## A

$30^{\circ}-60^{\circ}-90^{\circ}$ right triangles finding sine and cosine of $30^{\circ}, 496$ finding tangent with, 489 side lengths of, 471, 473
$30^{\circ}-\mathbf{6 0} 0^{\circ}-90^{\circ}$ Triangle Theorem (Thm. 9.5), 473
$\mathbf{4 5}^{\circ}-\mathbf{4 5}^{\circ}-\mathbf{9 0}$ (isosceles) right triangles
finding sine and cosine of $30^{\circ}, 496$
side lengths, 471, 472
in standard position, 462
$45^{\circ}-45^{\circ}-90^{\circ}$ Triangle Theorem
(Thm. 9.4), 472
AA, See Angle-Angle (AA) Similarity Theorem (Thm. 8.3)
AAS, See Angle-Angle-Side (AAS)
Absolute value, finding, 1
Acute angle, 39
Acute triangle
in circumscribed circle, 311
classifying by angles, 232
classifying by Pythagorean inequalities, 467
orthocenter of, 322
Addition Property of Equality, 92
Adjacent angles, 48-49
Adjacent arcs, 539
Ailles rectangle, 476
Algebraic Properties of Equality, 92
Algebraic reasoning, 91-95, 117
distributive property, 93
other properties of equality, 94
properties of equality, 92
Alternate exterior angles, 128
Alternate Exterior Angles Converse (Thm. 3.7), 139
Alternate Exterior Angles Theorem
(Thm. 3.3), 132
exploring converses, 137
Alternate interior angles, 128
Alternate Interior Angles Converse (Thm. 3.6), 139
proving theorems about parallel lines, 140
Alternate Interior Angles Theorem
(Thm. 3.2), 132
exploring converses, 137
proof of, 134
Altitude of cone, 642
Altitude of triangle
defined, 321
examples of segments and points in triangles, 300, 323
using, 319, 321-323, 351
"and" (intersection), 694-695
Angle(s)
and arc measures in circles, 561-563
circumscribed, 564
classifying, and types of, 39
congruent, 40
construction, copying an angle, 40
corresponding (See Corresponding angles)
defined, 38
diagram interpretation, 51
finding angle measures, 47, 49, 50 (See also Angle measures)
inscribed, 553-555, 584
measuring and constructing, 37-42, 58
naming, 38
pairs of, describing, 47-51, 58
adjacent angles, 48-49
complementary angles, 48-49
linear pair, 50
supplementary angles, 48-49
vertical angles, 50
pairs of, formed by transversals, 128
alternate exterior angles, 128
alternate interior angles, 128
consecutive interior angles, 128
corresponding angles, 128
proof of Symmetric Property of Angle Congruence, 102, 110
Properties of Angle Congruence (Thm. 2.2), 101
of triangles, 231-235, 290
angle measures of triangles, 233-235
classifying triangles by sides and angles, 232-233
relating to sides, 335, 337-338
Angle Addition Postulate (Post. 1.4), 41
Angle-Angle-Side (AAS)
congruence, 271, 273
identifying congruent triangles, 271
using Law of Sines to solve triangle, 510
Angle-Angle-Side (AAS) Congruence Theorem (Thm. 5.11), 271
Angle-Angle (AA) Similarity Theorem (Thm. 8.3), 428
proof of, 428
triangle similarity theorems compared, 439
using, 429-430
Angle bisector(s)
construction, bisecting an angle, 42 defined, 42
examples of segments and points in triangles, 300, 323
finding angle measures, 42
points on, 301
proportionality in triangle, 449
using, 304-305
Angle Bisector Theorem (Thm. 6.3), 304
converse of, 304
Angle measures
in kite, 401
of polygons
exterior, 362-363
interior, 360-362
in regular polygons, 611
in rhombus, 390
of triangles, 233-235
types of angles, 39
using properties of equality with, 94
Angle of depression, 497
Angle of elevation, 490
Angle of rotation, 190
Angle-Side-Angle (ASA)
congruence, 270, 272, 273
copying a triangle using ASA, 272
using Law of Sines to solve triangle, 510
Angle-Side-Angle (ASA) Congruence Theorem (Thm. 5.10), 270
Angles Inside the Circle Theorem (Thm. 10.15), 563
Angles Outside the Circle Theorem (Thm. 10.16), 563
Another Way
corresponding angles, 132
probability, sample space and outcomes, 668
segments of secants and tangents, 572
sketching a diagram, 86
solving right triangle, 503
Table of Trigonometric Ratios, 502
triangles and Laws of Cosines or Sines, 511
Apothem of regular polygon, 609, 611
Arc Addition Postulate (Post. 10.1), 539
Arc length, 595-596

Arc measures, 537-541, 583
finding, 538-539
finding from angle relationships in circles, 562-563
finding with congruent chords, 546
identifying congruent arcs, 540
of intercepted arc, 555
of minor and major arcs, 538
proving circles are similar, 541
Area
of circle, 601-602
in coordinate plane, 29-33, 57 finding, 31
finding after dilation, 416
of kite, 610
of regular polygons, 609-610, 612-613, 657
of rhombus, 610
of sectors, 601, 604-605
of similar polygons, 421
of triangle, square, and rectangle, 31
of triangle, using trigonometric ratios, 508
using to find probability, 670
Areas of Similar Polygons (Thm. 8.2), 421
Arithmetic mean, compared to geometric mean, 477, 484
Arithmetic sequence, $n$th term of, 63
ASA, See Angle-Side-Angle (ASA)
Auxiliary line, 234
Axiom(s), 12
Axis of revolution, 620


Base
of cone, 642
of isosceles triangle, 252
of solid, 618
Base angles
of isosceles triangle, 252
of trapezoid, 398
Base Angles Theorem (Thm. 5.6), 252
converse of, 252
using, 253
Bases (of trapezoid), 398
Basics of geometry, 1
angles, describing pairs of, 47-51, 58
angles, measuring and constructing, 37-42, 58
midpoint and distance formulas, 19-23, 57
perimeter and area in coordinate plane, 29-33, 57
points, lines, and planes, 3-7, 56
segments, measuring and constructing, $11-15,56$

Bayes' Theorem, 690
Between, 14
Biconditional statement(s)
defined, and writing, 69
and definitions, 230
triangles, equilateral and equiangular, 253
Binomial distribution(s), 707-710, 716
constructing, 710
defined, 709
interpreting, 710
Binomial experiments, 709
Binomials, multiplying, 527
Birthday problem, 706
Bisecting angles, 42, See also Angle bisector(s)
Bisecting segments, See Segment bisector(s)
Bisector, perpendicular, See Perpendicular bisector(s)
Bisectors of triangles, 309-314, 350
angle bisectors of triangle, 309
circumcenter of triangle, 310-312
circumscribing circle about triangle, 311-312
incenter of triangle, 313-314
inscribing circle within triangle, 314
perpendicular bisectors of triangle, 309

## C

Cavalieri, Bonaventura, 626
Cavalieri's Principle, 626
Center of arc, 40
Center of circle, 528, 530
Center of dilation, 208
Center of regular polygon, 609, 611
Center of rotation, 190
Center of sphere, 648
Center of symmetry, 193
Central angle of circle
defined, 537, 538
and inscribed angles, 553
Central angle of regular polygon, 611
Centroid (of triangle)
defined, 320
examples of segments and points in triangles, 323
finding, 321
Centroid Theorem (Thm. 6.7), 320
Ceva's Theorem, 452
Chord of a sphere, 648
Chord(s) of circles, 545-548, 583
defined, 530
intersection with tangent on circle, 562
perpendicular to diameter, 545
using congruent chords
to find arc measure, 546
to find circle's radius, 548
using diameter, 547
using perpendicular bisectors, 547
Circle(s), 526, See also Diameter; Radius, of circle
angle relationships in circles, 561-565, 584-585
finding angle and arc measures, 562-563
using circumscribed angles, 564
arc measures, 537-541, 583
identifying congruent arcs, 540
proving circles are similar, 541
area of, 601-602, 656
chords, 545-548, 583
circumference of, 594
circumscribed about triangle, 311-312
in coordinate plane, 575-578, 586
equations of circles, 576-577
writing coordinate proofs involving circles, 578
defined, 528, 530
drawing by using string, 529
inscribed angles, 553-555, 584
inscribed polygons, 553, 556-557, 584
lines and segments that intersect circles, 529-533, 582
and tangents, 529-533
radius, finding, 532
relationships with tangent circles, 528
segment relationships in circles, 569-572, 585
Circular arc, 537
Circular cone, 642, See also Cones
Circumcenter of triangle
circumscribing circle about triangle, 312
defined, 310
examples of segments and points in triangles, 323
finding, 312
types of triangles with circumscribed circles, 311

