Reference

Properties

Properties of Equality

Addition Property of Equality
If $a = b$, then $a + c = b + c$.

Subtraction Property of Equality
If $a = b$, then $a - c = b - c$.

Multiplication Property of Equality
If $a = b$, then $a \cdot c = b \cdot c$, $c \neq 0$.

Division Property of Equality
If $a = b$, then $\frac{a}{c} = \frac{b}{c}$, $c \neq 0$.

Reflexive Property of Equality
$a = a$

Symmetric Property of Equality
If $a = b$, then $b = a$.

Transitive Property of Equality
If $a = b$ and $b = c$, then $a = c$.

Substitution Property of Equality
If $a = b$, then a can be substituted for b (or b for a) in any equation or expression.

Properties of Segment and Angle Congruence

Reflexive Property of Congruence
For any segment AB, $\overline{AB} \cong \overline{AB}$.

For any angle A, $\angle A \cong \angle A$.

Symmetric Property of Congruence
If $\overline{AB} \cong \overline{CD}$, then $\overline{CD} \cong \overline{AB}$.

If $\angle A \cong \angle B$, then $\angle B \cong \angle A$.

Transitive Property of Congruence
If $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$, then $\overline{AB} \cong \overline{EF}$.

If $\angle A \cong \angle B$ and $\angle B \cong \angle C$, then $\angle A \cong \angle C$.

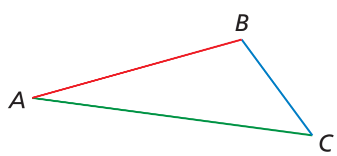
Other Properties

Transitive Property of Parallel Lines
If $p \parallel q$ and $q \parallel r$, then $p \parallel r$.

Distributive Property
Sum
$a(b + c) = ab + ac$
Difference
$a(b - c) = ab - ac$

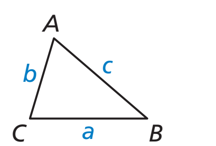
Triangle Inequalities

Triangle Inequality Theorem

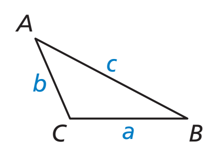


$AB + BC > AC$
$AC + BC > AB$
$AB + AC > BC$

Pythagorean Inequalities Theorem



If $c^2 < a^2 + b^2$, then $\triangle ABC$ is acute.



If $c^2 > a^2 + b^2$, then $\triangle ABC$ is obtuse.

Formulas

Coordinate Geometry

Slope

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$

Slope-intercept form

$y = mx + b$

Point-slope form

$y - y_1 = m(x - x_1)$

Standard form of a linear equation

$Ax + By = C$

Midpoint Formula

$\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$

Standard equation of a circle

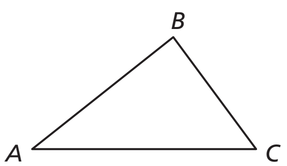
$(x - h)^2 + (y - k)^2 = r^2$, with center (h, k) and radius r

Distance Formula

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

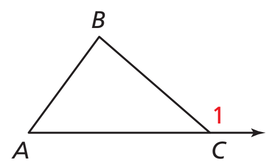
Polygons

Triangle Sum Theorem



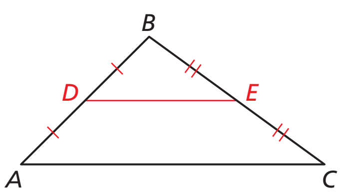
$m\angle A + m\angle B + m\angle C = 180^\circ$

Exterior Angle Theorem



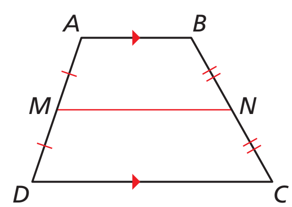
$m\angle 1 = m\angle A + m\angle B$

Triangle Midsegment Theorem



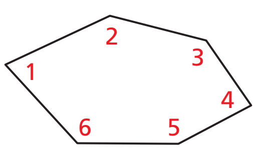
$\overline{DE} \parallel \overline{AC}$, $DE = \frac{1}{2}AC$

