\angle PQR$.

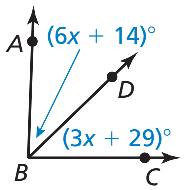
34. $m\angle RQS = 71°$. Find $m\angle PQS$ and $m\angle PQR$.

35. $m\angle PQR = 124°$. Find $m\angle PQS$ and $m\angle RQS$.

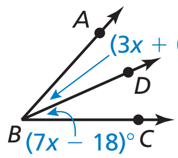
36. $m\angle PQR = 119°$. Find $m\angle PQS$ and $m\angle RQS$.

In Exercises 37–40, $\overrightarrow{BD}$ bisects $\angle ABC$. Find $m\angle ABD$, $m\angle CBD$, and $m\angle ABC$.

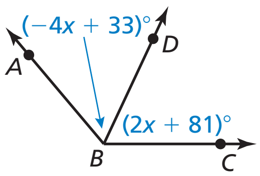
37.



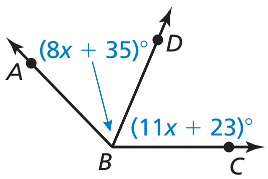
38.



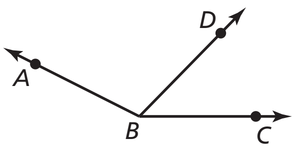
39.



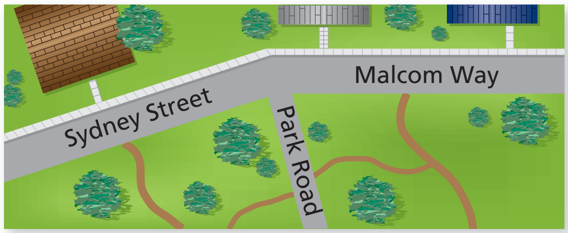
40.



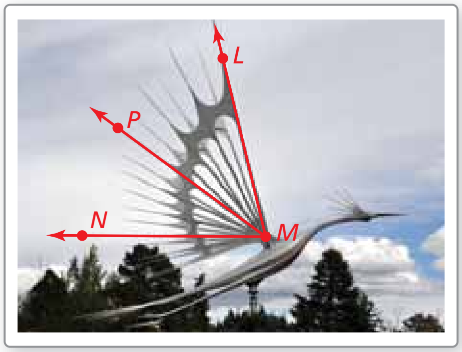
41. **WRITING** Explain how to find $m\angle ABD$ when you are given $m\angle ABC$ and $m\angle CBD$.



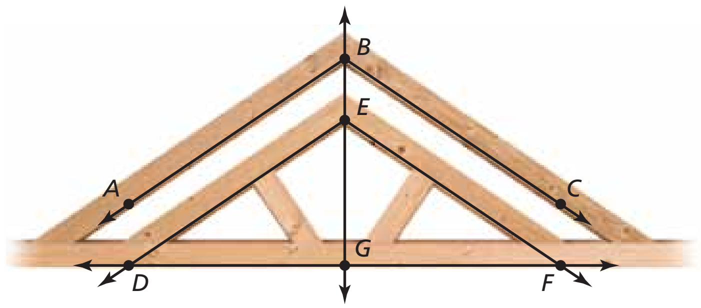
42. **ANALYZING RELATIONSHIPS** The map shows the intersections of three roads. Malcom Way intersects Sydney Street at an angle of 162°. Park Road intersects Sydney Street at an angle of 87°. Find the angle at which Malcom Way intersects Park Road.



43. **ANALYZING RELATIONSHIPS** In the sculpture shown in the photograph, the measure of $\angle LMN$ is 76° and the measure of $\angle PMN$ is 36°. What is the measure of $\angle LMP$?



USING STRUCTURE **In Exercises 44–46, use the diagram of the roof truss.**



44. In the roof truss, $\overrightarrow{BG}$ bisects $\angle ABC$ and $\angle DEF$, $m\angle ABC = 112°$, and $\angle ABC \cong \angle DEF$. Find the measure of each angle.

a. $m\angle DEF$ **b.** $m\angle ABG$

c. $m\angle CBG$ **d.** $m\angle DEG$

45. In the roof truss, $\angle DGF$ is a straight angle and $\overrightarrow{GB}$ bisects $\angle DGF$. Find $m\angle DGE$ and $m\angle FGE$.

46. Name an example of each of the four types of angles according to their measures in the diagram.

47. **MATHEMATICAL CONNECTIONS** In $\angle ABC$, $\overrightarrow{BX}$ is in the interior of the angle, $m\angle ABX$ is 12 more than 4 times $m\angle CBX$, and $m\angle ABC = 92°$.

a. Draw a diagram to represent the situation.

b. Write and solve an equation to find $m\angle ABX$ and $m\angle CBX$.

48. **THOUGHT PROVOKING** The angle between the minute hand and the hour hand of a clock is 90°. What time is it? Justify your answer.

49. **ABSTRACT REASONING** Classify the angles that result from bisecting each type of angle.

a. acute angle **b.** right angle
c. obtuse angle **d.** straight angle

50. **ABSTRACT REASONING** Classify the angles that result from drawing a ray in the interior of each type of angle. Include all possibilities and explain your reasoning.

a. acute angle **b.** right angle
c. obtuse angle **d.** straight angle

51. **CRITICAL THINKING** The ray from the origin through (4, 0) forms one side of an angle. Use the numbers below as x- and y-coordinates to create each type of angle in a coordinate plane.

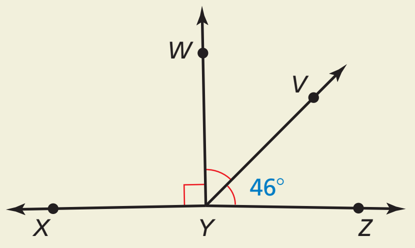
−2	−1	0	1	2

a. acute angle **b.** right angle
c. obtuse angle **d.** straight angle

52. **MAKING AN ARGUMENT** Your friend claims it is possible for a straight angle to consist of two obtuse angles. Is your friend correct? Explain your reasoning.

53. **CRITICAL THINKING** Two acute angles are added together. What type(s) of angle(s) do they form? Explain your reasoning.

54. **HOW DO YOU SEE IT?** Use the diagram.