## Circumcenter Theorem

(Thm. 6.5), 310
Circumference, area, and volume, 590
arc length, 593, 595-596, 656
areas (See also Area)
of circles and sectors, 601-605, 656
of polygons, 609-613
circumference, 593-597, 656
surface areas
of cones, 641-642, 659-660
of spheres, 647-649, 660
three-dimensional figures, 617-620, 657-658
volumes
of cones, 641, 643-644, 659-660
of prisms and cylinders, 625-630, 658
of pyramids, 635-638, 659
of spheres, 647, 650-651, 660
Circumference, of circle, 594
Circumscribed angle, 564
Circumscribed Angle Theorem (Thm. 10.17), 564
Circumscribed circle, 556-557
Classifying
angles, 39
lines, pairs of, 124, 125
polygons, 30, 31, 361
quadrilaterals, $358,389,402$
solids, 617, 618
triangles by sides and angles, 232-233
Clockwise rotation, 190
Coin flip, 668, 699, 707
Coincident lines, example of, 124, 125
Collinear points, 4
Combination(s), 669, 702-703, 716
counting, 702
defined, 702
finding probability using, 703
formula, 702-703
Common Errors
adjacent angles, 48
angle approximation, 565
angle names and angle measures, 49
angle symbol compared to less than symbol, 337
angles and vertex, 254
area of semicircle, 605
calculator, inverse sine feature, 511
conditional statement and contrapositive, 67
diameter of sphere, 649
geometric mean of right triangle, 481
indirect proofs, 337
linear pair of angles, 50
naming an angle, 38
pay attention to units, 596
probability
and binomial distribution, 710
overlapping events, 695
protractor scales, 39
rays, 5
transformation order, 192
triangle congruence, 271
triangles, proportional, 479
triangles, redrawing, 255
write ratio of volumes, 630
Common external tangent, 531
Common internal tangent, 531
Common tangent, 531
Compass, 13
Complement of event, 669-670
Complementary angles
defined, 48-49
proving cases, 107
sine and cosine of, 494
Completing the square
solving quadratic equations by, 527
in standard equation of circle, 577
Component form of vector, 174
Composite solids, volumes, 630, 638, 644, 651
Composition of rigid motions, 239-240
Composition of transformations, 176
Composition Theorem (Thm. 4.1), 176
Compositions
performing, 176
performing with rotations, 192
Compound event(s), 694-695
Compound inequalities, writing, 299
Concave polygons, 30
Concentric circles, 531
Concept Summary
Interpreting a Diagram, 51
Segments, Lines, Rays, and Points in Triangles, 323
Triangle Congruence Theorems, 273
Triangle Similarity Theorems, 439
Types of Proofs, Symmetric
Property of Angle Congruence, 110
Ways to Prove a Quadrilateral Is a Parallelogram, 379
Writing a Two-Column Proof, 102
Conclusion, in conditional statement, 66
Concurrent lines, rays, or segments, 310
Conditional probability
comparing, 687
defined, 677
finding a table, 679
finding with conditional relative frequencies, 686
Conditional relative frequency, 685-686
Conditional statement(s), 65-70, 116
biconditional statements, 69
defined, 66
in if-then form, 66
negation, 66
related conditionals, 67
true or false determination, 65
truth tables, 70
using definitions, 68
writing, 66-67
Cones
frustum of, 646
lateral surface of, 642
surface area of, 641-642
volume of, 641, 643-644, 659-660
Congruence and transformations, 199-203, 223
congruence transformations, 201
identifying congruent figures, 200
reflections in intersecting lines, 199, 203
reflections in parallel lines, 199, 202
using theorems about, 202-203
Congruence, properties of, 101-102
Congruence transformation, 201
Congruent angles, 40
Congruent arcs, identifying, 540
Congruent Central Angles Theorem (Thm. 10.4), 540
Congruent circles, 540
Congruent Circles Theorem (Thm. 10.3), 540
Congruent Complements Theorem (Thm. 2.5), 107
Congruent Corresponding Chords Theorem (Thm. 10.6), 546
Congruent figures
defined, 200
using properties of, 241
Congruent Parts of Parallel Lines Corollary, 374
Congruent polygons, 239-242, 290
using corresponding parts, 240-241
using Third Angles Theorem (Thm. 5.4), 242
Congruent segments, 13
Congruent Supplements Theorem (Thm. 2.4), 107
Congruent triangles, 228
angles of triangles, 231-235, 290
congruent polygons, 239-242, 290
coordinate proofs, 283-286, 294
equilateral and isosceles triangles, 251-255, 291
proving triangle congruence
by ASA and AAS, 269-273, 292-293
by SAS, 245-248, 291
by SSS, 261-265, 292
using, 277-280, 293

## Conjecture

defined, 76
making and testing, 77
reasoning with, 75
writing, about isosceles triangles, 251
writing, on angles of triangle, 231
Consecutive integers, 77
Consecutive interior angles, 128
Consecutive Interior Angles Converse (Thm. 3.8), 139
Consecutive Interior Angles Theorem
(Thm. 3.4), 132
exploring converses, 137
Consecutive vertices, 360
Constant of proportionality, 597
Construction(s)
bisecting a segment, 21
bisecting an angle, 42
centroid of triangle, 320
circumscribing circle about triangle, 312
copying a segment, 13
copying a triangle
using ASA, 272
using SAS, 248
using SSS, 264
copying an angle, 40
defined, 13
of a dilation, 210
of equilateral triangle, 254
inscribing circle within triangle, 314
parallel lines, 139
perpendicular bisector, 149
perpendicular line, 149
point along directed line segment, 447
proving, 280
square inscribed in circle, 557
tangent to circle, 533
Contingency table, 684
Contradiction, Proof by, 336
Contrapositive
defined, of conditional statement, 67 truth table for, 70
Contrapositive of the Triangle Proportionality Theorem, 447

## Converse

defined, of conditional statement, 67
truth table for, 70
Converses of theorems
Alternate Exterior Angles Converse (Thm. 3.7), 139
Alternate Interior Angles Converse (Thm. 3.6), 139

Converse of the Angle Bisector Theorem (Thm. 6.4), 304
Converse of the Base Angles Theorem (Thm. 5.7), 252
Converse of the Hinge Theorem (Thm. 6.13), 344
Converse of the Perpendicular Bisector Theorem (Thm. 6.2), 302
Converse of the Pythagorean Theorem (Thm. 9.2), 466
Converse of the Triangle Proportionality Theorem (Thm. 8.7), 446
Corresponding Angles Converse (Thm. 3.5), 138
Isosceles Trapezoid Base Angles Converse (Thm. 7.15), 399
Parallelogram Diagonals Converse (Thm. 7.10), 378
Parallelogram Opposite Angles Converse (Thm. 7.8), 376
Parallelogram Opposite Sides Converse (Thm. 7.7), 376
Perpendicular Chord Bisector Converse (Thm. 10.8), 546
Convex polygons, 30
Coordinate (of point), 12
Coordinate plane
circles in, 575-578, 586
classifying triangle in, 233
dilating figures in, 207, 209
midsegments in, 330
parallelograms in, 371, 380, 392
perimeter and area in, 29-33, 57
placing figures in, 284
reflecting figures in, 181
rotating figures in, 189, 191
translating a figure in, 175
trapezoid in, 398
trapezoid midsegment, 400
Coordinate proof(s), 283-286, 294
applying variable coordinates, 285
defined, 284
placing figure in coordinate plane, 284
writing, 283, 284, 286
Coordinate Rule for Dilations, 209
Coordinate Rules for Reflections, 183
Coordinate Rules for Rotations about the Origin, 191
Coplanar circles, 528, 531
Coplanar points, 4
Corollaries
Congruent Parts of Parallel Lines Corollary, 374
Corollary to the Base Angles Theorem (Cor. 5.2), 253