Trapezoid Midsegment Theorem



$\overline{MN} \parallel \overline{AB}$, $\overline{MN} \parallel \overline{DC}$, $MN = \frac{1}{2}(AB + CD)$

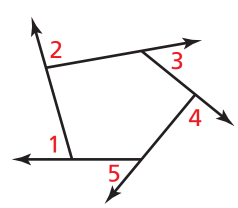
Polygon Interior Angles Theorem



$n = 6$

$m\angle 1 + m\angle 2 + \cdots + m\angle n = (n - 2) \cdot 180^\circ$

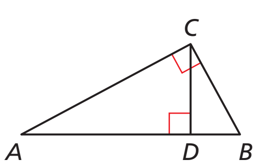
Polygon Exterior Angles Theorem



$n = 5$

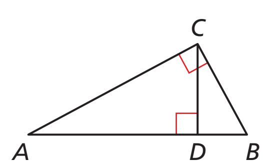
$m\angle 1 + m\angle 2 + \cdots + m\angle n = 360^\circ$

Geometric Mean (Altitude) Theorem



$CD^2 = AD \cdot BD$

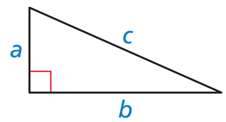
Geometric Mean (Leg) Theorem



$CB^2 = DB \cdot AB$ $\quad AC^2 = AD \cdot AB$

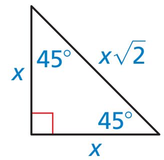
Right Triangles

Pythagorean Theorem



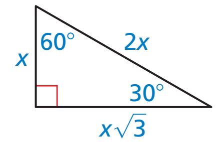
$a^2 + b^2 = c^2$

45°-45°-90° Triangles



hypotenuse $=$ leg $\cdot \sqrt{2}$

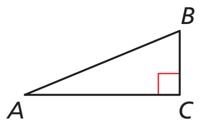
30°-60°-90° Triangles



hypotenuse $=$ shorter leg $\cdot$ 2
longer leg $=$ shorter leg $\cdot \sqrt{3}$

Trigonometry

Ratios



$\sin A = \dfrac{BC}{AB}$ $\quad$ $\cos A = \dfrac{AC}{AB}$ $\quad$ $\tan A = \dfrac{BC}{AC}$

$\sin^{-1}\dfrac{BC}{AB} = m\angle A$ $\quad$ $\cos^{-1}\dfrac{AC}{AB} = m\angle A$ $\quad$ $\tan^{-1}\dfrac{BC}{AC} = m\angle A$

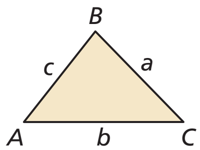
Conversion between degrees and radians
$180° = \pi$ radians

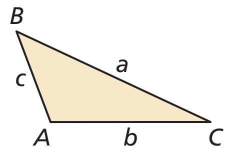
Sine and cosine of complementary angles
Let A and B be complementary angles. Then the following statements are true.

$\sin A = \cos(90° - A) = \cos B$ $\quad$ $\sin B = \cos(90° - B) = \cos A$

$\cos A = \sin(90° - A) = \sin B$ $\quad$ $\cos B = \sin(90° - B) = \sin A$

Any Triangle





Area

$\text{Area} = \dfrac{1}{2}bc \sin A$

$\text{Area} = \dfrac{1}{2}ac \sin B$

$\text{Area} = \dfrac{1}{2}ab \sin C$

Law of Sines

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

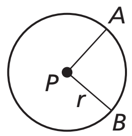
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Law of Cosines

$a^2 = b^2 + c^2 - 2bc \cos A$
$b^2 = a^2 + c^2 - 2ac \cos B$
$c^2 = a^2 + b^2 - 2ab \cos C$

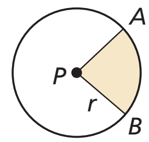
Circles

Arc length



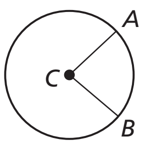
Arc length of $\overset{\frown}{AB} = \dfrac{m\overset{\frown}{AB}}{360^\circ} \cdot 2\pi r$