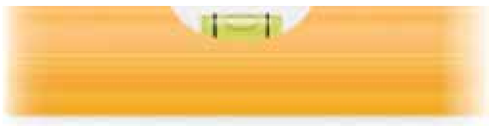
a. Is it possible for $\angle XYZ$ to be a straight angle? Explain your reasoning.

b. What can you change in the diagram so that $\angle XYZ$ is a straight angle?

55. **WRITING** Explain the process of bisecting an angle in your own words. Compare it to bisecting a segment.

56. **ANALYZING RELATIONSHIPS** $\overrightarrow{SQ}$ bisects $\angle RST$, $\overrightarrow{SP}$ bisects $\angle RSQ$, and $\overrightarrow{SV}$ bisects $\angle RSP$. The measure of $\angle VSP$ is 17°. Find $m\angle TSQ$. Explain.

57. **ABSTRACT REASONING** A bubble level is a tool used to determine whether a surface is horizontal, like the top of a picture frame. If the bubble is not exactly in the middle when the level is placed on the surface, then the surface is not horizontal. What is the most realistic type of angle formed by the level and a horizontal line when the bubble is not in the middle? Explain your reasoning.



Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Solve the equation. *(Skills Review Handbook)*

58. $x + 67 = 180$

59. $x + 58 = 90$

60. $16 + x = 90$

61. $109 + x = 180$

62. $(6x + 7) + (13x + 21) = 180$

63. $(3x + 15) + (4x - 9) = 90$

64. $(11x - 25) + (24x + 10) = 90$

65. $(14x - 18) + (5x + 8) = 180$

1.6 Describing Pairs of Angles

Essential Question

How can you describe angle pair relationships and use these descriptions to find angle measures?

EXPLORATION 1 Finding Angle Measures

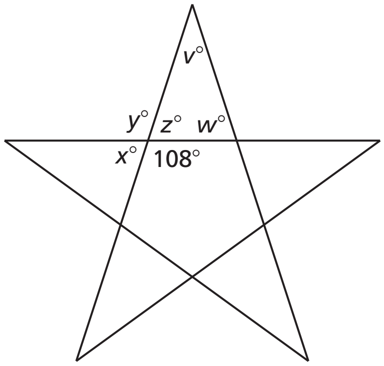
Work with a partner. The five-pointed star has a regular pentagon at its center.

a. What do you notice about the following angle pairs?

$x°$ and $y°$

$y°$ and $z°$

$x°$ and $z°$



b. Find the values of the indicated variables. Do not use a protractor to measure the angles.

$x =$

$y =$

$z =$

$w =$

$v =$

Explain how you obtained each answer.

EXPLORATION 2 Finding Angle Measures

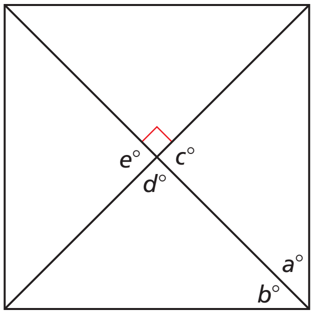
Work with a partner. A square is divided by its diagonals into four triangles.

a. What do you notice about the following angle pairs?

$a°$ and $b°$

$c°$ and $d°$

$c°$ and $e°$



b. Find the values of the indicated variables. Do not use a protractor to measure the angles.

$c =$

$d =$

$e =$

Explain how you obtained each answer.

ATTENDING TO PRECISION

To be proficient in math, you need to communicate precisely with others.

Communicate Your Answer

3. How can you describe angle pair relationships and use these descriptions to find angle measures?

4. What do you notice about the angle measures of complementary angles, supplementary angles, and vertical angles?

1.6 Lesson

Core Vocabulary

complementary angles, *p. 48*
supplementary angles, *p. 48*
adjacent angles, *p. 48*
linear pair, *p. 50*
vertical angles, *p. 50*

Previous
vertex
sides of an angle
interior of an angle
opposite rays

What You Will Learn

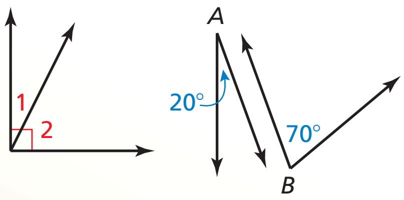
- Identify complementary and supplementary angles.
- Identify linear pairs and vertical angles.

Using Complementary and Supplementary Angles

Pairs of angles can have special relationships. The measurements of the angles or the positions of the angles in the pair determine the relationship.

Core Concept

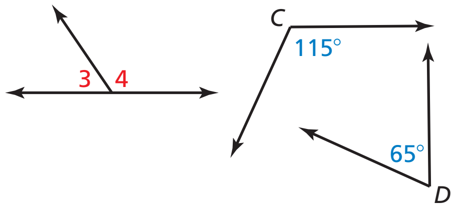
Complementary and Supplementary Angles



$\angle 1$ and $\angle 2$ $\qquad$ $\angle A$ and $\angle B$

complementary angles

Two positive angles whose measures have a sum of 90°. Each angle is the *complement* of the other.



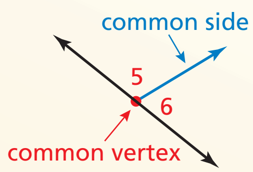
$\angle 3$ and $\angle 4$ $\qquad$ $\angle C$ and $\angle D$

supplementary angles

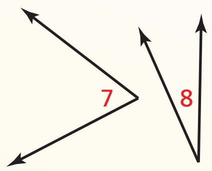
Two positive angles whose measures have a sum of 180°. Each angle is the *supplement* of the other.

Adjacent Angles

Complementary angles and supplementary angles can be *adjacent angles* or *nonadjacent angles*. **Adjacent angles** are two angles that share a common vertex and side, but have no common interior points.



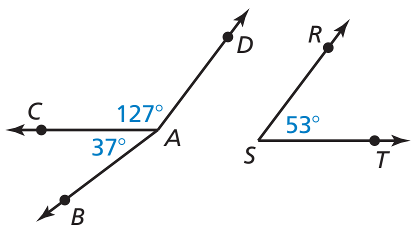
$\angle 5$ and $\angle 6$ are adjacent angles.



$\angle 7$ and $\angle 8$ are nonadjacent angles.