Corollary to the Converse of Base Angles Theorem (Cor. 5.3), 253
Corollary to the Polygon Interior Angles Theorem (Cor. 7.1), 361
Corollary to the Triangle Sum
Theorem (Cor. 5.1), 235
Rectangle Corollary (Cor. 7.3), 388
Rhombus Corollary (Cor. 7.2), 388
Square Corollary (Cor. 7.4), 388
Corollary to a theorem, defined, 235
Corresponding angles
in congruent polygons, 240-241
defined, 128
Corresponding Angles Converse
(Thm. 3.5), 138
constructing parallel lines, 139
Corresponding Angles Theorem
(Thm. 3.1), 132
exploring converses, 137
Corresponding lengths, in similar polygons, 419
Corresponding part(s)
defined, in congruent polygons, 240-241
of similar polygons, 418
Corresponding sides, in congruent polygons, 240-241
Cosine ratio, 493-497, 520-521
of $45^{\circ}$ and $30^{\circ}$ angles, 496
of complementary angles, 494
defined, 494
finding leg lengths, 495
inverse, 502
Counterclockwise rotation, 190
Counterexample, 77
Cross section(s), 619
Cube, 617
Customary units of length, 2
Cylinders, volume, 625-627, 629-630, 658

## D

Deductive reasoning, 75-79, 116
compared to inductive reasoning, 78, 79
defined, 78
using correct logic, 64
using with laws of logic, 78
Defined terms of geometry, 5
Definitions
and biconditional statements, 230
writing as conditional statement, 68

## Degrees

converting between radians and, 597
measure of angle, 39

Density, 628
Dependent events, 675-678, 714
comparing to independent events, 678
defined, 675, 677
determination of, 675
probability of, 677-678
Diagonal of polygon, 360
Diagrams
identifying postulates from, 85
interpreting, 51, 83
sketching and interpreting, 86
using for congruent triangles, 265
Diameter
chord perpendicular to, 545,547
defined, 530
of sphere, 648, 649
Die roll, 668
Dilation(s), 207-211, 224
comparing triangles after, 417
constructing, 210
coordinate rule for, 209
defined, 208
finding perimeter and area after, 416
identifying, 208
negative scale factor, 210
performing, in coordinate plane, 207, 209
scale factor, 208, 415
Directed line segment
constructing point along, 447
defined, 156
Directrix, of parabola, 722
Disjoint events, 694, 715
Distance (between points)
defined, 12
finding minimum distance, 185
using circumference and arc length to find, 596
Distance Formula, 23
using, 229
Distance from a point to a line
defined, 148
finding, 159
Distributive Property, 93
Division Property of Equality, 92
Dodecagon, 363
Dodecahedron, 617
Dynamic geometry software
basic drawings of lines, segments, and rays, 3
calculating sine and cosine ratios, 493
constructing chords, 545
drawing perpendicular bisector, 300
drawing triangles, 245
side lengths and angle measures, 172

## E

Edge (of polyhedron), 617, 618
Endpoints, 5
Enlargement, 208
Equations
of circles, writing and graphing, 576-577
of lines, writing, 123
of perpendicular line, 299
solving with variables on both sides, 229
writing for perpendicular bisectors, 305
Equations of parallel and perpendicular lines, 155-159, 166
distance from point to line, 159
identifying lines, 157
partitioning a directed line segment, 156
writing, 155, 158, 166
Equiangular polygon, 361
Equiangular triangle, classifying, 232
Equidistant (point), 302
Equidistant Chords Theorem (Thm. 10.9), 548
Equilateral polygon, 361
Equilateral triangle
classifying, 232
constructing, 254
using, 254-255
Equivalent statements, 67
Event(s)
compound, 694-695
defined, 668
probability of complement of, 669-670
Experimental probability, 671, 675
Exterior Angle Inequality Theorem, 342
Exterior angle measures of polygons, 362-363
Exterior Angle Theorem (Thm. 5.2), 234
Exterior angles, 233
Exterior of the angle, 38
External segment, 571
External Tangent Congruence Theorem (Thm. 10.2), 532


Faces (of polyhedron), 617, 618
Factorial numbers, $n, 700$
Favorable outcomes, 669
Flawed reasoning, 64
Flow proof, 106

Flowchart proof
concept summary of, 110
defined, 106
matching reasons in, 105
Focus, of parabola, 721-725
defined, 722
Formulas
arc length, 595
circle
area of, 601-602
circumference of, 594
combinations, 702
cone
surface area of, 642
volume of, 643
cylinder, volume, 627
density, 628
Distance Formula, 23
kite area, 610
Midpoint Formula, 22
permutations, 701
population density, 603
prism, volume, 626
pyramid, volume, 636
Pythagorean Theorem (Thm. 9.1), 464
regular polygon, area, 612
rhombus area, 610
sectors, area, 601, 604-605
sphere
surface area of, 648
volume of, 650
spherical cap, volume, 654
Frequency(ies)
probability and two-way tables, 684-686
Fundamental Counting Principle, 700


## Geometric mean

compared to arithmetic mean, 477, 484
defined, 477, 480
using, 480-481
Geometric Mean (Altitude) Theorem (Thm. 9.7), 480
Geometric Mean (Leg) Theorem (Thm. 9.8), 480
Geometric probability, 670
Geometric relationships, proving, 105-110, 118
Glide reflection(s), 184
Golden ratio, 426
Graph theory, 276
Graphing calculator
combinations, 702
permutations, 701
Graphing a circle, 577
Great circle, 648

## H

Heads and tails, 668
Hinge Theorem (Thm. 6.12), 344
converse of, 344
using, 345

## Histograms

analyzing, 707
making, 665
Horizontal component, 174
Horizontal lines, 157
Horizontal stretch, and nonrigid transformation, 211
Hypotenuse (of right triangle), 264
Hypotenuse-Leg (HL) Congruence Theorem (Thm. 5.9), 264-265, 273
Hypothesis, in conditional statement, 66


Icosahedron, 617
If-then form, of conditional statement, 66
Image, 174
Incenter of triangle
defined, 313
examples of segments and points in triangles, 323
inscribing circle within triangle, 314 using, 313
Incenter Theorem (Thm. 6.6), 313
Independent events, 675-678, 714
comparing to dependent events, 678
defined, 675-676
determination of, 675-676
probability of, 676-677
Indirect measurement
of river, 277,279
using geometric mean of right triangle, 481
Indirect proof
defined, 336
used in Triangle Larger Angle
Theorem (Thm. 6.10), 337
writing, 336, 352
Indirect reasoning, 336
Inductive reasoning, 75-79, 116
compared to deductive reasoning, 78, 79
defined, 76
using with conjecture, 76-77
Inferring the truth, 64
Initial point, of vector, 174

Inscribed angle(s)
defined, 553, 554
finding measure of angle, 555
finding measure of intercepted arc, 555
Inscribed Angles of a Circle Theorem (Thm. 10.11), 555
Inscribed polygon(s)
constructing square inscribed in circle, 557
defined, 553, 556
Inscribed Quadrilateral Theorem (Thm. 10.13), 556
Inscribed Right Triangle Theorem (Thm. 10.12), 556
Intercepted arc, 553, 554
Interior angle measures of polygons, 360-362
finding unknown interior angle measure, 361
number of sides of polygon, 361
sum of angle measures, 360
Interior angles, 233
Interior of the angle, 38
Intersecting lines
and circles, 562
example of, 124,125
reflections, 203
Intersection
defined, 6
of events, 694-695
of lines and planes, 3, 6
Inverse
defined, of conditional statement, 67
truth table for, 70
Inverse cosine, 502
Inverse of the Triangle Proportionality
Theorem, 447
Inverse operations, 92
Inverse sine, 502
Inverse tangent, 502
Inverse trigonometric ratios, 502
Isometry, 176
Isomorphic polygons, 276
Isosceles right triangle
side lengths, 471,472
in standard position, 462
Isosceles trapezoid
defined, 398
using properties of, 399
Isosceles Trapezoid Base Angles Converse (Thm. 7.15), 399
Isosceles Trapezoid Base Angles
Theorem (Thm. 7.14), 399
Isosceles Trapezoid Diagonals
Theorem (Thm. 7.16), 399

Isosceles triangle(s), 251-255, 291,
See also Isosceles right triangle
classifying, 232
median and altitude of, 323
using, 254-255
using Base Angles Theorem, 252-253
writing conjecture about, 251


Joint frequency, 684
Joint relative frequency, 685


Kite(s)
area of, 610
defined, 401
finding angle measures in, 401, 410
Kite Diagonals Theorem (Thm. 7.18), 401
Kite Opposite Angles Theorem
(Thm. 7.19), 401