Area of a sector



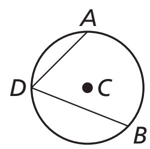
Area of sector $APB = \dfrac{m\overset{\frown}{AB}}{360^\circ} \cdot \pi r^2$

Central angles



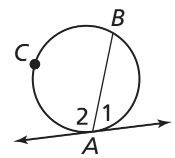
$m\angle ACB = m\overset{\frown}{AB}$

Inscribed angles



$m\angle ADB = \frac{1}{2}m\overset{\frown}{AB}$

Tangent and intersected chord

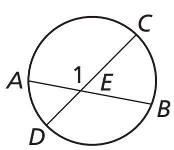


$m\angle 1 = \frac{1}{2}m\overset{\frown}{AB}$

$m\angle 2 = \frac{1}{2}m\overset{\frown}{BCA}$

Angles and Segments of Circles

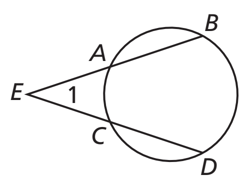
Two chords



$m\angle 1 = \frac{1}{2}\left(m\overset{\frown}{AC} + m\overset{\frown}{DB}\right)$

$EA \cdot EB = EC \cdot ED$

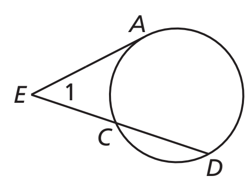
Two secants



$m\angle 1 = \frac{1}{2}\left(m\overset{\frown}{BD} - m\overset{\frown}{AC}\right)$

$EA \cdot EB = EC \cdot ED$

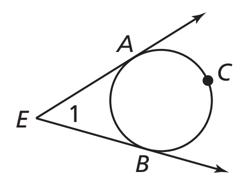
Tangent and secant



$m\angle 1 = \frac{1}{2}\left(m\overset{\frown}{AD} - m\overset{\frown}{AC}\right)$

$EA^2 = EC \cdot ED$

Two tangents



$m\angle 1 = \frac{1}{2}\left(m\overset{\frown}{ACB} - m\overset{\frown}{AB}\right)$

$EA = EB$

Probability and Combinatorics

Theoretical Probability $= \dfrac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}}$

Experimental Probability $= \dfrac{\text{Number of successes}}{\text{Number of trials}}$

Probability of the complement of an event
$P(\overline{A}) = 1 - P(A)$

Probability of independent events
$P(A \text{ and } B) = P(A) \cdot P(B)$

Probability of dependent events
$P(A \text{ and } B) = P(A) \cdot P(B \mid A)$

Probability of compound events
$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

Permutations

$_nP_r = \dfrac{n!}{(n-r)!}$

Combinations

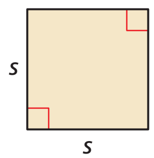
$_nC_r = \dfrac{n!}{(n-r)! \cdot r!}$

Binomial experiments

$P(k \text{ successes}) = {_nC_k}p^k(1-p)^{n-k}$

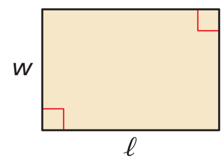
Perimeter, Area, and Volume Formulas

Square



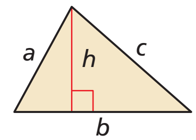
$P = 4s$
$A = s^2$

Rectangle



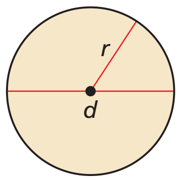
$P = 2\ell + 2w$
$A = \ell w$

Triangle



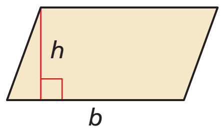
$P = a + b + c$
$A = \frac{1}{2}bh$

Circle



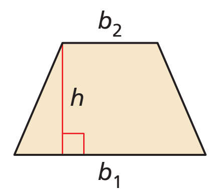
$C = \pi d$ or $C = 2\pi r$
$A = \pi r^2$