EXAMPLE 1 Identifying Pairs of Angles

In the figure, name a pair of complementary angles, a pair of supplementary angles, and a pair of adjacent angles.



SOLUTION

Because 37° + 53° = 90°, $\angle BAC$ and $\angle RST$ are complementary angles.

Because 127° + 53° = 180°, $\angle CAD$ and $\angle RST$ are supplementary angles.

Because $\angle BAC$ and $\angle CAD$ share a common vertex and side, they are adjacent angles.

COMMON ERROR

In Example 1, $\angle DAC$ and $\angle DAB$ share a common vertex and a common side. But they also share common interior points. So, they are *not* adjacent angles.

EXAMPLE 2 Finding Angle Measures

a. $\angle 1$ is a complement of $\angle 2$, and $m\angle 1 = 62°$. Find $m\angle 2$.

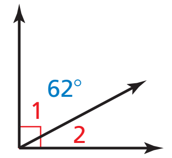
b. $\angle 3$ is a supplement of $\angle 4$, and $m\angle 4 = 47°$. Find $m\angle 3$.

COMMON ERROR

Do not confuse angle names with angle measures.

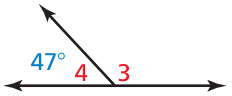
SOLUTION

a. Draw a diagram with complementary adjacent angles to illustrate the relationship.



$$m\angle 2 = 90° - m\angle 1 = 90° - 62° = 28°$$

b. Draw a diagram with supplementary adjacent angles to illustrate the relationship.

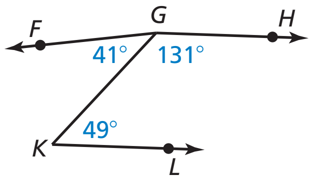


$$m\angle 3 = 180° - m\angle 4 = 180° - 47° = 133°$$

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

In Exercises 1 and 2, use the figure.



1. Name a pair of complementary angles, a pair of supplementary angles, and a pair of adjacent angles.

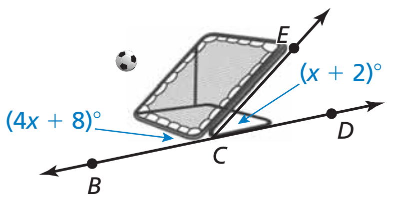
2. Are $\angle KGH$ and $\angle LKG$ adjacent angles? Are $\angle FGK$ and $\angle FGH$ adjacent angles? Explain.

3. $\angle 1$ is a complement of $\angle 2$, and $m\angle 2 = 5°$. Find $m\angle 1$.

4. $\angle 3$ is a supplement of $\angle 4$, and $m\angle 3 = 148°$. Find $m\angle 4$.

EXAMPLE 3 Real-Life Application

When viewed from the side, the frame of a ball-return net forms a pair of supplementary angles with the ground. Find $m\angle BCE$ and $m\angle ECD$.



SOLUTION

Step 1 Use the fact that the sum of the measures of supplementary angles is 180°.

$m\angle BCE + m\angle ECD = 180°$	Write an equation.
$(4x + 8)° + (x + 2)° = 180°$	Substitute angle measures.
$5x + 10 = 180$	Combine like terms.
$x = 34$	Solve for x.

Step 2 Evaluate the given expressions when $x = 34$.

$$m\angle BCE = (4x + 8)° = (4 \cdot 34 + 8)° = 144°$$

$$m\angle ECD = (x + 2)° = (34 + 2)° = 36°$$

▶ So, $m\angle BCE = 144°$ and $m\angle ECD = 36°$.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

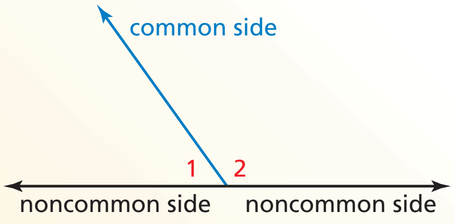
5. $\angle LMN$ and $\angle PQR$ are complementary angles. Find the measures of the angles when $m\angle LMN = (4x - 2)°$ and $m\angle PQR = (9x + 1)°$.

Using Other Angle Pairs

Core Concept

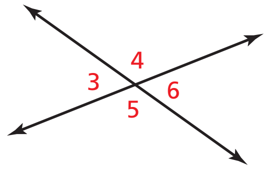
Linear Pairs and Vertical Angles

Two adjacent angles are a **linear pair** when their noncommon sides are opposite rays. The angles in a linear pair are supplementary angles.



$\angle 1$ and $\angle 2$ are a linear pair.

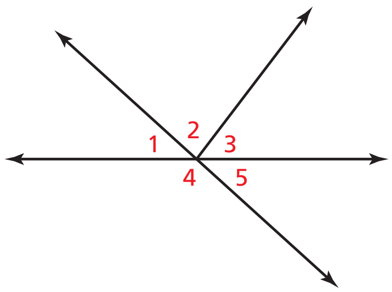
Two angles are **vertical angles** when their sides form two pairs of opposite rays.



$\angle 3$ and $\angle 6$ are vertical angles.
$\angle 4$ and $\angle 5$ are vertical angles.

EXAMPLE 4 Identifying Angle Pairs

Identify all the linear pairs and all the vertical angles in the figure.



COMMON ERROR

In Example 4, one side of $\angle 1$ and one side of $\angle 3$ are opposite rays. But the angles are not a linear pair because they are *nonadjacent*.

SOLUTION

To find vertical angles, look for angles formed by intersecting lines.

▶ $\angle 1$ and $\angle 5$ are vertical angles.

To find linear pairs, look for adjacent angles whose noncommon sides are opposite rays.

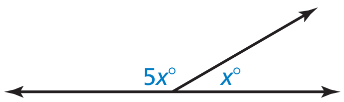
▶ $\angle 1$ and $\angle 4$ are a linear pair. $\angle 4$ and $\angle 5$ are also a linear pair.

EXAMPLE 5 Finding Angle Measures in a Linear Pair

Two angles form a linear pair. The measure of one angle is five times the measure of the other angle. Find the measure of each angle.

SOLUTION

Step 1 Draw a diagram. Let $x°$ be the measure of one angle. The measure of the other angle is $5x°$.



Step 2 Use the fact that the angles of a linear pair are supplementary to write an equation.