Lateral surface of cone, 642
Law of Cosines, 507-508, 511-512, 522
defined, 511
solving triangles
with SAS case, 511
with SSS case, 512
Law of Cosines (Thm. 9.10), 511
Law of Detachment, 78
Law of Sines, 507-510, 522
ambiguous case of, 515
areas of triangles, 508
defined, 509
solving triangles
with AAS case, 510
with ASA case, 510
with SSA case, 509
Law of Sines (Thm. 9.9), 509
Law of Syllogism, 78
Laws of Logic, 78
Legs
of isosceles triangle, 252
of right triangle
defined, 264
finding, with sine and cosine ratios, 495
finding, with tangent ratio, 489
of trapezoid, 398
Likelihoods, and probabilities, 666, 668
Line(s)
in coordinate plane, characteristics of, 124
intersecting in circles, 562
intersections with planes, 3
Line Intersection Postulate
(Post. 2.3), 84
Line-Point Postulate (Post. 2.2), 84
Plane Intersection Postulate
(Post. 2.7), 84
Plane-Line Postulate (Post. 2.6), 84 that intersect circles, 529-533, 582
Two Point Postulate (Post. 2.1), 84
undefined term, and naming, 4
writing equations of, 123
Line Intersection Postulate
(Post. 2.3), 84
Line of reflection, 182
Line of symmetry, 185
Line perpendicular to plane, 86
Line-Point Postulate (Post. 2.2), 84
Line segment(s), See also Segment(s) defined, 5 directed, partitioning, 156
Line symmetry, 185
Linear pair (of angles), 50
Linear Pair Perpendicular Theorem (Thm. 3.10), 150
Linear Pair Postulate (Post. 2.8), 108, 133
Lines, pairs of
classifying, 124, 125
identifying parallel and perpendicular lines, 126-127
Lines Perpendicular to a Transversal Theorem (Thm. 3.12), 150
Literal equations, rewriting, 63
Logic, deductive reasoning and flawed reasoning, 64
Logically equivalent statements, 70


Major arc, 538
Making Sense of Problems
circumcenter on right triangle, 312
inductive reasoning and deductive reasoning, 79
Marginal frequency, 684
Marginal relative frequency, 685
Measure of a major arc, 538
Measure of a minor arc, 538
Measure of an angle, 39
Measure of an Inscribed Angle Theorem (Thm. 10.10), 554
Measurement, indirect, 277, 279
Median of trapezoid, 400
Median of triangle, 319-321, 323, 351
defined, 320
examples of segments and points in triangles, 300, 323
Metric units of length, 2

Midpoint(s), 19-23, 57
defined, 20
Distance Formula, 23
of line segment, finding, 19
Midpoint Formula, 22
and segment bisectors, 20-21
Midpoint Formula, 22
using, 229
Midsegment of a trapezoid, 400
Midsegment of a triangle, 329-332, 351
defined, 330
examples of segments in triangles, 300
using in coordinate plane, 330
using Triangle Midsegment Theorem (Thm. 6.8), 331-332
Midsegment triangle, 330
Minor are, 537, 538
Modeling with Mathematics, Throughout. See for example:
basics of geometry
area of shed floor, 33
planes in sulfur hexafluoride, 7
circles, northern lights, 565
congruent triangles, angle measures, 235
probabilities and likelihoods, 666
reasoning and proofs, city street, 95
right triangles and trigonometry
angle of elevation and height of tree, 490
equilateral triangle road sign, 474
similarity, swimming pool, 420
three-dimensional figures, rectangular chest, 629
transformations, golf website, 177
triangles, neighborhood distances, 332
Multiplication Property of Equality, 92

Mutually exclusive events, 694

n factorial, 700
n-gon, 30
Negation, of conditional statement, 66
Negative scale factor, 210
Nets for three-dimensional solids, 592
Nonrigid transformation, 211
$\boldsymbol{n}$ th term, of arithmetic sequence, 63


Obtuse angle
defined, 39
trigonometric ratios for, 508

## Obtuse triangle

in circumscribed circle, 311
classifying by angles, 232
classifying by Pythagorean inequalities, 467
orthocenter of, 322
Octahedron, 617
Opposite of statement, See Negation, of Conditional Statement
Opposite rays, 5
Opposite Sides Parallel and Congruent Theorem (Thm. 7.9), 378
"or" (union), 694-695
Orthocenter of triangle
defined, 321
examples of segments and points in triangles, 323
finding, 322
type of triangle, and location, 322
Outcomes
defined, 668
favorable, 669
Overlapping events
defined, 694
finding probability of, 695, 715


Pairs of angles, See Angle(s), pairs of
Pairs of lines, See Lines, pairs of
Parabola(s)
directrix, 722-724
Distance Formula to write equation of, 722
equation of translation of, 725
focus of, 721-725
latus rectum, 728
satellite dishes and spotlights, 721
standard equations of, 723-724
Parabolic reflectors, 725
Paragraph proof
concept summary of, 110
defined, 108
Parallel and perpendicular lines, 122
equations of, 155-159, 166
identifying, 357
pairs of lines and angles, 125-128, 164
parallel lines and transversals, 131-134, 164
proofs with parallel lines, 137-141, 165
proofs with perpendicular lines, 147-151, 165
Parallel lines
constructing, 139
defined, and identifying, 126-127
example of, 124,125
identifying, slopes of, 157
proofs with, 137-141, 165
constructing parallel lines, 139
Corresponding Angles Converse (Thm. 3.5), 138
proving Alternate Interior Angles Converse, 140
Transitive Property of Parallel Lines (Thm. 3.9), 141
properties of, 132-134
Alternate Exterior Angles
Theorem (Thm. 3.3), 132
Alternate Interior Angles Theorem (Thm. 3.2), 132
Consecutive Interior Angles Theorem (Thm. 3.4), 132
Corresponding Angles Theorem (Thm. 3.1), 132
proportionality with three lines, 448
proving theorems about, 140
Reflections in Parallel Lines Theorem (Thm. 4.2), 202
and transversals, 131-134, 164
writing equations of, $155,158,166$
Parallel planes, 126
Parallel Postulate (Post. 3.1), 127
Parallelogram(s)
in coordinate plane, 371, 380, 392
defined, 368
diagonal lengths of, 378
identifying and verifying, 376-379
properties of, 367-370, 408
properties of diagonals, 390-391
properties of special parallelograms, 387-392, 409-410
side lengths of, 377
ways to prove quadrilateral is parallelogram, 379
writing two-column proof, 370
Parallelogram Consecutive Angles Theorem (Thm. 7.5), 369
Parallelogram Diagonals Converse (Thm. 7.10), 378
Parallelogram Diagonals Theorem (Thm. 7.6), 369
Parallelogram Opposite Angles Converse (Thm. 7.8), 376
Parallelogram Opposite Angles Theorem (Thm. 7.4), 368
Parallelogram Opposite Sides Converse (Thm. 7.7), 376
Parallelogram Opposite Sides Theorem (Thm. 7.3), 368
Partitioning a directed line segment, 156
Patterns, in dilation, 416

Percent, finding, 665
Performance Tasks
Bicycle Renting Stations, 349
Circular Motion, 581
Comfortable Horse Stalls, 55
Creating the Logo, 289
Induction and the Next Dimension, 115
Judging the Math Fair, 453
The Magic of Optics, 221
Navajo Rugs, 163
A New Dart Board, 713
Scissor Lifts, 407
Triathlon, 517
Water Park Renovation, 655

## Perimeter

in coordinate plane, 29-33, 57
finding, 31
finding after dilation, 416
of similar polygons, 420
of triangle, square, and rectangle, 31
Perimeters of Similar Polygons (Thm. 8.1), 420
Permutation(s), 699-701, 716
counting, 700
defined, 700
finding probability using, 701
formulas, 701
Perpendicular bisector(s)
constructing, 149
drawing, 300
examples of segments and points in triangles, 300, 323
points on, 301
using, 302-303
using chords of circles, 546-547
writing equations for, 305
Perpendicular Bisector Theorem (Thm. 6.1), 302
converse of, 302
Perpendicular Chord Bisector Converse (Thm. 10.8), 546
Perpendicular Chord Bisector Theorem (Thm. 10.7), 546
Perpendicular lines
defined, 68
equation of, 299
example of, 124
identifying, 127
slopes of, 157
proofs with, 147-151, 165
constructing perpendicular lines, 149
distance from point to line, 148
proving theorems about perpendicular lines, 150
writing equations of, $155,158,166$