Parallelogram



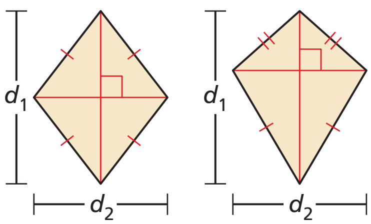
$A = bh$

Trapezoid



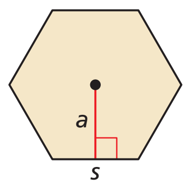
$A = \frac{1}{2}h(b_1 + b_2)$

Rhombus/Kite



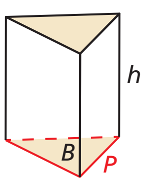
$A = \frac{1}{2}d_1 d_2$

Regular *n*-gon



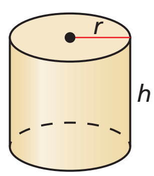
$A = \frac{1}{2}aP$ or $A = \frac{1}{2}a \bullet ns$

Prism



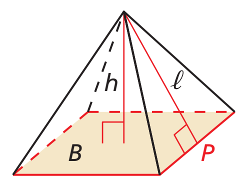
$L = Ph$
$S = 2B + Ph$
$V = Bh$

Cylinder



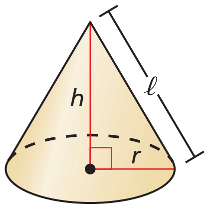
$L = 2\pi rh$
$S = 2\pi r^2 + 2\pi rh$
$V = \pi r^2 h$

Pyramid



$L = \frac{1}{2}P\ell$
$S = B + \frac{1}{2}P\ell$
$V = \frac{1}{3}Bh$

Cone

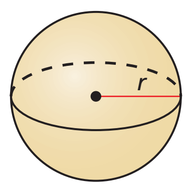


$L = \pi r\ell$

$S = \pi r^2 + \pi r\ell$

$V = \frac{1}{3}\pi r^2 h$

Sphere



$S = 4\pi r^2$

$V = \frac{4}{3}\pi r^3$

Other Formulas

Geometric mean

$x = \sqrt{a \cdot b}$

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a},$$

where $a \neq 0$ and $b^2 - 4ac \geq 0$

Density

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$$

Similar polygons or similar solids with scale factor $a : b$

Ratio of perimeters $= a : b$

Ratio of areas $= a^2 : b^2$

Ratio of volumes $= a^3 : b^3$

Conversions

U.S. Customary

1 foot = 12 inches
1 yard = 3 feet
1 mile = 5280 feet
1 mile = 1760 yards
1 acre = 43,560 square feet
1 cup = 8 fluid ounces
1 pint = 2 cups
1 quart = 2 pints
1 gallon = 4 quarts
1 gallon = 231 cubic inches
1 pound = 16 ounces
1 ton = 2000 pounds

U.S. Customary to Metric

1 inch = 2.54 centimeters
1 foot ≈ 0.3 meter
1 mile ≈ 1.61 kilometers
1 quart ≈ 0.95 liter
1 gallon ≈ 3.79 liters
1 cup ≈ 237 milliliters
1 pound ≈ 0.45 kilogram
1 ounce ≈ 28.3 grams
1 gallon ≈ 3785 cubic centimeters

Time

1 minute = 60 seconds
1 hour = 60 minutes
1 hour = 3600 seconds
1 year = 52 weeks

Temperature

$C = \frac{5}{9}(F - 32)$

$F = \frac{9}{5}C + 32$

Metric

1 centimeter = 10 millimeters
1 meter = 100 centimeters
1 kilometer = 1000 meters
1 liter = 1000 milliliters
1 kiloliter = 1000 liters
1 milliliter = 1 cubic centimeter
1 liter = 1000 cubic centimeters
1 cubic millimeter = 0.001 milliliter
1 gram = 1000 milligrams
1 kilogram = 1000 grams

Metric to U.S. Customary

1 centimeter ≈ 0.39 inch
1 meter ≈ 3.28 feet
1 meter ≈ 39.37 inches
1 kilometer ≈ 0.62 mile
1 liter ≈ 1.06 quarts
1 liter ≈ 0.26 gallon
1 kilogram ≈ 2.2 pounds
1 gram ≈ 0.035 ounce
1 cubic meter ≈ 264 gallons