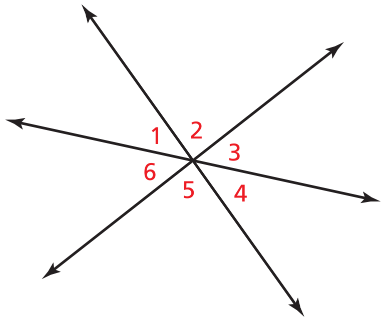
$x° + 5x° = 180°$	Write an equation.
$6x = 180$	Combine like terms.
$x = 30$	Divide each side by 6.

▶ The measures of the angles are $30°$ and $5(30°) = 150°$.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

6. Do any of the numbered angles in the figure form a linear pair? Which angles are vertical angles? Explain your reasoning.



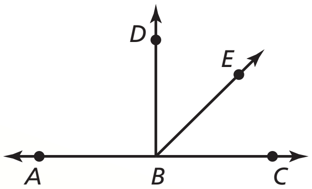
7. The measure of an angle is twice the measure of its complement. Find the measure of each angle.

8. Two angles form a linear pair. The measure of one angle is $1\frac{1}{2}$ times the measure of the other angle. Find the measure of each angle.

Concept Summary

Interpreting a Diagram

There are some things you can conclude from a diagram, and some you cannot. For example, here are some things that you ***can* conclude** from the diagram below.



YOU CAN CONCLUDE

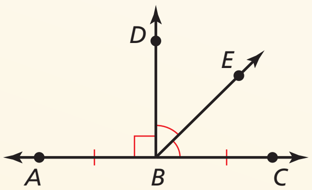
- All points shown are coplanar.
- Points A, B, and C are collinear, and B is between A and C.
- $\overleftrightarrow{AC}$, $\overrightarrow{BD}$, and $\overrightarrow{BE}$ intersect at point B.
- $\angle DBE$ and $\angle EBC$ are adjacent angles, and $\angle ABC$ is a straight angle.
- Point E lies in the interior of $\angle DBC$.

Here are some things you ***cannot* conclude** from the diagram above.

YOU CANNOT CONCLUDE

- $\overline{AB} \cong \overline{BC}$.
- $\angle DBE \cong \angle EBC$.
- $\angle ABD$ is a right angle.

To make such conclusions, the following information must be given.



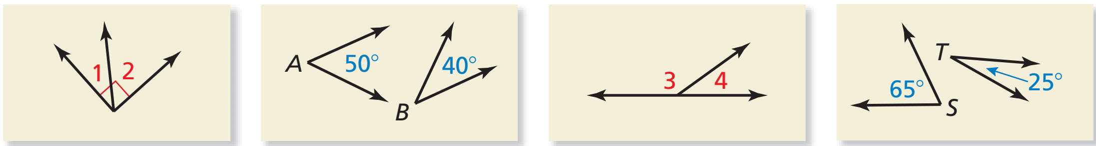
1.6 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

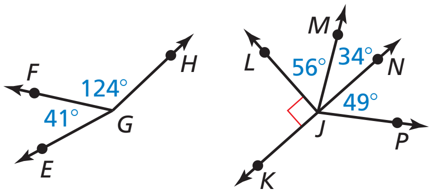
1. **WRITING** Explain what is different between adjacent angles and vertical angles.

2. **WHICH ONE DOESN'T BELONG?** Which one does *not* belong with the other three? Explain your reasoning.



Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, use the figure. *(See Example 1.)*



3. Name a pair of adjacent complementary angles.

4. Name a pair of adjacent supplementary angles.

5. Name a pair of nonadjacent complementary angles.

6. Name a pair of nonadjacent supplementary angles.

In Exercises 7–10, find the angle measure. *(See Example 2.)*

7. $\angle 1$ is a complement of $\angle 2$, and $m\angle 1 = 23°$. Find $m\angle 2$.

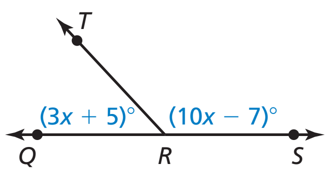
8. $\angle 3$ is a complement of $\angle 4$, and $m\angle 3 = 46°$. Find $m\angle 4$.

9. $\angle 5$ is a supplement of $\angle 6$, and $m\angle 5 = 78°$. Find $m\angle 6$.

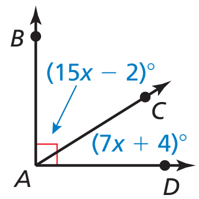
10. $\angle 7$ is a supplement of $\angle 8$, and $m\angle 7 = 109°$. Find $m\angle 8$.

In Exercises 11–14, find the measure of each angle. *(See Example 3.)*

11.



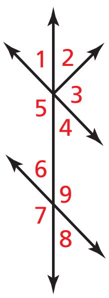
12.



13. $\angle UVW$ and $\angle XYZ$ are complementary angles, $m\angle UVW = (x - 10)°$, and $m\angle XYZ = (4x - 10)°$.

14. $\angle EFG$ and $\angle LMN$ are supplementary angles, $m\angle EFG = (3x + 17)°$, and $m\angle LMN = \left(\frac{1}{2}x - 5\right)°$.

In Exercises 15–18, use the figure. *(See Example 4.)*



15. Identify the linear pair(s) that include $\angle 1$.

16. Identify the linear pair(s) that include $\angle 7$.

17. Are $\angle 6$ and $\angle 8$ vertical angles? Explain your reasoning.

18. Are $\angle 2$ and $\angle 5$ vertical angles? Explain your reasoning.

In Exercises 19–22, find the measure of each angle. *(See Example 5.)*

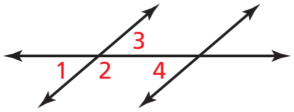
19. Two angles form a linear pair. The measure of one angle is twice the measure of the other angle.

20. Two angles form a linear pair. The measure of one angle is $\frac{1}{3}$ the measure of the other angle.

21. The measure of an angle is nine times the measure of its complement.

22. The measure of an angle is $\frac{1}{4}$ the measure of its complement.

ERROR ANALYSIS In Exercises 23 and 24, describe and correct the error in identifying pairs of angles in the figure.



23. ✗ $\angle 2$ and $\angle 4$ are adjacent.

24. ✗ $\angle 1$ and $\angle 3$ form a linear pair.