Perpendicular Postulate (Post. 3.2), 127
Perpendicular Transversal Theorem (Thm. 3.11), 150
Plane(s), See also Parallel planes
intersections with lines, 3
Plane Intersection Postulate (Post. 2.7), 84
Plane-Line Postulate (Post. 2.6), 84
Plane-Point Postulate (Post. 2.5), 84
Three Point Postulate (Post. 2.4), 84
undefined term, and naming, 4
Plane Intersection Postulate (Post. 2.7), 84
Plane-Line Postulate (Post. 2.6), 84
Plane-Point Postulate (Post. 2.5), 84
Platonic solids, 617
Point(s)
Line Intersection Postulate (Post. 2.3), 84
Line-Point Postulate (Post. 2.2), 84
Plane-Line Postulate (Post. 2.6), 84
Plane-Point Postulate (Post. 2.5), 84
Three Point Postulate (Post. 2.4), 84
Two Point Postulate (Post. 2.1), 84
undefined term, and naming, 4
Point of concurrency
defined, 310
examples of segments and points in triangles, 323
Point of tangency, 530
Polar coordinate system, 196
Polygon(s)
angle measures in, 611
angles of, 359-363, 408
exterior angle measures of, 362-363
interior angle measures of, 360-362
area of, 29, 609-613, 657
classifying types of, 30, 31, 361
congruent, 239-242, 290
using corresponding parts, 240-241
using Third Angles Theorem
(Thm. 5.4), 242
convex compared to concave, 30
drawing regular, 37
inscribed, 553, 556-557, 584
similar, 417-422 (See also Similar polygons)
Polygon Exterior Angles Theorem (Thm. 7.2), 362
Polygon Interior Angles Theorem
(Thm. 7.1), 360
Polyhedron, 617, 618
Population density, 603

Postulate, defined, 12
Postulates
Angle Addition Postulate (Post. 1.4), 41
Arc Addition Postulate (Post. 10.1), 539
Line Intersection Postulate (Post. 2.3), 84
Line-Point Postulate (Post. 2.2), 84
Linear Pair Postulate (Post. 2.8), 108
Parallel Postulate (Post. 3.1), 127
Perpendicular Postulate (Post. 3.2), 127
Plane Intersection Postulate (Post. 2.7), 84
Plane-Line Postulate (Post. 2.6), 84
Plane-Point Postulate (Post. 2.5), 84
Protractor Postulate (Post. 1.3), 39
Reflection Postulate (Post. 4.2), 184
Rotation Postulate (Post. 4.3), 192
Ruler Postulate (Post. 1.1), 12
Segment Addition Postulate (Post. 1.2), 14-15
Three Point Postulate (Post. 2.4), 84
Translation Postulate (Post. 4.1), 176
Two Point Postulate (Post. 2.1), 84
Volume Addition Postulate, 633
Postulates and diagrams, 83-86, 117
diagrams, sketching and interpreting, 83, 86
identifying postulates from a diagram, 85
Line Intersection Postulate (Post. 2.3), 84
Line-Point Postulate (Post. 2.2), 84
Plane Intersection Postulate (Post. 2.7), 84
Plane-Line Postulate (Post. 2.6), 84
Plane-Point Postulate (Post. 2.5), 84
Three Point Postulate (Post. 2.4), 84
Two Point Postulate (Post. 2.1), 84
Precision, Attending to
exactly two answers, 669
probabilities, 710
rounding trigonometric ratios and lengths, 488
standard position for right triangle, 462
on statement of problem, 314
use $\pi$ key on calculator, 594
Preimage, 174
Prime notation, 174
Prisms, volume, 625-627, 629-630, 658
Probability, 664
binomial distributions, 707-710, 716
of complements of events, 669-670
conditional (See Conditional probability)
disjoint and overlapping events, 693-696, 715
experimental , 671, 675
frequencies, 684-686
geometric, 670
independent and dependent events, 675-679, 714
permutations and combinations, 699-703, 716
sample spaces, 667-671, 714
theoretical, 668-670
two-way tables, 683-687, 715
Probability distribution(s)
constructing, 708
defined, 708
interpreting, 709
Probability experiment, 668
Probability of an event
defined, 668
and likelihoods, 666, 668
Probability of complement of event, 669-670
Probability of compound events, 694-695
Probability of dependent events, 677-678
Probability of independent events, 676-677
Proof(s), See also Reasoning and proofs
with congruent triangles, that triangles are congruent, 242
constructions, 280
defined, 100
with parallel lines, 137-141, 165
constructing parallel lines, 139
Corresponding Angles Converse (Thm. 3.5), 138
proving Alternate Interior Angles Converse (Thm. 3.6), 140
proving theorems about parallel lines, 134
Transitive Property of Parallel Lines (Thm. 3.9), 141
with perpendicular lines, 147-151, 165
constructing perpendicular lines, 149
distance from point to line, 148
proving theorems about perpendicular lines, 150
proving statements about segments and angles, 99-102, 118
flowchart proof, 106, 110
paragraph proof, 108, 110
two-column proofs, 100, 102, 110
using properties of congruence, 101
types of, 110
writing coordinate proofs involving circles, 578
Proof by Contradiction, 336
Proofs of theorems
Angle-Angle-Side (AAS) Congruence Theorem (Thm. 5.11), 271
Angle-Angle (AA) Similarity Theorem (Thm. 8.3), 428
Angle-Side-Angle (ASA) Congruence Theorem (Thm. 5.10), 270
Base Angles Theorem (Thm. 5.6), 252
Circumcenter Theorem (Thm. 6.5), 310
Converse of the Hinge Theorem (Thm. 6.13), 345
Kite Diagonals Theorem (Thm. 7.18), 401
Parallelogram Diagonals Theorem (Thm. 7.6), 370
Parallelogram Opposite Sides Converse (Thm. 7.7), 376
Parallelogram Opposite Sides Theorem (Thm. 7.3), 368
Perpendicular Bisector Theorem (Thm. 6.1), 302
Perpendicular Transversal Theorem (Thm. 3.11), 150
Rhombus Diagonals Theorem (Thm. 7.11), 390
Side-Angle-Side (SAS) Congruence Theorem (Thm. 5.5), 246
Side-Side-Side (SSS) Congruence Theorem (Thm. 5.8), 262
Side-Side-Side (SSS) Similarity Theorem (Thm. 8.4), 437
Similar Circles Theorem (Thm. 10.5), 541
Slopes of Parallel Lines (Thm. 3.13), 439
Slopes of Perpendicular Lines (Thm. 3.14), 440
Symmetric Property of Angle Congruence, 102, 110
Symmetric Property of Segment Congruence, 101
Triangle Larger Angle Theorem (Thm. 6.10), 337

Triangle Midsegment Theorem (Thm. 6.8), 331
Triangle Sum Theorem (Thm. 5.1), 234

## Properties

Addition Property of Equality, 92
Algebraic Properties of Equality, 92
of congruence, 101-102
Distributive Property, 93
Division Property of Equality, 92
Multiplication Property of Equality, 92
of parallel lines, 132-134
Reflexive Property, 94
Substitution Property of Equality, 92
Subtraction Property of Equality, 92
Symmetric Property, 94
Transitive Property, 94
Properties of Angle Congruence (Thm. 2.2), 101
Properties of Segment Congruence
(Thm. 2.1), 101
Properties of Triangle Congruence (Thm. 5.3), 241
Proportionality, 445-449, 456
finding relationships, 445
ratios forming, 415
of three parallel lines, 448
with triangle angle bisector, 449
of triangles, 446-447
Proportions, solving
Protractor Postulate (Post. 1.3), 39
Pyramids
frustum of, 640
net for, 592
volumes of, 635-638, 659
Pythagorean Inequalities Theorem
(Thm. 9.3), 467
Pythagorean Theorem (Thm. 9.1), 463-467, 518
classifying triangles as acute or obtuse, 467
in Distance Formula, 23
proving without words, 463
using, 464-465
using converse of, 466
Pythagorean Theorem (Thm. 9.1), 464
Pythagorean triple, 464


Quadratic equations, solving by
completing the square, 527
Quadrilateral
area of, 29
classifications of, 358, 389, 402
identifying special, 402
with inscribed angles, 553
Quadrilaterals and other polygons, 356
angles of polygons, 359-363, 408
properties of parallelograms, 367-371, 408
properties of special parallelograms, 387-392, 409-410
properties of trapezoids and kites, 397-402, 410
proving quadrilateral is a parallelogram, 375-380, 409

## R

Radians, measuring angles in, 597
Radicals, using properties of, 461
Radius
of arc, 40
of circle
defined, 530
finding with congruent chords, 548
finding with segments, 572
of regular polygon, 611
of sphere, 648
Random variable, 708
Ratios, forming a proportionality, 415
Ray(s), and naming, 5
Reading
abbreviations: sin, cos, hyp., 494
abbreviations: tan, opp., adj., 488
approximately equal to, 23
biconditionals, 390
bisect, 20
bisector of circle arc, 546
circles, radius and diameter, 530
circum- prefix, 311
compound inequality, 339
contradiction, 336
corresponding lengths, 419
dilation scale factor, 208
inverse tangent, 502
negative reciprocals, 157
parallelogram notation, 201
raked stage, 504
scale factors, 211
statement of proportionality, 418
trapezoid midsegment, 400
triangle altitudes, 322
triangle area formula, 321
triangle classifications, 232
triangle notation, 31
two-way table, 684

## Reading Diagrams

center of circle circumscribed about polygon, 612
congruent angles, 40
congruent segments, 13
rely on marked information, 402
right angle and right triangle, 23
Real-life problems, Throughout. See for example:
basics of geometry
angles in ball-return net, 49
planes in sulfur hexafluoride, 7
circles, graphs of, earthquake and seismograph, 578
circumference and distance traveled, 596
congruent triangles
bench with diagonal support, 263
sign on barn, 248
parallel and perpendicular lines
in neighborhood layout, 151
sunlight angles, 134
probability
adults with pets, 671
diagnostic test for diabetes, 696
reasoning and proofs, percent raise, 93
relationships within triangles
biking, 346
bridge, 303
circumcenter or incenter for
lamppost placement, 314
distance in city, 311
soccer goal, 305
right triangles and trigonometry
angle of depression and skiing on mountain, 497
angle of elevation and height of tree, 490
equilateral triangle road sign, 474
roof height, 479
skyscrapers and support beams, 465
solving right triangles and raked stage, 504
step angle of dinosaurs, 512
similarity
height of flagpole, 430
triangles and shoe rack, 447
three-dimensional figures, rectangular dresser, 629
transformations
finding minimum distance, 185
golf website, 177
scale factor and length of image, 211
Reasoning and proofs, 62
algebraic reasoning, 91-95
conditional statements, 65-70, 116
inductive and deductive reasoning, 75-79, 116
postulates and diagrams, 83-86, 117
proving geometric relationships, 105-110, 118
proving statements about segments and angles, 99-102, 118
Reasoning, visual, of similar triangles, 429

## Rectangle

defined, 388
diagonal lengths in, 391
perimeter and area, 31
Rectangle Corollary (Cor. 7.3), 388
Rectangle Diagonals Theorem (Thm. 7.13), 391
Reduction, 208
Reflection(s), 181-185, 222
coordinate rules for, 183
defined, 182
glide reflections, 184
in horizontal and vertical lines, 182
in line $y=x$ or $y=-x, 183$
performing, 182-183
triangle in coordinate plane, 181
triangle using reflective device, 181
Reflection Postulate (Post. 4.2), 184
Reflections in Intersecting Lines
Theorem (Thm. 4.3), 203
Reflections in Parallel Lines Theorem
(Thm. 4.2), 202
Reflexive Property, 94
triangle congruence, 241
Regular polygon
angle measures in, 611
areas of, 609-610, 612-613, 657
defined, 361
Related conditional statements, 67
Relationships between special parallelograms, 389
Relationships within triangles, 298
bisectors of triangles, 309-341, 350
indirect proof and inequalities in one triangle, 335-339, 352
inequalities in two triangles, 343-346, 352
medians and altitudes of triangles, 319-323, 351
perpendicular and angle bisectors, 301-305, 350
triangle midsegments, 329-332, 351
Relative frequencies, finding,
conditional, 685-686
joint and marginal, 685-686

## Remember

complete the square, 577
convex polygon, 360
distance between points, 148
dodecagon, 363

Fundamental Counting Principle, 700
inverse operations, 92
order of operations, 93
perimeter and area in coordinate plane, 31
perpendicular lines, 126
polygon in coordinate plane, 371
radical in simplest form, 472
slope-intercept form, 158
slope of line, 156
slopes, product of, 183
system of linear equations in two variables, 159
Triangle Inequality Theorem (Thm. 6.11), 467
triangle side lengths, 473
Revolution, solids of, 620
Rhombus
angle measures in, 390
area of, 610
defined, 388
Rhombus Corollary (Cor. 7.2), 388
Rhombus Diagonals Theorem (Thm. 7.11), 390
Rhombus Opposite Angles Theorem
(Thm. 7.12), 390
Right angle, 39
Right Angles Congruence Theorem (Thm. 2.3), 106
Right cone, 642
Right Triangle Similarity Theorem (Thm. 9.6), 478

## Right triangles

in circumscribed circle, 311
classifying, 232
orthocenter of, 322
similar, 477-481, 519
identifying, 478-479
using geometric mean, 480-481
solving, 501-504, 521
using inverse trigonometric ratios, 502
special, side lengths of
$30^{\circ}-60^{\circ}-90^{\circ}$ triangle, 471,473
isosceles ( $45^{\circ}-45^{\circ}-90^{\circ}$ ), 471, 472
standard position for, 462
Right triangles and trigonometry, 460
cosine ratio, 493-497, 520-521
Law of Cosines, 507-508, 511-512, 522

Law of Sines, 507-510, 522
Pythagorean Theorem (Thm. 9.1), 463-467, 518
similar right triangles, 477-481, 519
sine ratio, 493-497, 520-521
solving right triangles, 501-504, 521
special right triangles, 471-474, 518
tangent ratio, 487-490, 520
Rigid motion
defined, 176
using in congruent polygons, 239-240
Rotation(s), 189-193, 223
in coordinate plane, 189, 191
coordinate rules for rotations about the origin, 191
defined, 190
direction, clockwise or counterclockwise, 190
performing, 190-191
performing compositions with, 192
Rotation Postulate (Post. 4.3), 192
Rotational symmetry, 193
Ruler Postulate (Post. 1.1), 12
Rules, proved and unproved, 12


Same-Side Interior Angles Theorem, See Consecutive Interior Angles Theorem (Thm. 3.4)
Sample space, 667-671, 714
defined, 668
finding, 667-668
SAS, See Side-Angle-Side (SAS)

## Scale factor

defined, 208
of dilation, 415
negative, 210
of similar solids, 630
units in, and finding, 211
Scalene triangle, classifying, 232
Secant, 530
Secant segment, 571-572
Sector of circle
area of, 601, 604-605, 656
defined, 604
Segment(s)
construction, bisecting a segment, 21
defined, and naming, 5
finding length of, 19, 20
finding midpoint of, 19
length in proportional triangles, 446
measuring and constructing, 11-15, 56
congruent segments, 13
Ruler Postulate (Post. 1.1), 12
Segment Addition Postulate (Post. 1.2), 14-15
partitioning a directed line segment, 156
proof of Symmetric Property of
Segment Congruence, 101
Properties of Segment Congruence (Thm. 2.1), 101
relationships in circles, 569-572, 585
chords, secants, and tangents, 570-572
that intersect circles, 529-533, 582

## Segment Addition Postulate

(Post. 1.2), 14-15
Segment bisector(s), 20
construction bisecting a segment, 21
of perpendicular bisector, 149
defined, 20
and midpoints, 20-21
Segments of a chord, 570
Segments of Chords Theorem (Thm. 10.18), 570
Segments of Secants and Tangents Theorem (Thm. 10.20), 572
Segments of Secants Theorem (Thm. 10.19), 571
Semicircle, 538
Side-Angle-Side (SAS)
congruence, 245-248, 273
construction, copying a triangle using SAS, 248
and properties of shapes, 247
using Law of Cosines to solve triangle, 511
Side-Angle-Side (SAS) Congruence Theorem (Thm. 5.5), 246
Side-Angle-Side (SAS) Similarity Theorem (Thm. 8.5), 438
triangle similarity theorems compared, 439
Side-Side-Angle (SSA), 264
special case for right triangles, 264-265
using Law of Sines to solve triangle, 509
Side-Side-Side (SSS)
congruence, 261-265, 273
construction, copying a triangle using SSS, 264
using, 262-264
using Law of Cosines to solve triangle, 512
Side-Side-Side (SSS) Congruence Theorem (Thm. 5.8), 262
Side-Side-Side (SSS) Similarity Theorem (Thm. 8.4), 436
proof of, 437
triangle similarity theorems compared, 439
using, 436-437