In Exercises 25 and 26, the picture shows the Alamillo Bridge in Seville, Spain. In the picture, $m\angle 1 = 58°$ and $m\angle 2 = 24°$.

25. Find the measure of the supplement of $\angle 1$.

26. Find the measure of the supplement of $\angle 2$.

27. **PROBLEM SOLVING** The arm of a crossing gate moves 42° from a vertical position. How many more degrees does the arm have to move so that it is horizontal?

A. 42° **B.** 138°

C. 48° **D.** 90°

28. **REASONING** The foul lines of a baseball field intersect at home plate to form a right angle. A batter hits a fair ball such that the path of the baseball forms an angle of 27° with the third base foul line. What is the measure of the angle between the first base foul line and the path of the baseball?

29. **CONSTRUCTION** Construct a linear pair where one angle measure is 115°.

30. **CONSTRUCTION** Construct a pair of adjacent angles that have angle measures of 45° and 97°.

31. **PROBLEM SOLVING** $m\angle U = 2x°$, and $m\angle V = 4m\angle U$. Which value of x makes $\angle U$ and $\angle V$ complements of each other?

A. 25 **B.** 9 **C.** 36 **D.** 18

MATHEMATICAL CONNECTIONS In Exercises 32–35, write and solve an algebraic equation to find the measure of each angle based on the given description.

32. The measure of an angle is 6° less than the measure of its complement.

33. The measure of an angle is 12° more than twice the measure of its complement.

34. The measure of one angle is 3° more than $\frac{1}{2}$ the measure of its supplement.

35. Two angles form a linear pair. The measure of one angle is 15° less than $\frac{2}{3}$ the measure of the other angle.

CRITICAL THINKING In Exercises 36–41, tell whether the statement is *always*, *sometimes*, or *never* true. Explain your reasoning.

36. Complementary angles are adjacent.

37. Angles in a linear pair are supplements of each other.

38. Vertical angles are adjacent.

39. Vertical angles are supplements of each other.

40. If an angle is acute, then its complement is greater than its supplement.

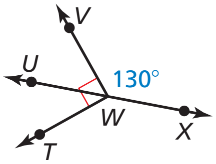
41. If two complementary angles are congruent, then the measure of each angle is 45°.

42. **WRITING** Explain why the supplement of an acute angle must be obtuse.

43. **WRITING** Explain why an obtuse angle does not have a complement.

44. THOUGHT PROVOKING Sketch an intersection of roads. Identify any supplementary, complementary, or vertical angles.

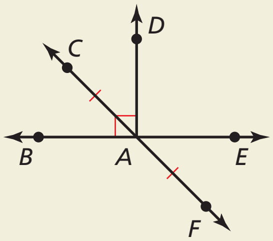
45. ATTENDING TO PRECISION Use the figure.



a. Find $m\angle UWV$, $m\angle TWU$, and $m\angle TWX$.

b. You write the measures of $\angle TWU$, $\angle TWX$, $\angle UWV$, and $\angle VWX$ on separate pieces of paper and place the pieces of paper in a box. Then you pick two pieces of paper out of the box at random. What is the probability that the angle measures you choose are supplementary? Explain your reasoning.

46. HOW DO YOU SEE IT? Tell whether you can conclude that each statement is true based on the figure. Explain your reasoning.



a. $\overline{CA} \cong \overline{AF}$.

b. Points C, A, and F are collinear.

c. $\angle CAD \cong \angle EAF$.

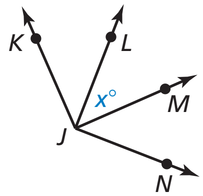
d. $\overline{BA} \cong \overline{AE}$.

e. $\overleftrightarrow{CF}$, $\overleftrightarrow{BE}$, and $\overleftrightarrow{AD}$ intersect at point A.

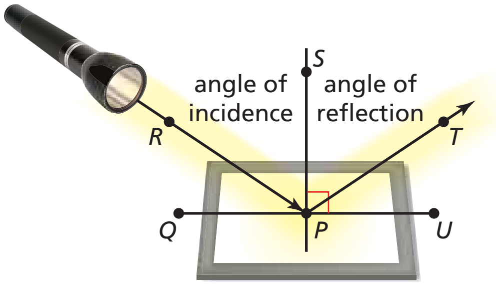
f. $\angle BAC$ and $\angle CAD$ are complementary angles.

g. $\angle DAE$ is a right angle.

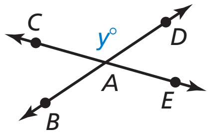
47. REASONING $\angle KJL$ and $\angle LJM$ are complements, and $\angle MJN$ and $\angle LJM$ are complements. Can you show that $\angle KJL \cong \angle MJN$? Explain your reasoning.



48. MAKING AN ARGUMENT Light from a flashlight strikes a mirror and is reflected so that the angle of reflection is congruent to the angle of incidence. Your classmate claims that $\angle QPR$ is congruent to $\angle TPU$ regardless of the measure of $\angle RPS$. Is your classmate correct? Explain your reasoning.



49. DRAWING CONCLUSIONS Use the figure.



a. Write expressions for the measures of $\angle BAE$, $\angle DAE$, and $\angle CAB$.

b. What do you notice about the measures of vertical angles? Explain your reasoning.

50. MATHEMATICAL CONNECTIONS Let $m\angle 1 = x°$, $m\angle 2 = y_1°$, and $m\angle 3 = y_2°$. $\angle 2$ is the complement of $\angle 1$, and $\angle 3$ is the supplement of $\angle 1$.

a. Write equations for y_1 as a function of x and for y_2 as a function of x. What is the domain of each function? Explain.

b. Graph each function and describe its range.

51. MATHEMATICAL CONNECTIONS The sum of the measures of two complementary angles is 74° greater than the difference of their measures. Find the measure of each angle. Explain how you found the angle measures.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Determine whether the statement is *always*, *sometimes*, or *never* true. Explain your reasoning. *(Skills Review Handbook)*

52. An integer is a whole number.

53. An integer is an irrational number.

54. An irrational number is a real number.

55. A whole number is negative.

56. A rational number is an integer.

57. A natural number is an integer.

58. A whole number is a rational number.

59. An irrational number is negative.

1.4–1.6 What Did You Learn?

Core Vocabulary

angle, *p. 38*
vertex, *p. 38*
sides of an angle, *p. 38*
interior of an angle, *p. 38*
exterior of an angle, *p. 38*
measure of an angle, *p. 39*
acute angle, *p. 39*
right angle, *p. 39*
obtuse angle, *p. 39*
straight angle, *p. 39*
congruent angles, *p. 40*
angle bisector, *p. 42*
complementary angles, *p. 48*
supplementary angles, *p. 48*
adjacent angles, *p. 48*
linear pair, *p. 50*
vertical angles, *p. 50*

Core Concepts

Section 1.4

Classifying Polygons, *p. 30*
Finding Perimeter and Area in the Coordinate Plane, *p. 31*

Section 1.5

Postulate 1.3 Protractor Postulate, *p. 39*
Types of Angles, *p. 39*
Postulate 1.4 Angle Addition Postulate, *p. 41*
Bisecting Angles, *p. 42*

Section 1.6

Complementary and Supplementary Angles, *p. 48*
Adjacent Angles, *p. 48*
Linear Pairs and Vertical Angles, *p. 50*
Interpreting a Diagram, *p. 51*

Mathematical Practices

1. How could you explain your answers to Exercise 33 on page 36 to a friend who is unable to hear?
2. What tool(s) could you use to verify your answers to Exercises 25–30 on page 44?
3. Your friend says that the angles in Exercise 28 on page 53 are supplementary angles. Explain why you agree or disagree.

Performance Task

Comfortable Horse Stalls

The plan for a new barn includes standard, rectangular horse stalls. The architect is sure that this will provide the most comfort for your horse because it is the greatest area for the stall. Is that correct? How can you investigate to find out?

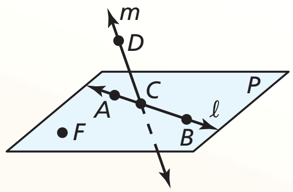
To explore the answers to this question and more, go to ***BigIdeasMath.com***.

1 Chapter Review

Dynamic Solutions available at *BigIdeasMath.com*

1.1 Points, Lines, and Planes (pp. 3–10)

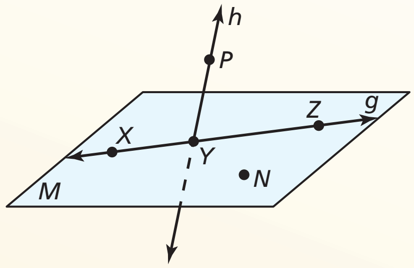
Use the diagram at the right. Give another name for plane *P*. Then name a line in the plane, a ray, a line intersecting the plane, and three collinear points.



You can find another name for plane *P* by using any three points in the plane that are not on the same line. So, another name for plane *P* is plane *FAB*.

A line in the plane is $\overleftrightarrow{AB}$, a ray is $\overrightarrow{CB}$, a line intersecting the plane is $\overleftrightarrow{CD}$, and three collinear points are *A*, *C*, and *B*.

Use the diagram.



1. Give another name for plane *M*.

2. Name a line in the plane.

3. Name a line intersecting the plane.

4. Name two rays.

5. Name a pair of opposite rays.

6. Name a point not in plane *M*.

1.2 Measuring and Constructing Segments (pp. 11–18)

a. Find *AC*.

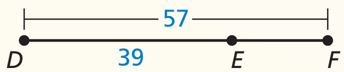
$AC = AB + BC$ Segment Addition Postulate (Postulate 1.2)

$= 12 + 25$ Substitute 12 for *AB* and 25 for *BC*.

$= 37$ Add.

▶ So, $AC = 37$.

b. Find *EF*.



$DF = DE + EF$ Segment Addition Postulate (Postulate 1.2)

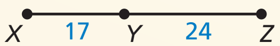
$57 = 39 + EF$ Substitute 57 for *DF* and 39 for *DE*.

$18 = EF$ Subtract 39 from each side.

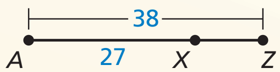
▶ So, $EF = 18$.

Find *XZ*.

7.



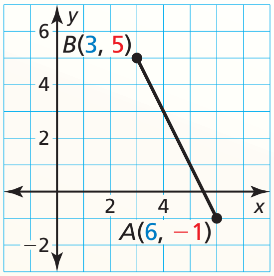
8.



9. Plot $A(8, -4)$, $B(3, -4)$, $C(7, 1)$, and $D(7, -3)$ in a coordinate plane. Then determine whether $\overline{AB}$ and $\overline{CD}$ are congruent.

1.3 Using Midpoint and Distance Formulas (pp. 19–26)

The endpoints of $\overline{AB}$ are $A(6, -1)$ and $B(3, 5)$. Find the coordinates of the midpoint M. Then find the distance between points A and B.



Use the Midpoint Formula.

$$M\left(\frac{6+3}{2}, \frac{-1+5}{2}\right) = M\left(\frac{9}{2}, 2\right)$$

Use the Distance Formula.

$$\begin{aligned} AB &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} && \text{Distance Formula} \\ &= \sqrt{(3-6)^2 + [5-(-1)]^2} && \text{Substitute.} \\ &= \sqrt{(-3)^2 + 6^2} && \text{Subtract.} \\ &= \sqrt{9 + 36} && \text{Evaluate powers.} \\ &= \sqrt{45} && \text{Add.} \\ &\approx 6.7 \text{ units} && \text{Use a calculator.} \end{aligned}$$

▶ So, the midpoint is $M\left(\frac{9}{2}, 2\right)$, and the distance is about 6.7 units.

Find the coordinates of the midpoint M. Then find the distance between points S and T.

10. $S(-2, 4)$ and $T(3, 9)$

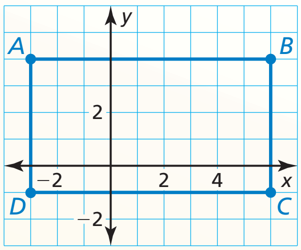
11. $S(6, -3)$ and $T(7, -2)$

12. The midpoint of $\overline{JK}$ is $M(6, 3)$. One endpoint is $J(14, 9)$. Find the coordinates of endpoint K.

13. Point M is the midpoint of $\overline{AB}$ where $AM = 3x + 8$ and $MB = 6x - 4$. Find AB.

1.4 Perimeter and Area in the Coordinate Plane (pp. 29–36)

Find the perimeter and area of rectangle $ABCD$ with vertices $A(-3, 4)$, $B(6, 4)$, $C(6, -1)$, and $D(-3, -1)$.