## Sides

classifying triangles by, 232-233
defined, of angle, 38
finding side lengths in special right triangles, 471-474, 518
lengths of, 338, 339
of polygons, 30
relating to angles of triangle, 335, 337-338
using side similarity to prove triangle similarity, 435-438, 455
Similar arcs, 541
Similar Circles Theorem (Thm. 10.5), 541
Similar figures
defined, 216
identifying, 171
proving similarity, 218
right triangles (See Triangle similarity, right triangles)
triangles (See Triangle similarity)
Similar polygons, 417-422, 454
areas of, 421
comparing triangles after dilation, 417
corresponding lengths, 419
corresponding parts of, 418
determining if polygons are similar, 422
perimeters of, 420
Similar solids
defined, 630
finding volume of, 630, 638, 644
Similarity, 414
proportionality theorems, 445-449, 456
proving slope criteria using similar triangles, 439-440
proving triangle similarity
by AA, 427-430, 454
by SAS, 438, 455
by SSS, 435-437, 455
similar polygons, 417-422, 454
and transformations, 215-218, 224, 418
and dilations, 215
and rigid motions, 215
Similarity statements, 418
Similarity transformations, 216-217
Sine ratio, 493-497, 520-521
of $45^{\circ}$ and $30^{\circ}$ angles, 496
of complementary angles, 494
defined, 494
finding leg lengths, 495
inverse, 502

## Sketching

diagram, 86
intersections of lines and planes, 6
solids of revolution, 620
Skew lines, 126
Slant height (of right cone), 642
Slope-intercept form, 158
Slope of line
defined, 156
finding, 123
proving criteria using similar triangles, 439-440
Slopes of Parallel Lines (Thm. 3.13), 157-158
proof of, 439
Slopes of Perpendicular Lines (Thm. 3.14), 157-158
proof of, 440
Solids, See Three-dimensional figures
Solids of revolution, 620
Solve a right triangle, 503
Spheres
diameter of, 648, 649
surface area of, 647-649, 660
volumes of, 650-651, 660
Spherical cap, 654
Spherical geometry, 88, 136, 154, 258, $268,308,348,426$

## Square

defined, 388
perimeter and area, 31
Square Corollary (Cor. 7.4), 388
SSA, See Side-Side-Angle (SSA)
SSS, See Side-Side-Side (SSS)
Standard equation of a circle, 576-577
Standard position for right triangle, 462
Straight angle, 39
Straightedge, 13
Structure, Throughout. See for example:
in corresponding parts of similar polygons, 418
in dilation, 416
to solve multi-step equation, 357
Study Skills
Analyze Your Errors: Type of Error, and Corrective Action, 145
Form a Weekly Study Group, Set Up Rules, 485
Keep Your Mind Focused during Class, 385
Keeping a Positive Attitude, 197
Keeping Your Mind Focused, 27
Keeping Your Mind Focused While Completing Homework, 551

Kinesthetic Learners, 623
Making a Mental Cheat Sheet, 691
Rework Your Notes, 327
Take Control of Your Class Time, 433
Use Your Preferred Learning Modality Visual, 259
Using the Features of Your Textbook to Prepare for Quizzes and Tests, 89
Substitution Property of Equality, 92
Subtend, 554
Subtraction Property of Equality, 92
Success of trial, 671
Supplementary angles
defined, 48-49
proving cases, 107
Surface area
of cones, 641-642
of prism, 591
of spheres, 647-649, 660

## Syllogism

example of, 64
Law of Syllogism, 78
Symmetric Property, 94
proof of angle congruence, 102, 110
proof of segment congruence, 101
triangle congruence, 241
Symmetry, rotational, 193


## Tangent(s)

constructing to a circle, 533
defined, 530
finding radius of circle, 532
using properties of, 532-533
Tangent and Intersected Chord Theorem (Thm. 10.14), 562
Tangent circles
defined, 528, 531
drawing and identifying common tangents, 531
Tangent Line to Circle Theorem
(Thm. 10.1), 532
Tangent ratio, 487-490, 520
calculating, 487
defined, 488
finding, 488-489
inverse, 502
Tangent segment, 571-572
Terminal point, of vector, 174
Tetrahedron, 617
Theorem, 101
Theorems. Definition page, see main entry for additional pages.
$30^{\circ}-60^{\circ}-90^{\circ}$ Triangle Theorem
(Thm. 9.5), 473
$45^{\circ}-45^{\circ}-90^{\circ}$ Triangle Theorem (Thm. 9.4), 472
Alternate Exterior Angles Converse (Thm. 3.7), 139
Alternate Exterior Angles Theorem (Thm. 3.3), 132
Alternate Interior Angles Converse (Thm. 3.6), 139
Alternate Interior Angles Theorem (Thm. 3.2), 132
Angle-Angle-Side (AAS) Congruence Theorem (Thm. 5.11), 271
Angle-Angle (AA) Similarity Theorem (Thm. 8.3), 428
Angle Bisector Theorem (Thm. 6.3), 304
Angle-Side-Angle (ASA) Congruence Theorem (Thm. 5.10), 270
Angles Inside the Circle Theorem (Thm. 10.15), 563
Angles Outside the Circle Theorem (Thm. 10.16), 563
Areas of Similar Polygons (Thm. 8.2), 421
Base Angles Theorem (Thm. 5.6), 252
Centroid Theorem (Thm. 6.7), 320
Ceva's Theorem, 452
Circumcenter Theorem (Thm. 6.5), 310
Circumscribed Angle Theorem (Thm. 10.17), 564
Composition Theorem (Thm. 4.1), 176
Congruent Central Angles Theorem (Thm. 10.4), 540
Congruent Circles Theorem (Thm. 10.3), 540
Congruent Complements Theorem (Thm. 2.5), 107
Congruent Corresponding Chords Theorem (Thm. 10.6), 546
Congruent Supplements Theorem (Thm. 2.4), 107
Consecutive Interior Angles Converse (Thm. 3.8), 139
Consecutive Interior Angles Theorem (Thm. 3.4), 132
Contrapositive of the Triangle Proportionality Theorem, 447
Converse of the Angle Bisector Theorem (Thm. 6.4), 304
Converse of the Base Angles Theorem (Thm. 5.7), 252
Converse of the Hinge Theorem (Thm. 6.13), 344

Converse of the Perpendicular Bisector Theorem (Thm. 6.2), 302
Converse of the Pythagorean Theorem (Thm. 9.2), 466
Converse of the Triangle Proportionality Theorem (Thm. 8.7), 446
Corresponding Angles Converse (Thm. 3.5), 138
Corresponding Angles Theorem (Thm. 3.1), 132
Equidistant Chords Theorem (Thm. 10.9), 548
Exterior Angle Inequality Theorem, 342
Exterior Angle Theorem (Thm. 5.2), 234
External Tangent Congruence Theorem (Thm. 10.2), 532
Geometric Mean (Altitude) Theorem (Thm. 9.7), 480
Geometric Mean (Leg) Theorem (Thm. 9.8), 480
Hinge Theorem (Thm. 6.12), 344
Hypotenuse-Leg (HL) Congruence Theorem (Thm. 5.9), 264
Incenter Theorem (Thm. 6.6), 313
Inscribed Angles of a Circle Theorem (Thm. 10.11), 555
Inscribed Quadrilateral Theorem (Thm. 10.13), 556
Inscribed Right Triangle Theorem (Thm. 10.12), 556
Inverse of the Triangle Proportionality Theorem, 447
Isosceles Trapezoid Base Angles Converse (Thm. 7.15), 399
Isosceles Trapezoid Base Angles Theorem (Thm. 7.14), 399
Isosceles Trapezoid Diagonals Theorem (Thm. 7.16), 399
Kite Diagonals Theorem (Thm. 7.18), 401
Kite Opposite Angles Theorem (Thm. 7.19), 401
Law of Cosines (Thm. 9.10), 511
Law of Sines (Thm. 9.9), 509
Linear Pair Perpendicular Theorem (Thm. 3.10), 150
Lines Perpendicular to a Transversal Theorem (Thm. 3.12), 150
Measure of an Inscribed Angle Theorem (Thm. 10.10), 554
Opposite Sides Parallel and Congruent Theorem (Thm. 7.9), 378