Draw the rectangle in a coordinate plane. Then find the length and width using the Ruler Postulate (Postulate 1.1).

Length $AB = |-3 - 6| = 9$

Width $BC = |4 - (-1)| = 5$

Substitute the values for the length and width into the formulas for the perimeter and area of a rectangle.

$$\begin{aligned} P &= 2\ell + 2w \\ &= 2(9) + 2(5) \\ &= 18 + 10 \\ &= 28 \end{aligned} \qquad \begin{aligned} A &= \ell w \\ &= (9)(5) \\ &= 45 \end{aligned}$$

▶ So, the perimeter is 28 units, and the area is 45 square units.

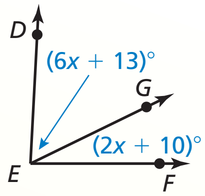
Find the perimeter and area of the polygon with the given vertices.

14. $W(5, -1)$, $X(5, 6)$, $Y(2, -1)$, $Z(2, 6)$

15. $E(6, -2)$, $F(6, 5)$, $G(-1, 5)$

1.5 Measuring and Constructing Angles (pp. 37–46)

Given that $m\angle DEF = 87°$, find $m\angle DEG$ and $m\angle GEF$.



Step 1 Write and solve an equation to find the value of x.

$m\angle DEF = m\angle DEG + m\angle GEF$	Angle Addition Postulate (Post. 1.4)
$87° = (6x + 13)° + (2x + 10)°$	Substitute angle measures.
$87 = 8x + 23$	Combine like terms.
$64 = 8x$	Subtract 23 from each side.
$8 = x$	Divide each side by 8.

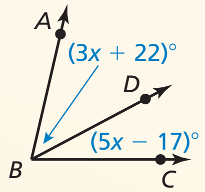
Step 2 Evaluate the given expressions when $x = 8$.

$$m\angle DEG = (6x + 13)° = (6 \cdot 8 + 13)° = 61°$$
$$m\angle GEF = (2x + 10)° = (2 \cdot 8 + 10)° = 26°$$

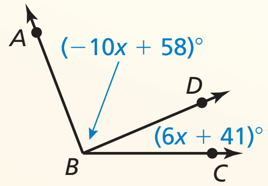
So, $m\angle DEG = 61°$, and $m\angle GEF = 26°$.

Find $m\angle ABD$ and $m\angle CBD$.

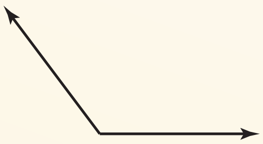
16. $m\angle ABC = 77°$



17. $m\angle ABC = 111°$



18. Find the measure of the angle using a protractor.

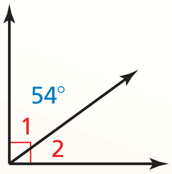


1.6 Describing Pairs of Angles (pp. 47–54)

a. $\angle 1$ is a complement of $\angle 2$, and $m\angle 1 = 54°$. Find $m\angle 2$.

Draw a diagram with complementary adjacent angles to illustrate the relationship.

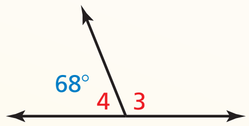
$$m\angle 2 = 90° - m\angle 1 = 90° - 54° = 36°$$



b. $\angle 3$ is a supplement of $\angle 4$, and $m\angle 4 = 68°$. Find $m\angle 3$.

Draw a diagram with supplementary adjacent angles to illustrate the relationship.

$$m\angle 3 = 180° - m\angle 4 = 180° - 68° = 112°$$



$\angle 1$ and $\angle 2$ are complementary angles. Given $m\angle 1$, find $m\angle 2$.

19. $m\angle 1 = 12°$

20. $m\angle 1 = 83°$

$\angle 3$ and $\angle 4$ are supplementary angles. Given $m\angle 3$, find $m\angle 4$.

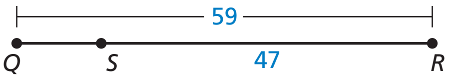
21. $m\angle 3 = 116°$

22. $m\angle 3 = 56°$

1 Chapter Test

Find the length of $\overline{QS}$. Explain how you found your answer.

1. 

2. 

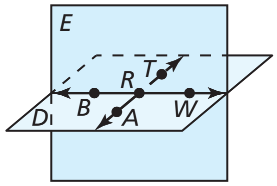
Find the coordinates of the midpoint M. Then find the distance between the two points.

3. $A(-4, -8)$ and $B(-1, 4)$

4. $C(-1, 7)$ and $D(-8, -3)$

5. The midpoint of $\overline{EF}$ is $M(1, -1)$. One endpoint is $E(-3, 2)$. Find the coordinates of endpoint F.

Use the diagram to decide whether the statement is true or false.



6. Points A, R, and B are collinear.

7. $\overleftrightarrow{BW}$ and $\overleftrightarrow{AT}$ are lines.

8. $\overrightarrow{BR}$ and $\overrightarrow{RT}$ are opposite rays.

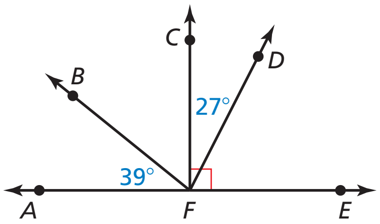
9. Plane D could also be named plane ART.

Find the perimeter and area of the polygon with the given vertices. Explain how you found your answer.

10. $P(-3, 4)$, $Q(1, 4)$, $R(-3, -2)$, $S(3, -2)$

11. $J(-1, 3)$, $K(5, 3)$, $L(2, -2)$

12. In the diagram, $\angle AFE$ is a straight angle and $\angle CFE$ is a right angle. Identify all supplementary and complementary angles. Explain. Then find $m\angle DFE$, $m\angle BFC$, and $m\angle BFE$.

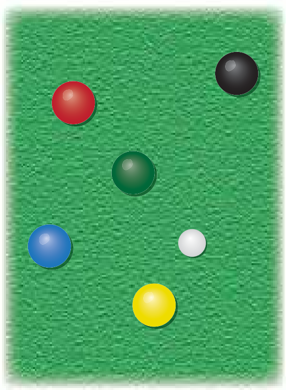


13. Use the clock at the left.

a. What is the measure of the acute angle created when the clock is at 10:00?

b. What is the measure of the obtuse angle created when the clock is at 5:00?

c. Find a time where the hour and minute hands create a straight angle.



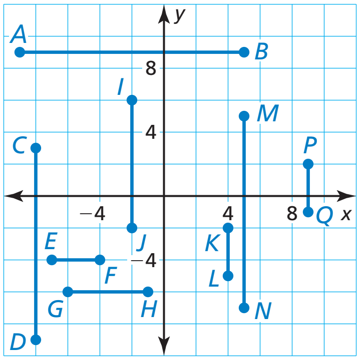
14. Sketch a figure that contains a plane and two lines that intersect the plane at one point.

15. Your parents decide they would like to install a rectangular swimming pool in the backyard. There is a 15-foot by 20-foot rectangular area available. Your parents request a 3-foot edge around each side of the pool. Draw a diagram of this situation in a coordinate plane. What is the perimeter and area of the largest swimming pool that will fit?

16. The picture shows the arrangement of balls in a game of boccie. The object of the game is to throw your ball closest to the small, white ball, which is called the *pallino*. The green ball is the midpoint between the red ball and the pallino. The distance between the green ball and the red ball is 10 inches. The distance between the yellow ball and the pallino is 8 inches. Which ball is closer to the pallino, the green ball or the yellow ball? Explain.

1 Cumulative Assessment

1. Use the diagram to determine which segments, if any, are congruent. List all congruent segments.



2. Order the terms so that each consecutive term builds off the previous term.

plane	segment	line	point	ray

3. The endpoints of a line segment are $(-6, 13)$ and $(11, 5)$. Which choice shows the correct midpoint and distance between these two points?

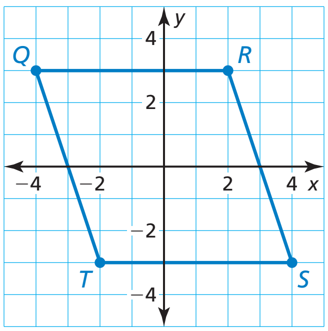
Ⓐ $\left(\frac{5}{2}, 4\right)$; 18.8 units

Ⓑ $\left(\frac{5}{2}, 9\right)$; 18.8 units

Ⓒ $\left(\frac{5}{2}, 4\right)$; 9.4 units

Ⓓ $\left(\frac{5}{2}, 9\right)$; 9.4 units

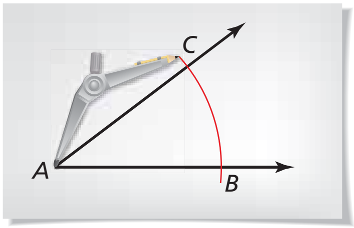
4. Find the perimeter and area of the figure shown.



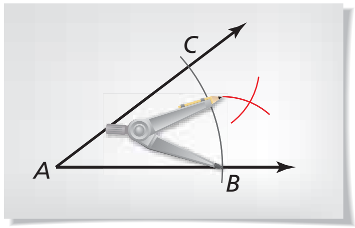
5. Plot the points $W(-1, 1)$, $X(5, 1)$, $Y(5, -2)$, and $Z(-1, -2)$ in a coordinate plane. What type of polygon do the points form? Your friend claims that you could use this figure to represent a basketball court with an area of 4050 square feet and a perimeter of 270 feet. Do you support your friend's claim? Explain.

6. Use the steps in the construction to explain how you know that $\overrightarrow{AG}$ is the angle bisector of $\angle CAB$.

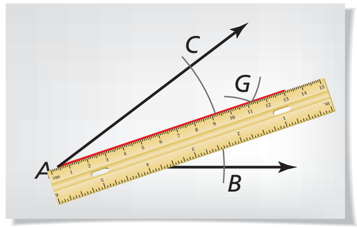
Step 1



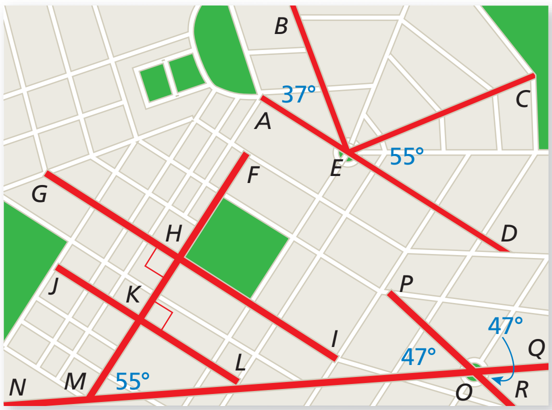
Step 2



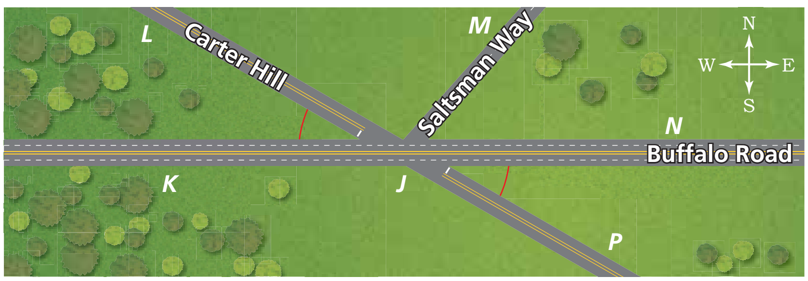
Step 3



7. The picture shows an aerial view of a city. Use the streets highlighted in red to identify all congruent angles. Assume all streets are straight angles.



8. Three roads come to an intersection point that the people in your town call Five Corners, as shown in the figure.



a. Identify all vertical angles.

b. Identify all linear pairs.

c. You are traveling east on Buffalo Road and decide to turn left onto Carter Hill. Name the angle of the turn you made.

$\angle KJL$	$\angle KJM$	$\angle KJN$	$\angle KJP$	$\angle LJM$
$\angle LJN$	$\angle LJP$	$\angle MJN$	$\angle MJP$	$\angle NJP$