Parallelogram Consecutive Angles Theorem (Thm. 7.5), 369
Parallelogram Diagonals Converse (Thm. 7.10), 378
Parallelogram Diagonals Theorem (Thm. 7.6), 369
Parallelogram Opposite Angles Converse (Thm. 7.8), 376
Parallelogram Opposite Angles Theorem (Thm. 7.4), 368
Parallelogram Opposite Sides Converse (Thm. 7.7), 376
Parallelogram Opposite Sides Theorem (Thm. 7.3), 368
Perimeters of Similar Polygons (Thm. 8.1), 420
Perpendicular Bisector Theorem (Thm. 6.1), 302
Perpendicular Chord Bisector Converse (Thm. 10.8), 546
Perpendicular Chord Bisector Theorem (Thm. 10.7), 546
Perpendicular Transversal Theorem (Thm. 3.11), 150
Polygon Exterior Angles Theorem (Thm. 7.2), 362
Polygon Interior Angles Theorem (Thm. 7.1), 360
Properties of Angle Congruence (Thm. 2.2), 101
Properties of Segment Congruence (Thm. 2.1), 101
Properties of Triangle Congruence (Thm. 5.3), 241
Pythagorean Inequalities Theorem (Thm. 9.3), 467
Pythagorean Theorem (Thm. 9.1), 464
Rectangle Diagonals Theorem (Thm. 7.13), 391
Reflections in Intersecting Lines Theorem (Thm. 4.3), 203
Reflections in Parallel Lines Theorem (Thm. 4.2), 202
Rhombus Diagonals Theorem (Thm. 7.11), 390
Rhombus Opposite Angles Theorem (Thm. 7.12), 390
Right Angles Congruence Theorem (Thm. 2.3), 106
Right Triangle Similarity Theorem (Thm. 9.6), 478
Segments of Chords Theorem (Thm. 10.18), 570
Segments of Secants and Tangents Theorem (Thm. 10.20), 572

Segments of Secants Theorem (Thm. 10.19), 571
Side-Angle-Side (SAS) Congruence Theorem (Thm. 5.5), 246
Side-Angle-Side (SAS) Similarity Theorem (Thm. 8.5), 438
Side-Side-Side (SSS) Congruence Theorem (Thm. 5.8), 262
Side-Side-Side (SSS) Similarity Theorem (Thm. 8.4), 436
Similar Circles Theorem (Thm. 10.5), 541
Slopes of Parallel Lines (Thm. 3.13), 157
Slopes of Perpendicular Lines (Thm. 3.14), 157
Tangent and Intersected Chord Theorem (Thm. 10.14), 562
Tangent Line to Circle Theorem (Thm. 10.1), 532
Third Angles Theorem (Thm. 5.4), 242
Three Parallel Lines Theorem (Thm. 8.8), 448
Transitive Property of Parallel Lines (Thm. 3.9), 141
Trapezoid Midsegment Theorem (Thm. 7.17), 400
Triangle Angle Bisector Theorem (Thm. 8.9), 449
Triangle Inequality Theorem (Thm. 6.11), 339
Triangle Larger Angle Theorem (Thm. 6.10), 337
Triangle Longer Side Theorem (Thm. 6.9), 337
Triangle Midsegment Theorem (Thm. 6.8), 331
Triangle Proportionality Theorem (Thm. 8.6), 446
Triangle Sum Theorem (Thm. 5.1), 233-234
Vertical Angles Congruence Theorem (Thm. 2.6), 108-110
Theoretical probability, 668-670
defined, 669
finding, 669, 675
Third Angles Theorem (Thm. 5.4), 242
Three-dimensional figures, 617-620, 657-658
classifying solids, 618
cross sections, 619
nets for, 592
Platonic solids, 617
solids of revolution, 620
Three Parallel Lines Theorem
(Thm. 8.8), 448
Three Point Postulate (Post. 2.4), 84
Tools, See Dynamic geometry software
Transformation(s), 170
congruence and, 199-203, 223
defined, 174
dilations, 207-211, 224
identifying, 171
reflections, 181-185, 222
rotations, 189-193, 223
similarity and, 215-218, 224, 418
translations, 173-177, 222
Transitive Property, 94
triangle congruence, 241
Transitive Property of Parallel Lines
(Thm. 3.9), 141
Translation(s), 173-177, 222
defined, 174
of figure in coordinate plane, 175
of figure using vector, 175
performing compositions, 176
performing translations, 174-175
of triangle in coordinate plane, 173
Translation Postulate (Post. 4.1), 176
Transversal(s)
angles formed by, 128
defined, 128
and parallel lines, 131-134, 164
Trapezoid(s), 397-400, 410
in coordinate plane, 398
defined, 398
isosceles, 398-399
making conjecture about, 397
midsegment of, 400
properties of, 398-399
Trapezoid Midsegment Theorem
(Thm. 7.17), 400
Tree diagram, 699
Trials of probability experiment, 671
Triangle(s), See also Right triangle
altitude of, 319, 321-323, 351
angles of, 231-235, 290
angle measures of triangles, 233-235
classifying triangles by sides and angles, 232-233
using angle-angle similarity, 428-430
area of, 1,31
using trigonometric ratios, 508
bisectors of (See Bisectors of triangles)
centroid of, 320-321, 323
circumcenter of, 310-312
classifying by Pythagorean inequalities, 467
classifying by sides and angles, 232-233
comparing measures in, 344-345 congruent (See Congruent triangles) construction, copying a triangle using SAS, 248
examples of segments, lines, rays, and points in, 300, 323
incenter of, 313-314
inequalities
in one triangle, 339
in two triangles, 343-346, 352
median of, 320-321
midsegments, 329-332, 351
perimeter of, 31
proportionality, 446-447, 449
proving congruence
by ASA and AAS, 269-273, 292-293
by SAS, 245-248, 291
by SSS, 261-265, 292
relating sides and angles, 335, 337-338, 352
Triangle Angle Bisector Theorem (Thm. 8.9), 449
Triangle Inequality Theorem (Thm. 6.11), 339
Triangle Larger Angle Theorem (Thm. 6.10), 337
Triangle Longer Side Theorem (Thm. 6.9), 337
Triangle Midsegment Theorem (Thm. 6.8), 331-332
Triangle Proportionality Theorem (Thm. 8.6), 446
contrapositive of, 447
converse of, 446
inverse of, 447
Triangle similarity
deciding if triangles are similar, 435
proving by AA, 427-430, 454
proving by SAS, 438, 455
proving by SSS, 436-437, 455
proving slope criteria using similar triangles, 439-440
right triangles, 477-481, 519
identifying, 478-479
using geometric mean, 480-481
Triangle Sum Theorem (Thm. 5.1), 233-234
Trigonometric ratio(s), See also Cosine ratio; Sine ratio; Tangent ratio
defined, 488
finding areas of triangles, 508
Trigonometry, See Right triangles and trigonometry
Truth table, 70
Truth value of statement, 70
Two-column proof
concept summary of, 102, 110
defined, 100
writing, 100, 102
writing for parallelograms, 370
Two-Point Postulate (Post. 2.1), 84
Two-way frequency table, 684
Two-way table(s), 683-687, 715
defined, 684
making, 684
and Venn diagram, 683

## U

Undefined terms of geometry, 4
Union of events, 694-695
Unit circle trigonometry, 462
Units of measure
converting between customary and metric units of length, 2
nonstandard units, to measure line segments, 11


Vector(s)
defined, 174
translating a figure using, 175
Venn diagram
classifying parallelograms, 389
classifying quadrilaterals, 358
reasoning with, 75
and two-way table, 683

## Vertex

of cone, 642
defined, of angle, 38
in polygons, 30
of polyhedron, defined, 617, 618
Vertex angle (of isosceles triangle), 252
Vertical angles, 50
Vertical Angles Congruence Theorem (Thm. 2.6), 108-110, 133
Vertical component, 174
Vertical lines, 157
Vertical stretch, and nonrigid transformation, 211
Volume(s)
of composite solid, 630, 638, 644, 651
of cones, 641, 643-644, 659-660
of cylinders, 625-627, 629-630, 658
defined, of solid, 626
and density, 628
of prisms, 625-627, 629-630, 658
of pyramids, 635-638, 659
of similar solids, 630, 638, 644
of spheres, 650-651, 660
of spherical cap, 654
Volume Addition Postulate, 633


Wheel of Theodorus, 476
Writing, Throughout. See for example: conjecture on angles of triangle, 231
conjecture on isosceles triangles, 251
a coordinate proof, 283, 284, 286
coordinate proofs involving circles, 578
an indirect proof, 336

